2074 Chaitra
2069 Chaitra
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2074 Chaitra

S.N. 1) What is a filter? What is the importance in communication? Explain ideal response and response of practical filter.

Solution

Filter is a frequency selective network. A filter is a system or electronic device that attenuates the unwanted signal, supress noise and passes signal of desired frequency range.

In communication filters are used for:

→ Radio and TV tuning to a particular station bandwidth

→ Band limiting of system before sampling.

→ In modem and speech processing.

→ Band pass filter: audio frequency range (20 Hz – 20 kHz)

→ Transmitter and receivers for certain frequency range.

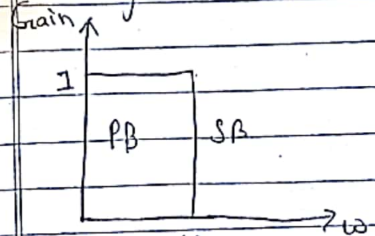

Fig: Ideal response

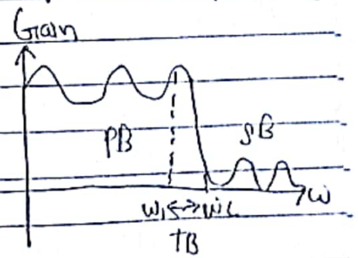

Fig: Response of practical filter

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i) Ideal filter response:

→ Ideal filter posses gain of unity in the pass band. i.e $|T| = 1$

→ Attenuation $(\alpha) = -20\log|T|$
$= 0\,dB$ for pass band

→ Infinite attenuation or gain of zero in the stop band.

→ The frequency point which separates PB and SB is called cut-off point.

→ No transition Band.

ii) Response of practical filter

→ Doesn't pose gain of unity in the pass-band.

→ Posses some attenuation in PB.

→ SB has little gain and finite attenuation.

→ Presence of transition band.

→ Ripple are present in PB and SB.

Q.N.2) Derive expression to calculate the order of Chebyshev low pass filter. Use this formula to find the order of chebyshev low pass filter having following specification.

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a) for pass band extending from $f = 0\,Hz$ to $f = 3.2\,KHz$, the attenuation should not exceed $0.4\,dB$.

b) For stop band extending from $f = 9.8\,KHz$ to $f = \infty$, the attenuation should not be less than $52\,dB$.

Solution

First part (see Q.N.2 of 2070 Chaitra)

Second part

Here,

$\alpha_{max} = 0.4\,dB$, $\omega_p = 2\pi * 3.2\times10^3\,rad/s$

$\alpha_{min} = 52\,dB$, $\omega_s = 2\pi * 9.8 * 10^3\,rad/s$

Order of Chebyshev low pass filter;

$$n = \frac{1}{\cosh^{-1}\left(\frac{\omega_s}{\omega_p}\right)} * \cosh^{-1}\sqrt{\frac{10^{\alpha_{min}/10}-1}{10^{\alpha_{max}/10}-1}}$$

$$= \frac{1}{\cosh^{-1}\left(\frac{9.8}{3.2}\right)} * \cosh^{-1}\sqrt{\frac{10^{52/10}-1}{10^{0.4/10}-1}}$$

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$= 4.39$

$\simeq 5$

$\therefore$ Minimum required order is 5.

Q.N.3) What is an all pass filter? What is its importance? Derive the transfer function of second order constant delay filter.

Solution

All pass filter is a type of filter which passes the signals of all frequency ranges given to it. It doesn't attenuate the input signals.

$H(\omega) = 1$, for all ω

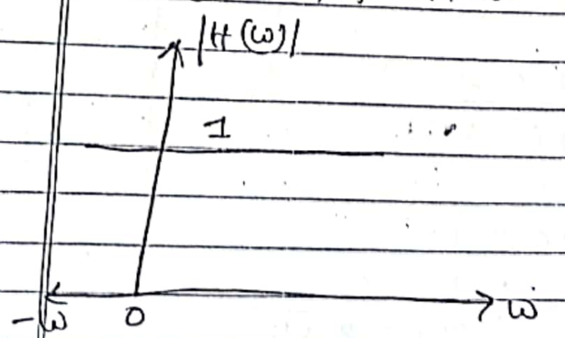

Figure: All pass filter.

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Importance of an All pass filter,

→ Used when we need delays and in case when we need phase equalization.

Second part

Already done in (Q.N.3 of 2073 Shrawan) Using STORCH method.

Q.N.4) What is frequency transformation? How can you convert a low pass filter in to a band stop filter using frequency transformation? Explain with suitable example.

Solution

→ Definition is given in answer of Q.N.4 of 2070 Chaitra & second part is done in Q.N.4 of 2071 Shrawan.

Q.N.5) What are the properties of RC Impedance function? Which of the following is valid RC impedance function? State with reason. Pick a valid RC impedance function and realize it using Foster-I and Cauer-I method.

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Solution

Properties of RC impedance function?

→ The poles and zeros lie on negative real axis.
→ Poles and zeros alternate on negative real axis.
→ Critical frequency near the origin must be pole and Critical frequency near to infinity must be zero.
→ The residue of poles are real and positive.

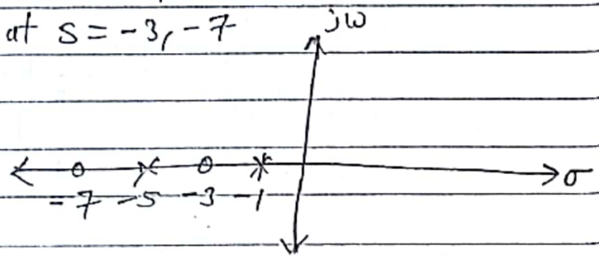
for, $Z(s) = \dfrac{s(s^2+2)}{(s^2+1)}$

Poles and zeros lies on imaginary axis. Not valid.

for $Z(s) = \dfrac{(s+3)(s+7)}{(s+1)(s+5)}$

Poles at $s = -1, -5$

Zeros at $s = -3, -7$



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Here, pole is near to origin & zero is near to infinity. Also, poles and zeros are alternating in negative real axis. Hence, this is valid RC impedance function.

for $Z(s) = \dfrac{(s+1)(s+5)}{(s+3)(s+7)}$

Zero lies near to Origin. Invalid.

for $Z(s) = \dfrac{(s+1)(s+3)}{(s+4)(s+5)}$

Zero lies nearer to origin. Invalid.

Using Foster-I method:

$$Z(s) = \frac{(s+3)(s+7)}{(s+1)(s+5)}$$

$$= \frac{s^2+10s+21}{s^2+6s+5}$$

$$= 1 + \frac{4s+16}{s^2+6s+5}$$

$$= 1 + \frac{4s+16}{(s+1)(s+5)}$$

$$= 1 + \frac{A}{s+1} + \frac{B}{s+5}$$

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Now,

$$A = \lim_{s \to -1} \left(\frac{4s+16}{s+5} \right) = 3$$

$$B = \lim_{s \to -5} \left(\frac{4s+16}{s+1} \right) = 1$$

$$\therefore \; Z(s) = 1 + \frac{3}{(s+1)} + \frac{1}{(s+5)}$$

$$= 1 + \frac{1}{\frac{s}{3} + \frac{1}{3}} + \frac{1}{s + \frac{1}{(1/5)}}$$

Comparing with,

$$Z(s) = R_\infty + \frac{1}{C_1 s + \frac{1}{R_1}} + \frac{1}{C_2 s + \frac{1}{R_2}}$$

We get,

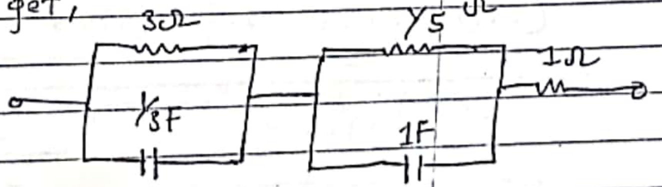

Fig: Foster - I Circuit.

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Using Cauer I method:

$$Z(s) = \frac{(s+3)(s+7)}{(s+1)(s+5)} = \frac{s^2+10s+21}{s^2+6s+5}$$

$$= \frac{s^2\left[1 + 10/s + 21/s^2\right]}{s^2\left[1 + 6/s + 5/s^2\right]}$$

for $S \to \infty$;

$$Z(s) = 1 \; [\text{Constant}]$$

$$
\begin{array}{l}
s^2+6s+5 \,)\, s^2+10s+21 \,(\, 1 \leftarrow R_1 \\
\qquad\qquad\quad -s^2+6s+5 \\
4s+16 \,)\, s^2+6s+5 \,(\, s/4 \leftarrow Y_2(s) \\
\qquad\qquad -s^2+4s \\
2s+5 \,)\, 4s+16 \,(\, 2 \leftarrow R_3 \\
\qquad\qquad -4s+10 \\
6 \,)\, 2s+5 \,(\, s/3 \leftarrow Y_4(s) \\
\qquad -2s \\
5 \,)\, 6 \,(\, 6/5 \leftarrow R_5 \\
\qquad -6 \\
\qquad\; \times
\end{array}
$$

$$Z(s) = R_1 + \frac{1}{Y_2(s) + \frac{1}{R_3 + \frac{1}{Y_4(s) + 1/R_5}}}$$

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i.e,

$$Z(s) = 1 + \cfrac{1}{\left(\frac{s}{4}\right) + \cfrac{1}{2 + \cfrac{1}{\left(\frac{s}{3}\right) + \cfrac{1}{(6/5)}}}}$$

First element is resistor,

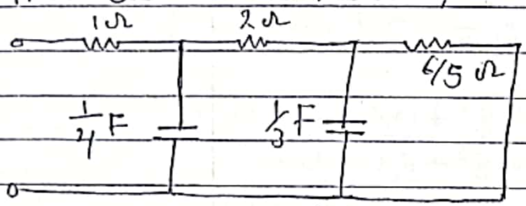

Fig:- Cauer-I network

Q.N.6) Define zeros of transmission. How zeros of transmission can be realized? Explain with suitable example.

Solution

(Explained in Q.N.6 of 2070 Chaitra).

Q.N.7) What information do you get from reflection coefficients? Design a third order Butterworth low pass filter using Resistively terminated lossless ladder with equal termination of 1Ω. (Use table 1)

Solution

Reflection coefficient is the ratio of power reflected back by the two port network to source to the available power at the network. So, reflection coefficient gives the information about how much power is reflected from network or blocked from it.

From the table, third order Butterworth transfer function is;

$$T_3(s) = \frac{1}{(s+1)(s+0.5+j0.86603)(s+0.5-j0.86603)}$$

$$= \frac{1}{(s+1)(s^2+s+0.25+0.75)}$$

$$= \frac{1}{(s+1)(s^2+s+1)} = \frac{1}{s^3+2s^2+2s+1}$$

We know,

$$Q(s) = \frac{1}{T(s)} = (s+1)(s^2+s+1)$$

Also, $\rho(s) = \dfrac{s^n}{Q(s)} = \dfrac{s^3}{s^3+2s^2+2s+1}$

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Now, $Z_{11}(s) = \dfrac{R_1[1 \pm s(s)]}{[1 \mp s(s)]}$

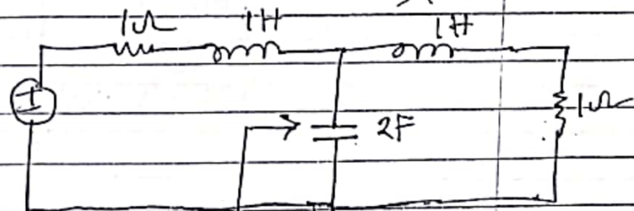
Here, $R_1 = 1\Omega$ (Given),

So, $Z_{11}(s) = \dfrac{1 \pm \left(\dfrac{s^3}{s^3+2s^2+2s+1}\right)}{1 \mp \left(\dfrac{s^3}{s^3+2s^2+2s+1}\right)}$

$= \dfrac{2s^3+2s^2+2s+1}{2s^2+2s+1}$ and $\dfrac{2s^2+2s+1}{2s^3+2s^2+2s+1}$

for $Z_{11}(s) = \dfrac{2s^3+2s^2+2s+1}{2s^2+2s+1}$,

$2s^2+2s+1 \,)\, 2s^3+2s^2+2s+1 \,(\, s$
$-2s^3 \pm 2s^2 \pm s$
$s+1 \,)\, 2s^2+2s+1 \,(\, s$
$-2s^2 \pm 2s$
$1 \,)\, s+1 \,(\, s$
$-s$
$1 \,)\, 1 \,(\, 1$
-1
$\times$



$Z_{11}(s)$

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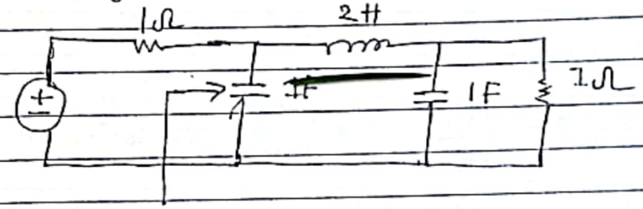
for $Z_{11}(s) = \dfrac{2s^2+2s+1}{2s^3+2s^2+2s+1}$

$Y_{11}(s) = \dfrac{2s^3+2s^2+2s+1}{2s^2+2s+1}$

Similarly,



$Y_{11}(S)$

Q.N.8) Draw the circuit diagram of Tow-Thomas low pass biquad circuit and derive it's transfer function. Design a second order low pass filter using Tow-Thomas biquad poles at $-450 \pm j\,893.03$ and DC gain of 1.5. The final circuit should consist practically realizable elements.

Solution
⇒ First Part is done in Q.N.8 of 2070 Chaitra.

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Second part:

$$\alpha \pm j\beta = -450 \pm j\,893.03$$

So, $\alpha = 450$, $\beta = 893.03$

$$\omega_0 = \sqrt{\alpha^2+\beta^2} = \sqrt{450^2+893.03^2}$$
$$= 1000 \text{ rad/s}$$

Also,

$$2\alpha = \omega_0/Q$$

$$\text{or, } Q = \frac{\omega_0}{2\alpha} = \frac{1000}{2*450} = 10/9$$

DC gain (H) = 1.5

The standard equation for second order low pass filter is;

$$T_{LP}(s) = \frac{-H\omega_0^2}{s^2+\left(\frac{\omega_0}{Q}\right)s+\omega_0^2}$$

& Tow-Thomas biquad eqn is,

$$T_{LP}(s) = \frac{-\left(1/R_3R_4C_1C_2\right)}{s^2+\left(\frac{1}{R_1C_1}\right)s+\left(\frac{1}{R_2R_4C_1C_2}\right)}$$

Now, Comparing two eqn's using tuning algorithm, we choose $\omega_0 = 1$ rad/s, $R_4 = 1\Omega$ and $C_1 = C_2 = 1F$ then,

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$$\omega_0^2 = \frac{1}{R_2R_4C_1C_2}$$

$$\text{or, } R_2 = \frac{1}{\omega_0^2R_4C_1C_2} = 1\Omega$$

$$\frac{\omega_0}{Q} = 1/R_1C_1$$

$$\text{or, } R_1 = Q = 10/9\ \Omega$$

$$H = R_2/R_3$$

$$\text{or, } R_3 = \frac{R_2}{H} = \frac{1}{1.5} = 2/3\ \Omega$$

To achieve $\omega_0 = 1000$ rad/s, do frequency scaling, $K_f = 1000$. To be practically realizable, choose $K_m = 10^4$.

Then,

$$R_1 = 10/9 * 10^4 = 11.11\ k\Omega$$
$$R_2 = R_4 = 1*10^4 = 10\ k\Omega$$
$$R_3 = 2/3 * 10^4 = 6.66\ k\Omega$$
$$C_1 = C_2 = \frac{1}{1000*10^4} = 0.1\ \mu F$$

Take,

$$R_5 = 10\ k\Omega$$

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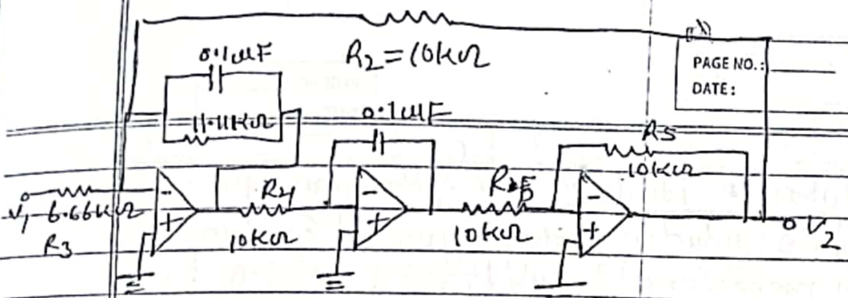
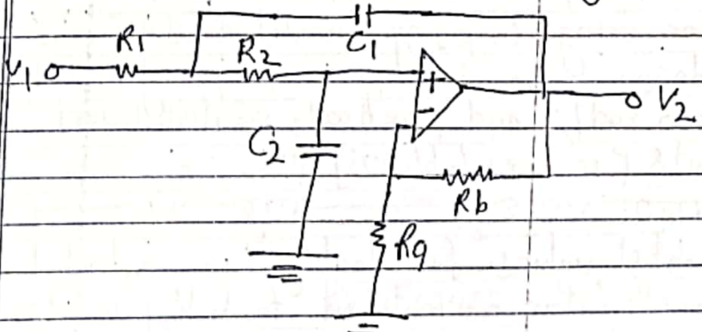


Fig:- Final Tow Thomas Circuit.

Q.N.9) How excess gain can be componcated in Sallon Key filter? Explain.

Solution

The circuit of sallen Key filter is,



To reduce the gain of Sallen Key LPF, replace R_1 by parallel combination of two resistors R_x and R_y.

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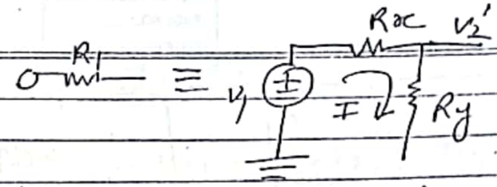

Here, $V_1 = I(R_x+R_y)$

or, $V_1 = \left(\frac{V_2'}{R_y}\right)(R_x+R_y)$

or, $\frac{V_2'}{V_1} = \frac{R_y}{R_x+R_y} = H$ ---- (i)

Also, $R_1 = \frac{R_x \cdot R_y}{R_x+R_y}$ ---- (ii)

Dividing eq (ii) by eq^n (i), gives,

$$\frac{R_1}{H} = \frac{R_x \cdot R_y/(R_x+R_y)}{R_y/(R_x+R_y)} = R_x$$

$$\therefore \boxed{R_x = R_1/H}$$

& from eq^n (i),

$$R_y = H(R_x+R_y)$$

or, $R_y(1-H) = HR_x$

or, $R_y = \frac{R_1}{1-H}$

$$\boxed{\therefore R_y = \frac{R_1}{1-H}}$$

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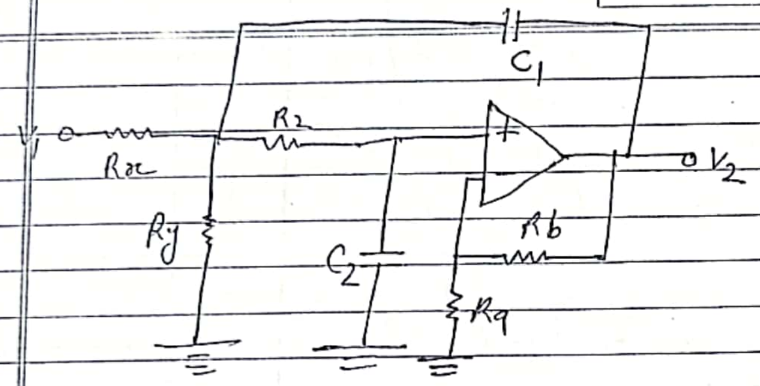

Fig:- Sallen Key Circuit for gain compensation.

Q.N.10) Define Sensitivity. Perform sensitivity analysis of Tow – Thomas Biquad low pass filter.

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Solution

Definition is in Q.N. 10 of 2070 Chaitra.

Second part
See (Q.N.9 of 2071 Chaitra)

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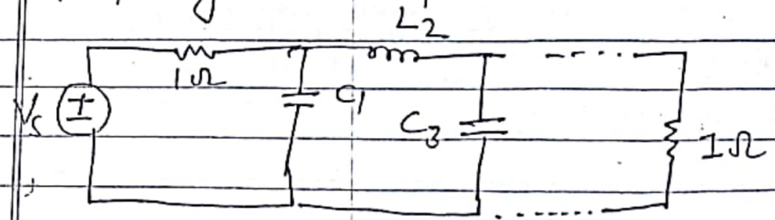
Q.N.11) What is ideal gyrator? How can you simulate inductor using gyrator? Explain with necessary derivation.

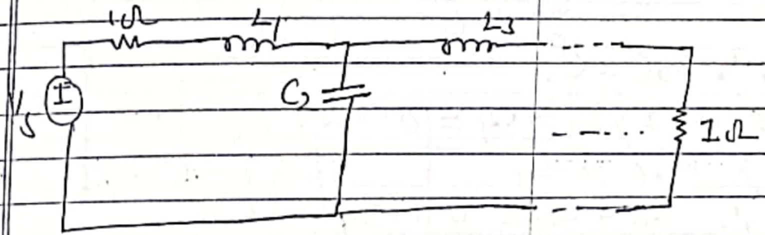
Solution
⇒ Ideal gyrator is the different name for Generalized Impedance Converter (GIC).
Sol ⁿ
See (Q.N.11 first part of 2069 chaitra)

Q.N.12) Design the fourth order Butterworth low pass filter using leapfrog simulation. In your final design the half power frequency should be 1000 rad/s and practically realizable elements [Refer table 2]

Table 2:
Elements values for doubly terminated Butterworth filter normalized to half power frequency of 1 rad/s.

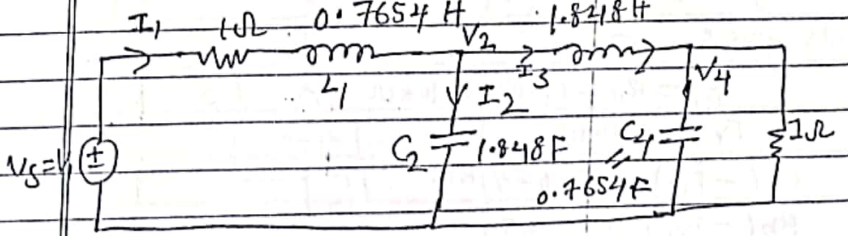


n	C_1	L_2	C_3	L_4	C_5
2	1.414	1.414			
3	1	2	1		
4	0.7654	1.848	1.848	0.7654	
5	0.618	1.618	2	1.618	0.618
n	L_1	C_2	L_3	C_4	L_5



Solution

From table,



Here,

$$I_1 = (V_1 - V_2)\, Y_1$$
$$V_2 = (I_1 - I_3)\, Z_2$$
$$I_3 = (V_2 - V_4)\, Y_3$$
$$V_4 = I_3 \cdot Z_4$$

Now,
$$-V_{I1} = (V_1 - V_2)(-T_{Y_1})$$
$$-V_2 = (-V_{I1} + V_{I3})\, T_{Z2}$$
$$V_{I3} = (-V_2 + V_4)(-T_{Y3})$$
$$V_4 = V_{I3} \cdot T_{Z4}$$

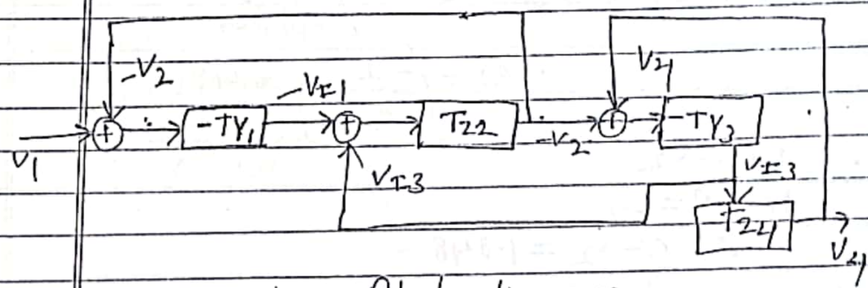


Fig: Block diagram

Design of $-T_{Y1}$:

$$-T_{Y_1} = \frac{-1}{\frac{1}{R_1} + L_1 s} = \frac{-1/L_1}{s + R_1/L_1}$$

$\Rightarrow$ lossy integrator, $T(s) = \dfrac{-1/R_aC}{s + \frac{1}{R_bC}}$

$\therefore R_aC = L_1$

or, $R_a = \dfrac{L_1}{C}$

let $C = 1F$, then,

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$R_a = 0.7654\,\Omega$

And, $R_bC = L_1/R_1$

or, $R_b = L_1 = 0.7654\,\Omega$

Design of T_{Z2}:

$T_{Z2} = \frac{1}{C_2 s}$ ⇒ Integrator + Unit gain amplifier

$$T(s) = \left(\frac{-1}{RCs}\right) * (-1)$$

$\therefore RC = C_2$

let $R = 1$,

$\therefore C = C_2 = 1.848$

Design of $-T_{Y3}$

$-T_{Y3} = \frac{-1}{L_3 s}$ ⇒ Integrator, $T(s) = -\frac{1}{RCs}$

$\therefore RC = L_3$

let $R = 1$,

$\therefore C = L_3 = 1.848$

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Design of T_{Z4}:

$$T_{Z4} = \frac{1}{1 + C_4 s} = \frac{1/C_4}{s + 1/C_4}$$

⇒ lossy integrator + Unity gain amplifier,

$$T(s) = \frac{-1/R_aC}{s + \frac{1}{R_b \cdot C}} * (-1)$$

$\therefore R_aC = C_4$

let $C = 1$; $R_a = C_4 = 0.7654$

& $R_b \cdot C = C_4$

$\therefore R_b = C_4 = 0.7654$

To achieve $\omega_0 = 1000$ rad/s and realize practical elements;

$K_f = 10^4$ & $K_m = 10^4$

This gives,

$R_1 = R_2 = 1 * 10^4 = 10\,k\Omega$

$C(-T_{Y1}) = 10\,nF$

$R_a(-T_{Y1}) = 7.654\,k\Omega$

$R_b(-T_{Y1}) = 7.654\,k\Omega$

$C(T_{Z2}) = 18.48\,nF$

$R(T_{Z2}) = 10\,k\Omega$, $R(-T_{Y3}) = 10\,k\Omega$

$C(-T_{Y3}) = 18.48\,nF$

$C(T_{Z4}) = 10\,nF$

$R_a(T_{Z4}) = 7.654\,k\Omega$

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$R_b\ (T_{24}) = 7.654\ K\Omega$

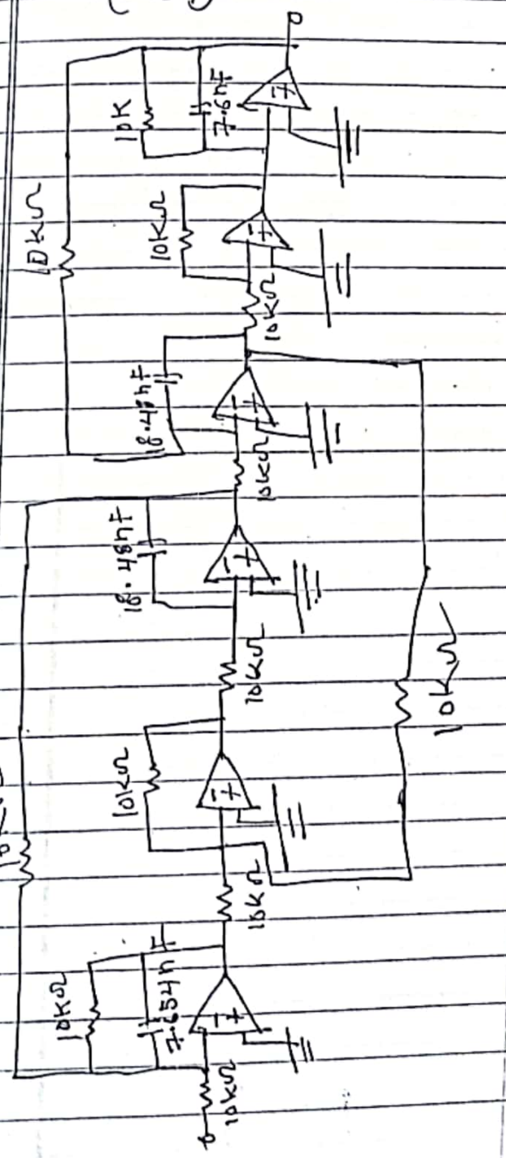

Fig:- Circuit diagram of implementing leapfrog simulation.

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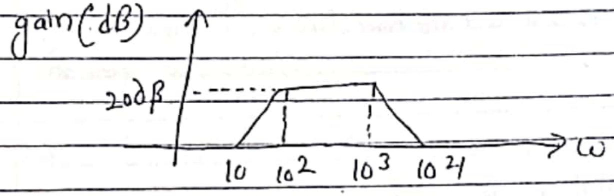
Q.N.B) What are the applications of switched Capacitor filter? Design a switched Capacitor filter for following requirements.



Solution
Already solved in Q.N.13 of 2070 Chaitra.

2073 Shrawan

S.N. 1) What is normalization and De-normalization? A low pass filter has half power frequency of ω_o rad/s. Derive formula to calculate the new value of the resistors, capacitors and inductors present in the low pass filter if you want to change its half power frequency to ω_n rad/s.

Solution

First part (See S.N. 7 (first part) 2071 Shawan).

Second part

To change half power frequency from ω_o rad/s to ω_n rad/s, We do frequency Scaling. The frequency Scaling factor,

$$K_f = \frac{\omega_n}{\omega_o}$$

Now, for resistor, initial impedance, $|Z_R| = R_{old}$

Since, resistance doesn't depend on frequency, after frequency Scaling,

$$|Z_R|_{new} = R_{new} = R_{old}$$

$$\boxed{\therefore R_{new} = R_{old}}$$

For inductor, initial impedance,

$$|Z_L| = \omega_o L_{old}$$

$$= K_f \cdot \omega_o \cdot \frac{L_{old}}{K_f}$$

$$= \omega_n \cdot \frac{L_{old}}{K_f} = \omega_n \cdot L_{new}$$

$$\boxed{L_{new} = \frac{L_{old}}{K_f}}$$

For Capacitor;

$$|Z_C| = \frac{1}{\omega_0 \cdot C_{old}} = \frac{1}{K_f \cdot \omega_0 \cdot C_{old}} \times K_f$$

$$= \frac{1}{K_f \, \omega_0 \left(\frac{C_{old}}{K_f}\right)}$$

$$= \frac{1}{\omega_n \cdot C_{new}}$$

$$\boxed{\therefore C_{new} = \frac{C_{old}}{K_f}}$$

S.N.2) What are the characteristics of Elliptical Response? Compare it with Chebyshev and inverse Chebyshev response.

Solution

The characteristics of Elliptical response are:-

i) Elliptical filter has equiripple in both pass band and stopband ($\alpha_1 = \alpha_2 = \alpha_{min}$).

ii) For the given specification, the order will be less than that of Butterworth and Chebyshev response.

iii) It has sharp cut-off i.e narrow transition bandwidth.

iv) The poles are on ellipse where as the zeros are at imaginary axis.

v) The magnitude-squared response of a low-pass elliptic filter is of the form:

$$|T_n(j\omega)|^2 = \frac{1}{1 + \varepsilon^2 \cdot R_n^2(\omega, L)}$$

where, $R_n^2(\omega, L)$ is called a Chebyshev rational function.

vi) Attenuation (α) = $10 \log\left[1 + \varepsilon^2 \cdot R_n^2(\omega, L)\right]$

At $\omega_p = 1$ rad/sec, $R_n(1) = 1$,

So, $\alpha_{max} = 10 \log\left[1 + \varepsilon^2\right]$

vii) $|T_n(j\omega)|^2$

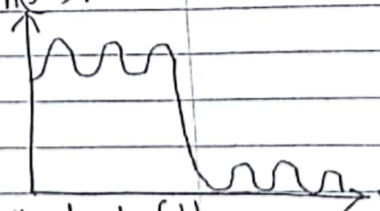


Fig:- Elliptical filter response

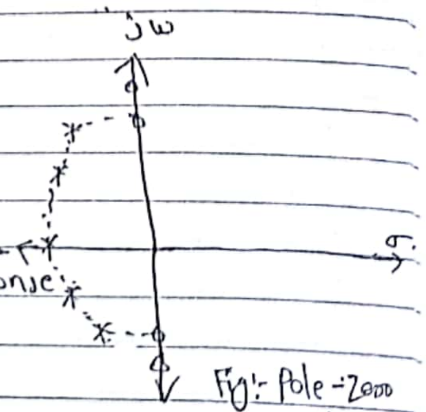


Fig:- Pole-Zero plot

Comparision of Elliptical response and Chebyshev response.

Elliptical response	Chebyshev response
i) It has equal ripple in both pass band and stop band.	i) It has ripple in Pass band only.
ii) For given Specification, Order of filter is less than Butterworth and chebyshev filter.	ii) For given Specification Order of filter is less than Butterworth filter only.
iii) Poles lies on ellipse while zeros lies on imaginary axis.	iii) Poles and zeros lie on ellipse, Centred at Origin.
iv) Transfer function: $\lvert T(s)\rvert^2 = \frac{1}{1+\varepsilon^2 R_n^2(\omega, L)}$	iv) Transfer function: $\lvert T(s)\rvert^2 = \frac{1}{1+\varepsilon^2 C_n^2(\omega)}$

Comparision of Elliptical response and Inverse Chebyshev response.

Elliptical response	Inverse Chebyshev response
i) It has equal ripple in both pass and stop band.	i) Equal ripple in stop-band and maximally flat in pass band.
ii) Order of filter is least among butterworth and Chebyshev.	ii) Order of filter is greater than elliptical filter.
iii) Poles lies on ellipse while zeros lies on imaginary axis.	iii) Poles and zeros lies on ellipse, Centred at origin.
iv) Transfer function: $\lvert T(s)\rvert^2 = \frac{1}{1+\varepsilon^2 R_n^2(\omega, L)}$	iv) Transfer function, $\lvert T(s)\rvert^2 = \frac{\varepsilon^2 C_n^2(1/\omega)}{1+\varepsilon^2 C_n^2(1/\omega)}$

Q.N.3) What is constant dely filter? Obtain the transfer function of second order constant delay filter.

Solution

First part (see. Q.N.3 (first part) 2069 Chaitra)

Second Part

Using STORCH method,

$$T_2(s) = \frac{a_0}{s^2+a_1 s+a_0} = \frac{1}{\cosh s + \sinh s}$$

Also,

$$\coth s = \frac{1}{s} + \cfrac{1}{\frac{3}{s} + \cfrac{1}{\frac{5}{s} + \cfrac{1}{\frac{7}{s} + \cdots}}}$$

For second order, take $n=2$; then take two steps,

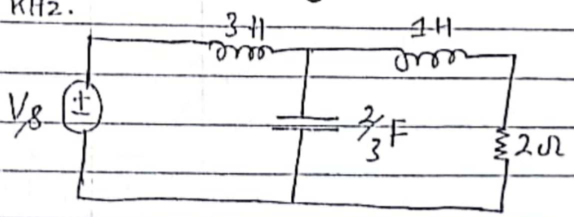
i.e., $\coth s = \frac{1}{s} + \frac{1}{(3/s)} = \frac{s^2+3}{3s} = \frac{\cosh s}{\sinh s}$

$\therefore \cosh s = s^2+3$ & $\sinh s = 3s$

So, $D_2(s) = \cosh s + \sinh s$
$= s^2 + 3s + 3$

$$\boxed{\therefore T_2(s) = \frac{D_2(0)}{D_2(s)} = \frac{3}{s^2+3s+3}}$$

Q.N.4) The following low pass filter has passband frequency ω_p of 1 rad/s. Transform it into a high pass filter having passband frequency of 2 kHz.



Solution

To transform low pass filter to high pass filter, replace 's' by '$\frac{\omega_c}{s}$',

Here, $\omega_c = 2\pi f$
$= 2\pi * 2 * 10^3$
$= 4000\pi$ rad/s

i) Resistor: $Z_R = R$

$\therefore Z_{R,new} = R$

$\therefore R(2) = 2\Omega$.

ii) Inductor: $Z_L = Ls$

$\therefore Z_{L,new} = \frac{L\omega_c}{s}$

$\therefore$ Inductor is replaced by Capacitor.

Hence, $L(3) = C = \frac{1}{L\omega_c} = \frac{1}{3 * 4000\pi} = 26.5\mu F$

$L(1) = C = \frac{1}{L\omega_c} = \frac{1}{1 * 4000\pi} = 79.577\mu F$

iii) Capacitor: $Z_c = \frac{1}{Cs}$

$\therefore Z_{c,new} = \frac{1}{C \cdot \omega_c/s} = \frac{s}{C\omega_c}$

$\therefore$ Capacitor is replaced by inductor,

Hence, $C(\frac{2}{3}) = L = \frac{1}{C\omega_c} = \frac{1}{\frac{2}{3} * 4000\pi}$
$= 119.36\mu H$

∴ The high-pass filter circuit is,

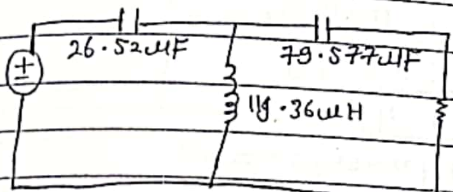


Fig:- High pass filter.

Q.N. 5) Which of the following functions are LC driving point impedance function and why?

$$Z(s) = \frac{s^4+10s^2+9}{s^3+4s} \quad \& \quad Z(s) = \frac{s^3+4s}{s^4+5s^2+6}$$

Also find the Foster parallel and cauer I form of the valid LC driving point impedance function.

Solution

To be a LC impedance function,

$$Z(s) = \frac{\text{even polynomial}}{\text{odd polynomial}}$$

And the succeding power of numerator and denominator must be differ by 2. Also, the highest and lowest power of denominator must be differ by Unity to the highest and lowest power of numerator.

All above conditions are satisfied by,

$$Z(s) = \frac{s^4+10s^2+9}{s^3+4s} \text{ only.}$$

Hence, it is LC driving point impedance function.

Now, Using Foster Parallel method;

$$Z(s) = \frac{s^4+10s^2+9}{s^3+4s}$$

$$
\begin{array}{r|l}
s^3+4s & s^4+10s^2+9 \;(\, s \\
 & -s^4 \pm 4s^2 \\
\hline
 & 6s^2+9
\end{array}
$$

$$\therefore Z(s) = s + \frac{6s^2+9}{s^3+4s} = Z_1(s) + Z_2(s)$$

Now,

$$Z_2(s) = \frac{6s^2+9}{s(s^2+4)} = \frac{A}{s} + \frac{Bs}{s^2+4}$$

$$\therefore A = Z_2(s)\cdot s \Big|_{s=0}$$

$$= \frac{6s^2+9}{s(s^2+4)}\cdot s \Big|_{s=0} = 9/4$$

$$B = \frac{6s^2+9}{s(s^2+4)}\cdot\frac{(s^2+4)}{s}\Big|_{s^2=-4} = \frac{15}{4}$$

$$\therefore Z_2(S) = \frac{9}{4s} + \frac{15}{4}\cdot\frac{s}{s^2+4}$$

$$\therefore Z(S) = s + \frac{9}{4s} + \frac{15s}{4(s^2+4)}$$

$$= s + \cfrac{1}{\frac{4}{9}s} + \cfrac{1}{\left(\frac{4}{15}\right)s + \cfrac{1}{\left(\frac{15}{16}\right)s}}$$

For Foster - II Comparing with,

$$Z(S) = C_1 s + \frac{1}{H_2 s} + \cfrac{1}{C_3 s + \cfrac{1}{H_4 s}}$$

gives,

$C_1 = 1F$, $H_2 = 4/9\,H$, $C_3 = \frac{4}{15}F$, $H_4 = \frac{15}{16}H$

And representing Z(S) equivalent to Y(S) gives;

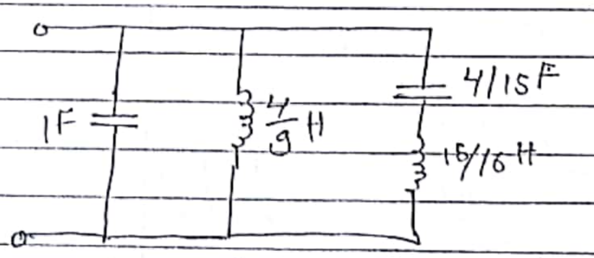


Fig:- Foster Parallel Circuit.

Using Cauer I method,

$$Z(S) = \frac{s^4+10s^2+9}{s^3+4s}$$

Numerator power, m = 4

Denominator power, n = 3

$\therefore m > n$

$s^3+4s\ )\ s^4+10s^2+9\ (\ s \leftarrow Z_1(S)$

$\quad -s^4 \pm 4s^2$

$6s^2+9\ )\ s^3+4s\ (\ \frac{1}{6}s \leftarrow Y_2(S)$

$\quad -s^3 \pm \frac{9}{6}s$

$\frac{5}{2}s\ )\ 6s^2+9\ (\ \frac{12}{5}s \leftarrow Z_3(S)$

$\quad -6s^2$

$9\ )\ \frac{5}{2}s\ (\ \frac{5}{18}s \leftarrow Y_4(S)$

$\quad -\frac{5}{2}s$

$\quad \times$

$$\therefore Z(S) = Z_1(S) + \cfrac{1}{Y_2(S) + \cfrac{1}{Z_3(S) + \cfrac{1}{Y_4(S)}}}$$

$$= s + \cfrac{1}{\left(\frac{1}{6}\right)s + \cfrac{1}{\left(\frac{12}{5}\right)s + \cfrac{1}{\left(\frac{5}{18}\right)s}}}$$

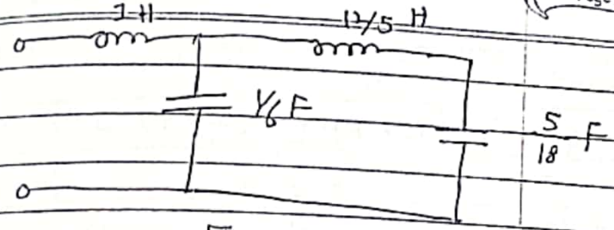


Fig:- Cauer – I network

Q.N.6) What is zero shifting by partial removal of pole? How can two-port passive circuits be synthesized using zero-shifting by partial pole removal? Explain.

Solution

The partial removal of pole means the removal of some fraction of total network that possibly could be removed without destroying positive, real nature of function being developed. Partial removal of poles relocates the zeros on jw-axis towards that pole. This process is called zero shifting by partial removal of pole.

lossless function can be written as;

$$z(s) = H\cdot s + \frac{K_0}{s} + \sum_{i=1}^{n} \frac{2K_i s}{s^2+\omega_0^2} \quad \text{--- eq}^n \text{(1)}$$

When pole at infinity

The first term on the RHS of eq^n(1) is due to contribution of pole at infinity. Suppose that, we substract some fraction of the term $H\cdot s$ from $z(s)$ by introducing a constant 'K_p' such that

$z_1(s) = z(s) - K_p\cdot Hs$, where $K_p < 1$

Here, $z_1(s)$ gives that the pole at infinity is partially removed.

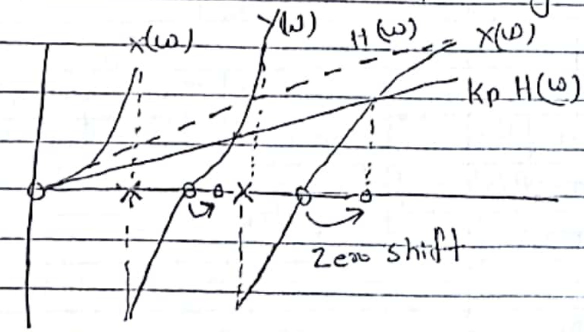
Since all zeros of $z_1(s)$ are again located on $s = j\omega$;

$X_1(\omega) = X(\omega) - K_p \cdot H(\omega)$, where $z_1(j\omega) = jX_1(\omega)$.

The zeros of $X_1(\omega)$ are value of 'ω' satisfying

$X(\omega) = K_p \cdot H\omega$

Plotting the response suggests that, zero is shifted toward pole, being weakened.



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Similarly, pole at zero is weakend by partially removal of $\frac{K_0}{s}$ of eq^n ①,

Poles at complex conjugate pair is partially removed by substracting third term of eq^{n}① (fractionally) with $Z(s)$.
By finding H and Kp. The two port network can by synthesized by partial removal of pole at desired location and zero shifting.

Q.N.7) What is Transmission and Reflection coefficient? How resistively terminated ladder network can be realized with finite transmission zeroes? Explain.

Solution:-
(see Q.N.7 (2071 Shawan))

Q.N.8) Draw the circuit diagram of Tow Thomas low pass filter and derive its transfer function. Realize following low pass filter using Tow Thomas biquad circuit.

Solution

First part
(see Q.N.8 (first part) 2071 Shawan)

Second Part
Transfer function of Biquad Circuit for LPF is,

$$T_{LP}(s) = \frac{-H\omega_0^2}{s^2+\left(\frac{\omega_0}{Q}\right)s+\omega_0^2}$$

$\therefore \omega_0 = 1000$ rad/s

$\frac{\omega_0}{Q} = 500$ or, $Q = \frac{1000}{500} = 2$

& $H = 2$ (Suppose)

Also, $$T_{LP}(s) = \frac{-1/R_3R_4C_1C_2}{s^2+\left(\frac{1}{R_1C_1}\right)\cdot s+\left(\frac{1}{R_2R_4C_1C_2}\right)}$$

Now, Using Tuning algorithm;
$C_1 = C_2 = 1F$ and $R_4 = 1\Omega$
$\omega_0 = 1$ rad/s

$\therefore \omega_0 = \sqrt{\frac{1}{R_2R_4C_1C_2}}$ or, $R_2 = \frac{1}{\omega_0^2}$
$= \frac{1}{1}$

$\therefore R_2 = 1\Omega$

Similarly, $\frac{\omega_0}{Q} = \frac{1}{R_1C_1}$

or $Q = R_1$

$\therefore R_1 = 2\Omega$

Now, $H = \frac{R_2}{R_3} = \frac{1}{R_3}$

or, $R_3 = \frac{1}{H} = \frac{1}{2} = 0.5\Omega$

We must realize the values at $\omega_0 = 1000$ rad/sec.

$\therefore K_f = 1000$,

For practical values, let $K_m = 1000$.

Then, $R_1 = 2 * 1000 = 2K\Omega$

$R_2 = 1 * 1000 = 1K\Omega$

$R_3 = 0.5 * 1000 = 0.5K\Omega$

$C_1 = \frac{1}{1000 * 1000} = 1\mu F$

$C_2 = \frac{1}{1000 * 1000} = 1\mu F$

$R_4 = 1 * 1000 = 1K\Omega$

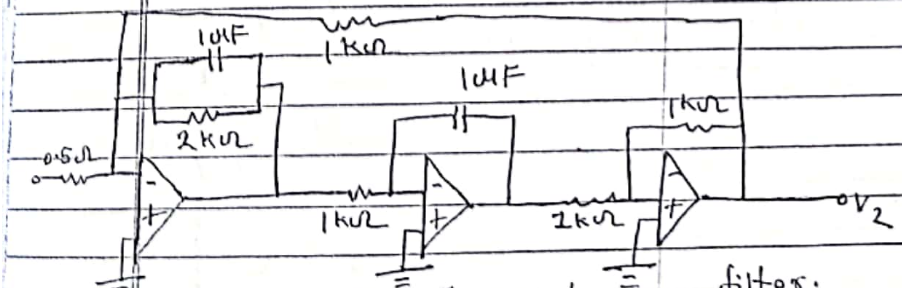


Fig:- Tow - Thomas low pass filter.

Q.N.9) How can the gain enhancement be performed in a Sallen-key Circuit? Explain with necessary diagram.

Solution!

(See Q.N.9 (2071 Shawan))//

Q.N.10) What is Sensitivity? Describe it's importance in filter design? Perform sensitivity analysis of quality factor in Tow Thomas low Pass filter.

Solution

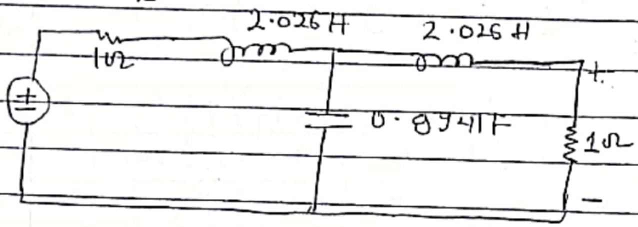
⇒ First part (see (Q.N.10) 2071 Shawan)

Q.N.11) What is GIFC? How a GIFC can be used to simulate a grounded inductor? Explain with necessary figures and derivations?

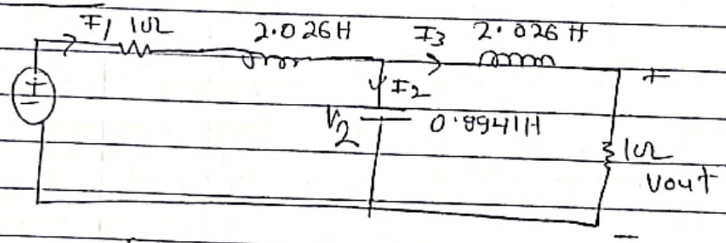
Solution

See (Q.N.11, first part of 2069 Chaitra).

S.N.12) The following circuit is a third-order chebyshev lowpass filter; simulate it using the leapfrog method. The final design should have $w_0 = 4000$ rad/s and practically realizable element values.



Solution



$I_1 = (V_1 - V_2)\, Y_1$

$V_2 = I_2 \cdot Z_2 = (I_1 - I_3)\, Z_2$

$I_3 = (V_2 - V_{out})\, Y_3$

$V_{out} = I_3 \cdot R_{out}$

Replace these eq^n^s as,

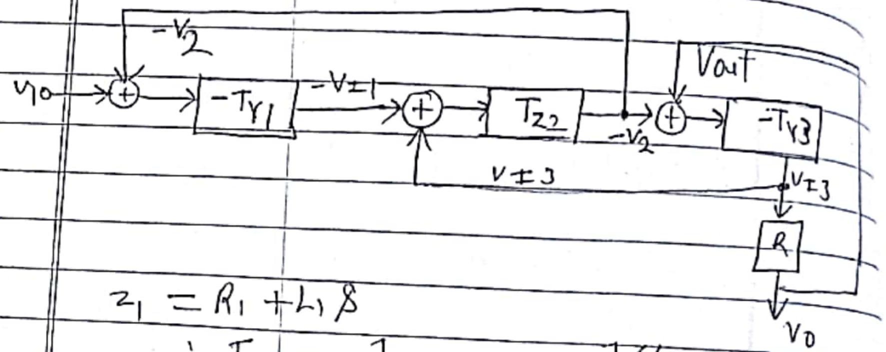
$-V_{I_1} = (V_1 - V_2)(-T_{Y_1})$

$-V_2 = (-V_{I_1} + V_{I_3}) \cdot T_{Z_2}$

$V_{I_3} = (-V_2 + V_{out}) \cdot (-T_{Y_3})$

$V_{out} = V_{I_3} \cdot R_{out}$

Block diagram



$Z_1 = R_1 + L_1 s$

$\therefore T_{Y_1} = \dfrac{1}{L_1 s + R_1} = \dfrac{1/L_1}{s + R_1/L_1}$

$\therefore -T_{Y_1} = \dfrac{-1/L_1}{s + R_1/L_1} \Rightarrow$ lossy integrator.

$T(s) = \dfrac{-1/R_aC}{s + \frac{1}{R_bC}}$

$\therefore R_aC = L_1$ & $R_b \cdot C = \dfrac{L_1}{R_1}$

or, $\boxed{R_a = \dfrac{L_1}{C}}$ & $\boxed{R_b = \dfrac{L_1}{R_1 C}}$

$T_{Z_2} = \dfrac{1}{C_2 s} \Rightarrow$ Integrator + Unity gain amplifier

$T(s) = -\dfrac{1}{RCs} * (-1)$

$\therefore RC = C_2$

$\boxed{\therefore C = C_2/R}$

$-T_{Y3} = -\frac{1}{L_3 s}$ ⇒ Integrator

$T(s) = -\frac{1}{RCs}$

$\therefore RC = L_3$

$\therefore \boxed{C = \frac{L_3}{R}}$

Now, to achieve $\omega_0 = 4000$ rad/s, do frequency scaling,

$K_f = 4000$

To get practical value of elements, do magnitude scaling, $K_m = 1000$

$\therefore R_1 = 1 * 1000 = 1k\Omega$

$R_0 = 1 * 1000 = 1k\Omega$

$L_1 = L_3 = \frac{2.026}{4000} \times 1000 = 0.5065\ H$

$C_2 = \frac{0.9941}{4000 * 1000} = 248.525\ nF$

Now, Design of $-T_{Y1}$;

let $C = 100\ \mu F$

$\therefore R_a = \frac{L_1}{C} = \frac{0.5065}{100 * 10^{-6}} = 5.065\ k\Omega$

$R_b = \frac{L_1}{CR_1} = \frac{0.5065}{100 \times 10^{-6} \times 1000} = 5.065\ \Omega$

Design of $-T_{Y3}$;

let $R = 1k\Omega$;

$\therefore C = \frac{L_3}{R} = \frac{0.5065}{1 \times 10^3} = 506.5\ \mu F$

Design of T_{Z2};

let $R = 1k\Omega$

$\therefore C = \frac{C_2}{R} = \frac{248.525 * 10^{-9}}{1 * 10^3} = 248.525\ pF$

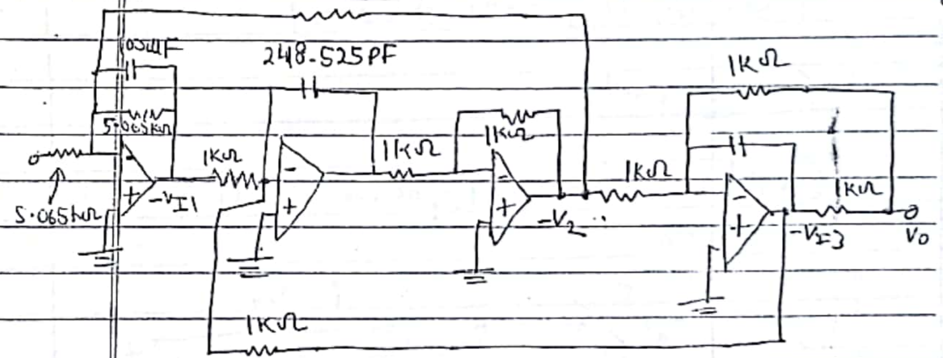


Fig:- Leapfrog Simulation.

Q.N.13) Why resistors are replaced by switched capacitor in IC technology? How can you simulate a resistor using a switched capacitor? Explain with necessary derivation. Also draw the switched capacitor equivalent circuit for inverting summer, lossy integration and non-inverting integrator.

Solution

First part and last part

(See Q.N.13, 2071 Shawan)

Second part

Consider Switched Capacitor circuit as shown below;

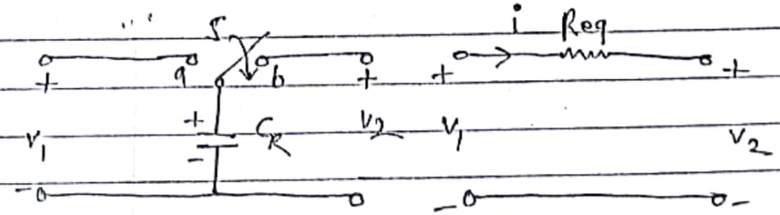


fig:- Switched Capacitor and its resistor equivalent.

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Case-I

When the switch (S) is connected to point (a) the charge stored in the capacitor,

$q_1 = C_R \cdot V_1$ .... (1) where

C_R = Switched Capacitor representing a resistor

Case II

When the switch (S) is connected to point (b), the charge remained in the capacitor,

$q_2 = C_R \cdot V_2$ ... (2)

So, the charge transferred to v_2 from v_1 is;

$\Delta q = q_1 - q_2$

$= C_R(V_1 - V_2)$ .... (3)

let switch (S) is flipped periodically, with clock period (T) such that clock frequency $f_c = \left(\frac{1}{T}\right)$ is so large compared to signal frequency $\omega = 2\pi f$ of two voltage sources V_1 and V_2.

i.e $f_c >> (\omega = 2\pi f)$

So that these signals can be assumed to be constant over period (T),

Now, current (i) $\approx \frac{\Delta q}{T} = \Delta q \cdot f_c$

$= C_R (V_1 - V_2) f_C$

$\therefore i = f_C \cdot C_R (V_1 - V_2) \quad \ldots (4)$

Again from second figure,

$$i = \frac{V_1 - V_2}{R_{eq}} \quad \ldots (5)$$

Comparing (4) & (5),

$$R_{eq} = 1/f_C \cdot C_R$$

Hence, resistor is replaced by the above circuit, where Capacitor value is,

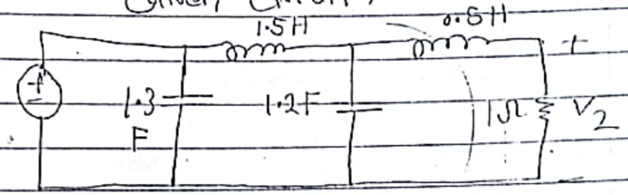
$$\frac{1}{R_{eq} \cdot f_C}.$$

2072 Chaitra

S.N.1) What is the significance of normalization & de-normalization in filter design? The following is a pass filter with $\omega p = 1$ rad/sec. Modify the circuit so that it becomes a low pass filter with a passband of 1000 rad/sec and a load resistance of 75 Ω.

Solution

Given Circuit,



Then, frequency Scaling factor (k_f) $= \frac{\omega p_{new}}{\omega p_{old}}$

$= 1000$

$R_{old} = 1\Omega$, $R_{ew} = 75\Omega$

We know, $R_{new} = K_m \cdot R_{old}$

or, $K_m = \frac{R_{enew}}{R_{old}} = \frac{75}{1}$

$= 75$

∴ Magnitude Scaling factor (K_m) $= 75$

∴ $R_{new} = 75\Omega$

$$C_{1,new} = \frac{C_{old}}{K_m \cdot K_f} = \frac{1.3}{75 * 1000} = 17.3\,\mu F$$

$$C_{1\,new} = \frac{1.2}{75 * 1000} = 16\,\mu F$$

$$L_{1,new} = \frac{K_m L_{new}}{K_f} = \frac{75 * 1.5}{1000} = 112.5\,mH$$

$$L_{2,new} = \frac{75 * 0.5}{1000} = 37.5\,mH$$

Then, the final circuit can be drawn as,

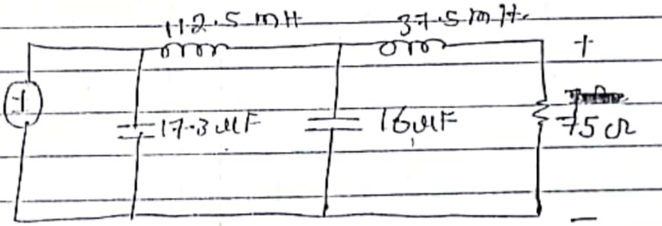


Fig:- Final Circuit.

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First part

(see in Q.N.1 of 2073 Shravan)

Q.N.2) Derive an expression to calculate the Order of Inverse Chebyshev low pass filter. Use this formula to estimate the order of Inverse Chebyshev low pass filter having following Specification.

$\alpha_{max} = 0.25\,dB$, $\omega_p = 1000\,rad/s$

$\alpha_{min} = 18\,dB$, $\omega_s = 1400\,rad/s$

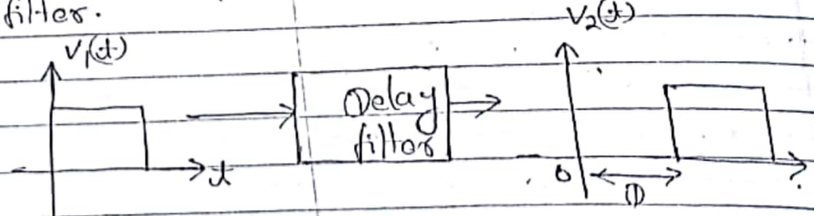
Solution

(see Q.N.2 of 2071 Chaitra.)

Q.N.3) What is delay equalization? How can it be done? Explain with necessary figures.

Solution

Delay equalization corresponds to adjusting the relative phases of different frequencies to achieve a constant group delay using all pass filter in series with an uncompensated filter.



let $v_1(t) = A \sin(\omega t + \phi) = A \cdot e^{j\phi}$ ---(1)

Then,

$v_2(t) = A \sin(\omega(t - D) + \phi) = A \sin(\omega t + \phi - \omega D)$
$= A \cdot e^{j(\phi - \omega D)}$ ---(2)

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$\therefore$ Transfer function,

$$T(S) = \frac{V_2}{V_1} = \frac{A \cdot e^{j(\phi - \omega D)}}{A \cdot e^{j\phi}}$$

$$= e^{-j\omega D} \quad \text{---} (3)$$

For normalization, put $D = 1$,

$\therefore T(S) = e^{-j\omega} = e^{-s} = \frac{1}{e^{s}}$

(Physically not realizable).

$|T(S)| = 1$, $\angle T(S) = -\omega D$ [from eqn (3)]

$\theta = -\omega D$

$\therefore D = -\theta/\omega$

So, Approximation. $D = \frac{-\Delta\theta}{\Delta\omega} = -\frac{d\theta}{d\omega}$

$$= \frac{-d(\theta)}{d\omega}$$

Q.N.4) What are the application of frequency Transformation in filter Design? How can you obtain a high pass filter from a given low pass filter? Explain with a suitable example.

Solution

First part (see Q.N.4 of 2071 Chaitra First part)

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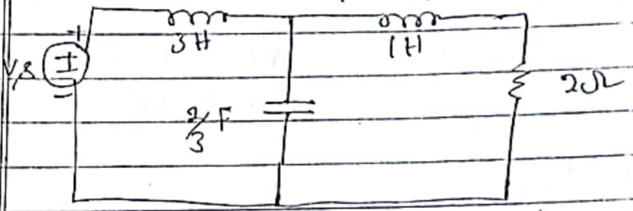
Second Part

Example, to obtain the high pass filter from the low pass filter, having passband frequency 2KHz. And the low pass filter has passband frequency ω_p of 1 rad/s.



To transform low pass filter to high pass filter replace 's' by '$\frac{\omega_c}{s}$',

Here, $\omega_c = 2\pi f$
$= 2\pi * 2 * 10^3$
$= 4000\pi$ rad/s

i) Resistor : $Z_R = R$

$\therefore Z_{R,new} = R$

$\therefore R(2) = 2\Omega$

ii) Inductor : $Z_L = Ls$

$\therefore Z_{L,new} = \frac{L \cdot \omega_c}{s}$

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∴ Inductor is replaced by Capacitor,

Hence, $L(3) = C = \frac{1}{\omega_c \cdot L} = \frac{1}{3 * 4000\pi}$

$= 26.52\,\mu F$

$L(1) = C = \frac{1}{L\omega_c} = \frac{1}{1 * 4000\pi}$

$= 79.577\,\mu F$

iii) Capacitor : $Z_c = \frac{1}{Cs}$

$Z_{c\,new} = \frac{1}{C \cdot \frac{\omega_c}{s}} = \frac{s}{C \cdot \omega_c}$

∴ Capacitor is replaced by inductor.

Hence,

$C\left(\frac{2}{3}\right) = L = \frac{1}{C\omega_c} = \frac{1}{\frac{2}{3} * 4000\pi}$

$= 119.36\,\mu H$

∴ The high-pass filter circuit is ;

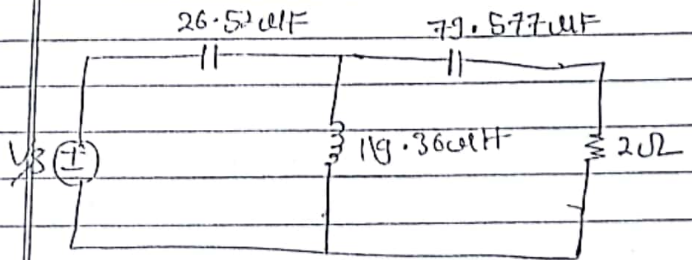

Fig:- high pass filter.

Q.N.5) Which of the following is LC lossless function and why? Pick one of the valid LC lossless functions and synthesize it using Foster and Cauer methods.

i) $Z_1(s) = \frac{s(s^2+4)(s^2+10)}{(s^2+2)(s^2+10)}$

ii) $Z_2(s) = \frac{(s^2+2)(s^2+10)}{s(s^2+5)}$

iii) $Z_3(s) = \frac{s^2+25}{s(s^2+5)(s^2+50)}$

Solution

Any function F(s) to be the LC lossless function which satisfies the following condition,

a) F(s) should be the ratio even to odd or odd to even; satisfied by all.

b) The poles and zeros must lie on imaginary axis; satisfies by all.

c) The poles and zeros must alternate each other; satisfied by $Z_2(s)$, only,

d) The highest (also lowest) power of numerator and denominator polynomial must be differ by Unity, satisfied by $Z_2(s)$.

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c) The succeeding power of numerator (also denominator) should differ by 2; satisfied by $Z_2(s)$.

$\therefore Z_2(s)$ is the LC lossless function.

i.e $Z_2(s) = \dfrac{(s^2+2)(s^2+10)}{s(s^2+5)} = \dfrac{s^4+12s^2+20}{s^3+5s}$

d) Synthesize using Foster method:

$$s^3+5s \,\big)\, s^4+12s^2+20 \,(\, s$$
$$-s^4 \pm 5s^2$$
$$7s^2+20$$

$\therefore Z_2(s) = s + \dfrac{7s^2+20}{s^3+5s}$

$= Z_3(s) + Z_4(s)$

Now, $Z_4(s) = \dfrac{7s^2+20}{s^3+5s} = \dfrac{7s^2+20}{s(s^2+5)}$

$= \dfrac{K_1}{s} + \dfrac{K_2 s}{s^2+5}$

$\therefore K_1 = Z_4(s)\cdot s \Big|_{s=0} = 4$

$K_2 = \dfrac{Z_4(s)\cdot(s^2+5)}{s}\Bigg|_{s^2=-5} = 3$

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$Z_2(s) = s + \dfrac{4}{s} + \dfrac{3s}{s^2+5} = Z_3(s) + Z_5(s) + Z_6(s)$

Now, $Z_6(s) = \dfrac{3s}{s^2+5} \Rightarrow Y_6(s) = \dfrac{s^2+5}{3s}$

$= \dfrac{s}{3} + \dfrac{5}{3s}$

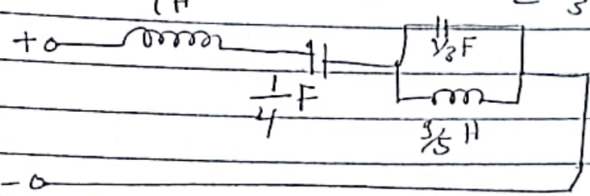

fig:- Foster - I Circuit.

b) Synthesize $Z_2(s)$ using Cauer method:

$$s^3+5s \,\big)\, s^4+12s^2+20 \,(\, s \rightarrow Z$$
$$-s^4 \pm 5s^2$$
$$7s^2+20 \,\big)\, s^3+5s \,(\, s/7 \rightarrow Y$$
$$s^3 \pm 20s/7$$
$$\tfrac{15}{7}s \,\big)\, 7s^2+20 \,(\, \tfrac{49s}{15} \rightarrow Z$$
$$-7s^2$$
$$20 \,\big)\, \tfrac{15}{7}s \,(\, \tfrac{3}{28}s \rightarrow Y$$
$$-\tfrac{15}{7}s$$
$$\times$$

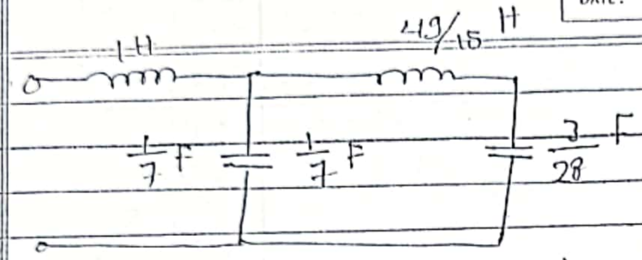

Fig:- Cauer – I Circuit.

S.N.6) Define transmission zeros. How zeros of transmission be realized? Explain with suitable example.

Solution
See (S.N.6 of 2070 Chaitra).

S.N.7) Design a third order Butterworth low pass filter using resistively terminated lossless ladder with unequal termination. $R_1 = 1\Omega$ and $R_2 = 4\Omega$ (Refer table 1).

Solution
From table 1, the third order Butterworth low pass filter has the transfer function,

$$T_3(s) = \frac{1}{(s+1)(s+0.5+j\,0.86603)(s+0.5-j\,0.86603)}$$

$$= \frac{1}{(s+1)(s^2+s+0.25+0.75)}$$

$$= \frac{1}{(s+1)(s^2+s+1)}$$

Now, for Butterworth filter, $|T(s)|^2 = |H(s)|^2$

$$\therefore H(s) = \frac{1}{(s+1)(s^2+s+1)}$$

We know,

$$D(s) = \frac{1}{T(s)} = (s+1)(s^2+s+1)$$

And,

$$\rho(s) = \frac{s^n}{D(s)} = \frac{s^3}{(s+1)(s^2+s+1)}$$

$$\therefore Z_{in} = R_1\left[\frac{1-\rho(s)}{1+\rho(s)}\right]^{\pm 1}$$

$$\text{or, } Z_{in} = 1.\left[\frac{1-\frac{s^3}{(s+1)(s^2+s+1)}}{1+\frac{s^3}{(s+1)(s^2+s+1)}}\right]^{\pm 1}$$

$$\text{or, } Z_{in} = \left[\frac{(s+1)(s^2+s+1)-s^3}{(s+1)(s^2+s+1)+s^3}\right]^{\pm 1}$$

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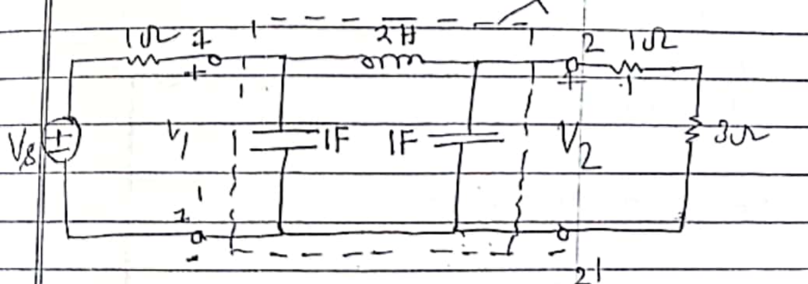
$$Z_{in} = \left[\frac{2s^2+2s+1}{2s^3+2s^2+2s+1}\right]^{\pm 1}$$

$$Z_{in} = \frac{2s^2+2s+1}{2s^3+2s^2+2s+1}, \quad Z_{in} = \frac{2s^3+2s^2+2s+1}{2s^2+2s+1}$$

Taking, $Z_{in} = \dfrac{2s^2+2s+1}{2s^3+2s^2+2s+1}$

$$\therefore Y_{in} = \frac{2s^3+2s^2+2s+1}{2s^2+2s+1}$$

$$
\begin{array}{l}
2s^2+2s+1 \,)\, 2s^3+2s^2+2s+1 \,(\, s \rightarrow Y \\
\quad -2s^3 \pm 2s^2 \pm s \\
\hline
\quad s+1 \,)\, 2s^2+2s+1 \,(\, 2s \rightarrow Z \\
\quad\quad -2s^2 \pm 2s \\
\hline
\quad\quad 1 \,)\, s+1 \,(\, s \rightarrow Y \\
\quad\quad\quad -s \\
\hline
\quad\quad\quad 1 \,)\, 1 \,(\, 1 \rightarrow Z \\
\quad\quad\quad\quad -1 \\
\hline
\quad\quad\quad\quad \times
\end{array}
$$



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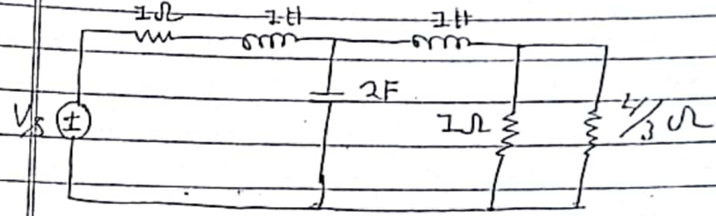
And, similarly,

Taking. $Z_{in} = \dfrac{2s^3+2s^2+2s+1}{2s^2+2s+1}$



Q.N.8) Realize the following transfer function by cascading two first-order sections using inverting op-amp configuration.

$$T(S) = \frac{12}{s^2+8s+12}$$

Solution

$$T(S) = \frac{12}{s^2+8s+12} = \frac{3}{(s+2)} \cdot \frac{4}{(s+6)}$$
$$= T_1(S) \cdot T_2(S)$$

Realization of $T_1(S)$:

$$T_1(S) = \frac{3}{s+2} = \frac{Z_2}{Z_1} = \frac{3/s}{1+2/s}$$

$$\therefore Z_2 = 3/s$$
$$Z_1 = 2/s + 1$$

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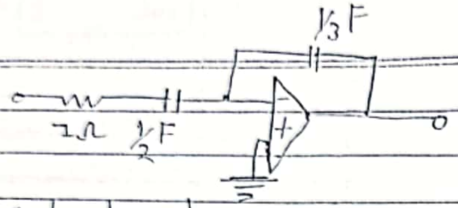
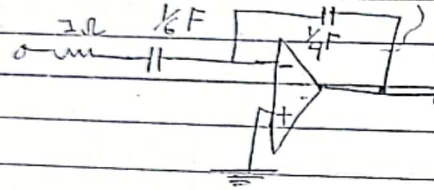
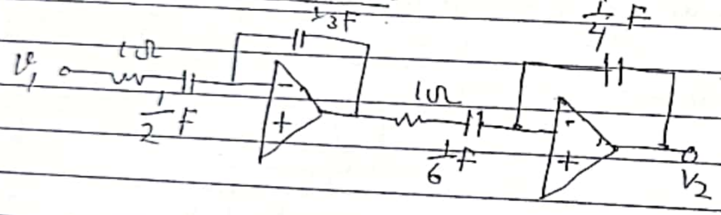


Realization of $T_2(S)$:

$$T_2(S) = \frac{4}{s+6} = \frac{Z_2}{Z_1} = \frac{4/s}{1+6/s}$$



∴ Realization of $T(S)$:



P-N-9) Design Sallen Key lowpass filter for fourth Order Butterworth filter. The final circuit should have $\omega_0 = 10{,}000$ rad/s have and practically realizable elements. (Refer table 1).

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Solution

Transfer function of 4th order Butterworth filter,

$$T_4(s) = \frac{1}{\left[(s+0.38268834)^2+(0.9238795)^2\right]\cdot\left[(s+0.9238795)^2+(0.3826834)^2\right]}$$

$$= \frac{1}{(s^2+0.7654s+1)(s^2+1.8478s+1)}$$

$$= \frac{1}{(s^2+0.7654s+1)} \cdot \frac{1}{(s^2+1.8478s+1)}$$

$$= T_1(S) \cdot T_2(S)$$

Realization of $T_1(S)$:

$$T_1(s) = \frac{1}{(s^2+0.7654s+1)}$$

Using Design – I:

$\omega_0 = 1$ rad/sec.

$\frac{\omega_0}{Q} = 0.7654 \Rightarrow Q = 1.3065$

$H = 1$

$K = 3 - \frac{1}{Q} = 3 - \frac{1}{1.3065} = 2.2346$

$1 + \frac{R_B}{R_A} = 2.2346 \quad \cdots (1)$

let $C_1 = C_2 = 1F$

$R_1 = R_2 = R = \frac{1}{\omega_0} = 1\Omega$

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$$R_1+R_2 = \frac{R_A \cdot R_B}{R_A+R_B} = 2 \quad \text{.... (2)}$$

From (1) & (2),

$R_A = 3.61901 \Omega$

$R_B = 4.46926 \Omega$

Realization of $T_2(S)$:

$$T_2(s) = \frac{1}{s^2+1.8478s+1}$$

$\therefore w_o = 1$ rad/sec.

$$\frac{w_o}{\rho} = 1.8478 \Rightarrow \rho = 0.54118$$

$\boxed{H=1}$

$$K = 3 - \frac{1}{\rho} = 3 - \frac{1}{0.5411} = 1.1522$$

$$1+\frac{R_B}{R_A} = 1.1522 \quad \text{---- (1)}$$

let $C_1 = C_2 = 1F$

And, $R_1 = R_1 = R = \frac{1}{w_o} = 1\Omega$

Now,

$$R_1+R_2 = \frac{R_A \cdot R_B}{R_A+R_B} \quad \text{.... (2)}$$

$$2 = \frac{R_A \cdot R_B}{R_A+R_B} \quad \text{....... (2)}$$

from (1) & (2)

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$R_A = 15.14 \Omega$

$R_B = 2.3044 \Omega$

To achieve $w_o = 10{,}000$ rad/sec; do frequency scaling, $K_f = 10{,}000$

$\therefore$ Elements of $T_1(S)$:

$R_1, R_2, R_A, R_B \rightarrow$ Same

$$C_1 = C_2 = \frac{1}{10{,}000} = 100 \mu F$$

Elements of $T_2(S)$:

$R_1, R_2, R_A, R_B \rightarrow$ same.

$$C_1 = C_2 = \frac{1}{10{,}000} = 100 \mu F$$

To achieve practically realizable element, do magnitude scaling, $K_m = 1000$.

$\therefore$ Element of $T_1(S)$

$R_1 = R_2 = 1 k\Omega$

$R_A = 3619.0 \Omega = 3.61910 k\Omega$

$R_B = 4.46926 k\Omega$

$C_1 = C_2 = 100 nF$

$\therefore$ Element of $T_2(S)$:

$R_1 = R_2 = 1 k\Omega$

$R_A = 15.14 k\Omega$

$R_B = 2.3044 k\Omega$

$C_1 = C_2 = 100 nF$

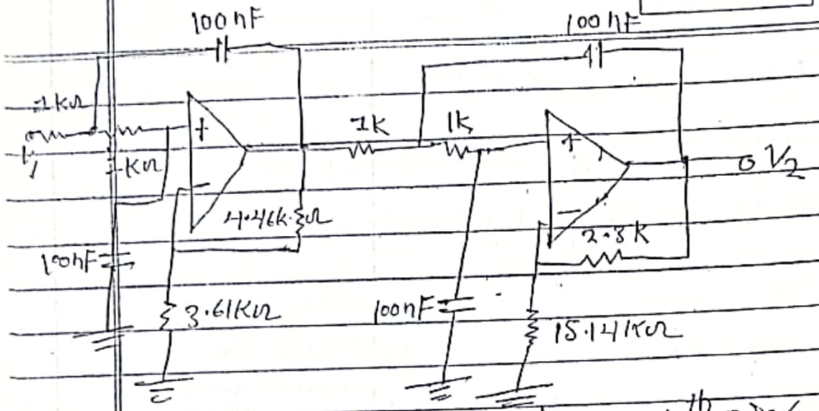

Fig:- Sallon and Key low pass 4th order Butterworth filter.

52 Q.N.10) What information do you get when the sensitivity of x with respect to y is −5? Perform sensitivity analysis for center frequency (ω_0) of the sallen Key low pass filter with respect to all the resistors and capacitor present in the circuit.

Solution

First Part

When the sensitivity of y with respect to x equal to −5, from this we understand that ($S_x^y = -5$) 1% change is x results 5% decrease in y or negative change in y.

Second Part

(See Q.N-10 second part of 2069 Chaitra.)

Q.N.11) Draw the Circuit diagram of an generalized Impedance Converter. Derive the relationship between Input and output Current. How can it be used to simulate a grounded FDNR?

Solution

The circuit diagram of an generalized impedance convertor (GIC; Antoniou's GIC) is,

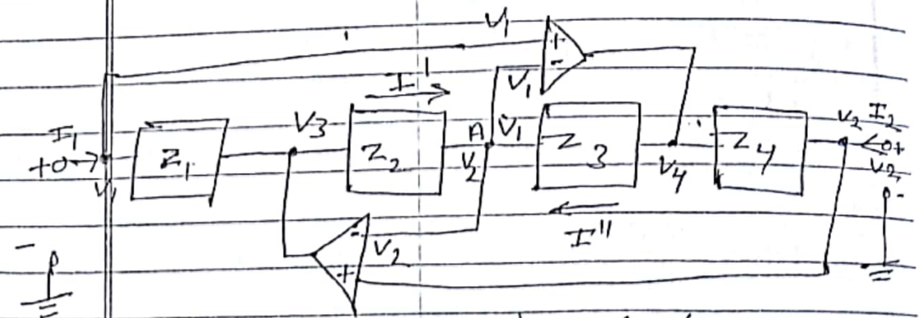

Fig:- Antoniou's GIC

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We have,

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} f_1(s) & 0 \\ 0 & f_2(s) \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$\therefore V_1 = f_1(s) V_2 \quad \text{---} (1)$$

$$I_1 = -f_2(s) \cdot I_2 \quad \text{---} (2)$$

From figure,

$$V_1 = V_2 \quad \text{---} (3)$$

$\because$ Op-amp has infinite impedance, current doesn't flow through op-amp.

$$\therefore I_1 = \frac{V_1 - V_3}{Z_1} \;\&\; I_2 = \frac{V_2 - V_4}{Z_4}$$

Applying KCL at node A,

$$I' + I'' = 0$$

$$\text{or, } \frac{V_3 - V_1}{Z_2} + \frac{V_4 - V_1}{Z_3} = 0$$

$$\text{or, } \frac{-I_1 Z_1}{Z_2} + \left(\frac{-I_2 Z_4}{Z_3}\right) = 0$$

$$\text{or, } \frac{-I_2 Z_4}{Z_3} = \frac{I_1 Z_1}{Z_2}$$

$$\boxed{\therefore I_1 = -I_2 \cdot \frac{Z_2 Z_4}{Z_1 \cdot Z_3}} \quad \text{---} (4)$$

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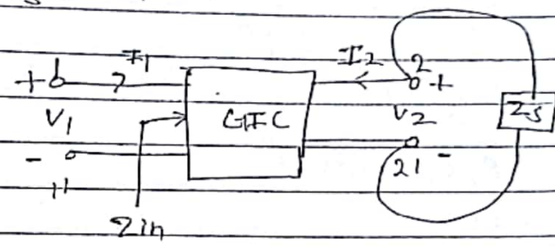
Hence, eqn (4) is the relation between input and output current.

If port-2 is terminated with impedance Z_5.



$$\therefore V_2 = -I_2 \cdot Z_5$$

$$\text{or, } -I_2 = \frac{V_2}{Z_5}$$

$$\therefore I_1 = -I_2 \cdot \frac{Z_2 Z_4}{Z_1 Z_3}$$

$$\text{or, } I_1 = \frac{V_2}{Z_5} \cdot \frac{Z_2 Z_4}{Z_1 Z_3}$$

$$\text{or, } I_1 = \frac{V_1}{Z_5} \cdot \frac{Z_2 Z_4}{Z_1 Z_3}$$

$$\text{or, } \frac{V_1}{I_1} = Z_5 \cdot \frac{Z_1 Z_3}{Z_2 \cdot Z_4}$$

$$\boxed{\text{or, } \frac{Z_{in}}{Z_5} = \frac{Z_1 Z_3}{Z_2 Z_4}}$$

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let Z_5 be the resistor of Resistance R_L,

let Z_1 and Z_3 be Capacitor,

let Z_2 and Z_4 be resistor.

$$\therefore Z_{in} = \frac{1}{C_1 R_2 C_3 R_4 s^2} \cdot R_L$$

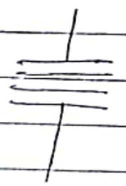
$$= \frac{R_L}{C_1 R_2 C_3 R_4 (j\omega)^2}$$

$$= \frac{-R_L}{C_1 R_2 C_3 R_4 \omega^2}$$

$$= \frac{-1}{D\omega^2}$$

where $D = \dfrac{C_1 R_2 C_3 R_4}{R_L}$

fig:- FDNR



P.N.17) Design a fourth Order Butterworth low pass filter having half power frequency of 4000 rad/s using Frequency dependent negative resistor (FDNR).

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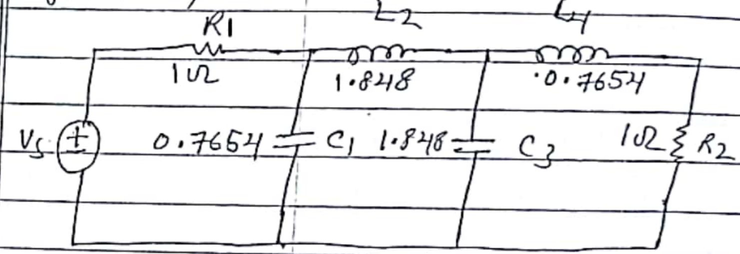
(Use table 2)

Solution

Circuit of fourth order Butterworth low pass filter is,



at $\omega_0 = 1$ rad/sec

Scale the frequency by the factor $K_f = 4000$.

$\therefore R_1 = R_1 = 1\Omega$, $R_2 = R_2 = 1\Omega$

$$C_1 = \frac{0.7654}{4000} = 191.35\ \mu F$$

$$C_3 = \frac{1.848}{4000} = 462\ \mu F$$

$$L_2 = \frac{1.848}{4000} = 462\ \mu H$$

$$L_4 = 191.35\ \mu H$$

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Now, Design Using FDNR,
Scale all element by $1/S$,

$\therefore$ Resistor(R) : $Z_R = R \rightarrow \frac{R}{S} \Rightarrow$ Capacitor $= \frac{1}{R}$

Inductor (L) : $Z = LS \rightarrow LS \cdot \frac{1}{S} = L \Rightarrow$ Resistor $= L$

Capacitor (C) : $Z_C = \frac{1}{CS} \rightarrow \frac{1}{CS} \cdot \frac{1}{S} = \frac{1}{CS^2} = \frac{1}{C\omega^2}$

$\Rightarrow$ FDNR, D = C

$\therefore$ The required Circuit is,

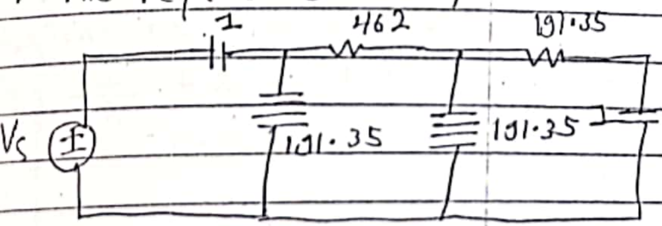

Fig: Simulation of Inductor Using FDNR

J.N.B) What is switched Capacitor filter? Design a switched Capacitor filter to realize the magnitude response given below,

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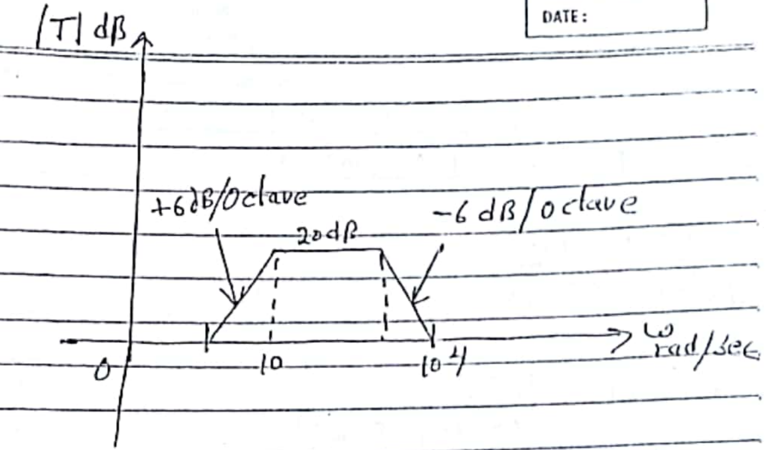

Solution

Any filter (active or passive) contain resistor. But, resistor takes large space on IC when fabricated. Therefore, the resistor is replaced by the circuit combination of MOSFET and Capacitor. These filters are called switched - Capacitor filter.

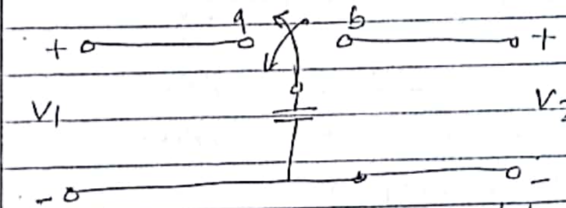

Fig :- Switched - Capacitor

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Here, initial gain $= 0\,dB \Rightarrow K = 1$,

Break points $= 0, 10^1, 10^3, 10^4$

Zeros : $0, 10^4$

poles : $10^1, 10^3$

$$\therefore T(S) = \frac{S(S+10^4)}{(S+10)(S+10^3)}$$

$$= \left(\frac{-1S}{S+10}\right) \cdot \left(\frac{-S+10^4}{S+10^3}\right)$$

$$= T_1(S) \cdot T_2(S)$$

Now,

let the impedance be the series combination of R and C,

$$\therefore T(S) = \frac{V_2}{V_1} = \frac{-Z_2}{Z_1} = \frac{-R_2 + \frac{1}{C_2 S}}{R_1 + \frac{1}{C_1 S}}$$

$$= \frac{-S + 1/R_2C_2}{S + 1/R_1C_1}$$

If Z_2 = Capacitor only, then,

$$T(S) = \frac{-1/C_2S}{R_1 + \frac{1}{C_1S}} = \frac{-C_1/C_2}{R_1C_1 S + 1}$$

$$= \frac{-1/R_1C_2}{S + 1/R_1C_1}$$

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$$\therefore T_1(S) = \frac{-1 \cdot S}{S+10} = -\frac{1/R_1C_2}{S + 1/R_1C_1}$$

$$\therefore \frac{1}{R_1C_2} = 1 \Rightarrow R_1C_2 = 1$$

And, $R_1C_1 = 10^{-1}$

let $C_1 = C_2 = 10pF$ and $f_c = 10KHz$

Then,

$$R_1 = 10^{-1}/(10 * 10^{-12}) = 10 * 10^9$$

$$C_a = \frac{1}{R_1 f_c} = 0.01\,pF$$

And,

$$T_2(S) = \frac{-S+10^4}{S+10^3} = \frac{-S + 1/R_2C_2}{S + \frac{1}{R_1C_1}}$$

$\therefore R_2C_2 = 10^4$

$R_1C_1 = 10^{-3}$

let $C_1 = C_2 = 10pF$ and $f_c = 10KHz$

$$\therefore R_1 = 10^{-4}/(10 * 10^{-12}) = 10 * 10^6$$

$$\frac{1}{C_b f_c} = 10 * 10^6$$

$$\therefore C_b = \frac{1}{10 * 10^3 * 10 * 10^6} = 10pF$$

And,

$$R_1 = 10^3 / (10 * 10^{-12}) = 100 * 10^6$$

$$\frac{1}{C_a f_c} = 100 * 10^6$$

$$\therefore C_a = \frac{1}{100 * 10^6 * 10 * 10^3} = 1pF$$

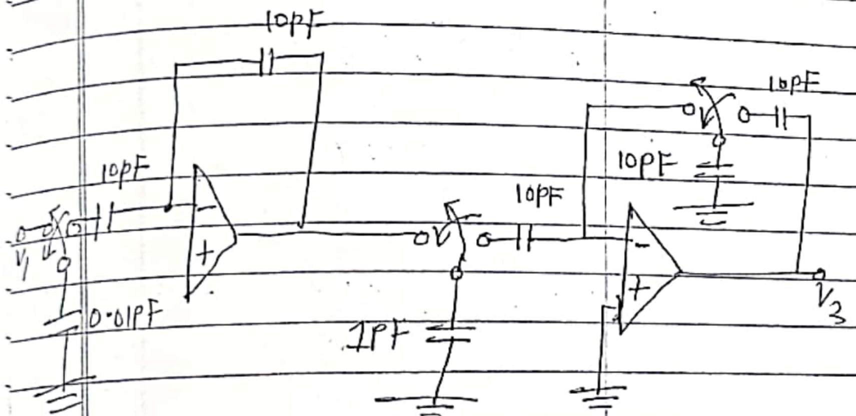


Fig:- Switched Capacitor filter.

2071 Shrawan

Q.N.1) What is normalization and denormalization? Explain the importance of normalization and denormalization in filter design with example.

Solution:

Normalization

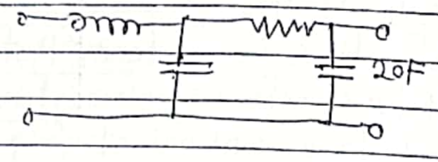
→ Normalization is a term that corresponds to filters whose component values are adjusted to a convenient frequency and impedance level. Normalization means converting the given specification to the standard values.

Denormalization:

→ Denormalization, in other word scaling, means to convert the circuit in to the required specification. Circuit elements are also scaled to get values that are easily available in the market. Two types of scaling are frequency scaling and magnitude scaling.

Second part

The circuit shown has cut-off of 10 rad/sec. Change cut-off frequency to 20 rad/sec and use inductor of 10H in the circuit.



Now,

We have, $\omega = 10$ rad/sec

and $\Omega = K_f \cdot \omega$

$20 = K_f \cdot 10$ & $K_f = 2$

Again, $L_{old} = 1$ Henry

$$L_{new} = \frac{L_{old}}{K_f} * K_m$$

or, $10 = \frac{1}{K_f} * K_m$

~~or $10 = \frac{1}{2} * K_m$~~

or, $K_m = 20$

Now scaling the ckt elements, we get,

$R_{new} = K_m \cdot R_{old} = 20 * 10 = 200\Omega$.

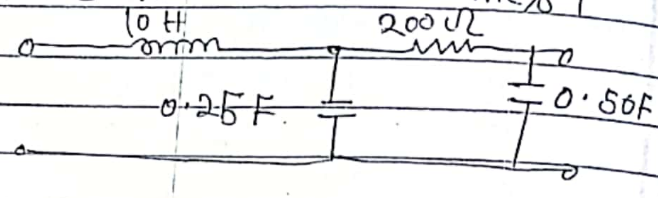
$$C_{new1} = \frac{C_{old}}{K_m \cdot K_f} = \frac{10}{20*2} = \frac{10}{40}$$

$= 0.25F$

$$C_{new2} = \frac{20}{20*2} = 0.5F$$

$$L_{new} = \frac{K_m}{K_f} * L_{old} = \frac{20}{2} * 1 = 10 \text{ Henry}$$

Then the new circuit becomes,



Q.N.2 Derive the relation to calculate the order of Chebyshev filter. Using this formula calculate the required order of Chebyshev filter for following lowpass filter specification.

$\alpha_{max} = 0.5$ dB, $\alpha_{min} = 20$ dB

$\omega_p = 1000$ rad/s, $\omega_s = 2000$ rad/s

Solution

We have, Transfer function of Chebyshev filter,

$$|T_n(s)|^2 = \frac{1}{1+\varepsilon^2 C_n^2(\omega)} \quad \text{--- (1)}$$

; $C_n(\omega) = \cos(n \cos^{-1}\omega)$ for $\omega \leq 1$

$C_n(\omega) = \cosh(n \cosh^{-1}\omega)$ for $\omega > 1$

And, Attenuation, $\alpha = -20 \log |T|$

or, $\alpha = 10 \log [1 + \epsilon^2 C_n^2(\omega)]$ ---- (2)

for $C_n(\omega) = \pm 1$, $\alpha = \alpha_{max}$

$\therefore \alpha_{max} = 10 \log [1 + \epsilon^2]$

$$\epsilon = [10^{\alpha_{max}/10} - 1]^{1/2} \text{ ---- (3)}$$

for $\omega = \omega_s$, $\alpha = \alpha_{min}$,

$\therefore \alpha_{min} = 10 \log [1 + \epsilon^2 C_n^2(\omega_s)]$

$$\therefore C_n(\omega_s) = \left[\frac{10^{\alpha_{min}/10} - 1}{10^{\alpha_{max}/10} - 1}\right]^{1/2}$$

$$\text{or, } \cosh(n \cosh^{-1} \omega_s) = \left[\frac{10^{\alpha_{min}/10} - 1}{10^{\alpha_{max}/10} - 1}\right]^{1/2}$$

$$\therefore n = \frac{\cosh^{-1}\left[(10^{\alpha_{min}/10} - 1)/(10^{\alpha_{max}/10} - 1)\right]^{1/2}}{\cosh^{-1}(\omega_s)}$$

After denormalization,

$$n = \frac{\cosh^{-1}\left[(10^{\alpha_{min}/10} - 1)/(10^{\alpha_{max}/10} - 1)\right]^{1/2}}{\cosh^{-1}\left(\frac{\omega_s}{\omega_p}\right)} \text{ ---- (4)}$$

Hence, eq^n (4) is the required value of the order ('n') of the Chebyshev filter.

Second part,

$\alpha_{max} = 0.5$ dB, $\alpha_{min} = 20$ dB

$\omega_p = 1000$ rad/s, $\omega_s = 2000$ rad/s

$$\text{And, } n = \frac{\cosh^{-1}\left[(10^{20/10} - 1)/(10^{0.5/10} - 1)\right]^{1/2}}{\cosh^{-1}\left(\frac{2000}{1000}\right)}$$

$$= 3.069$$

$\therefore$ The order of filter is 4.

Q.N.3) What are the characteristics of elliptic response? Compare it with that of Inverse Chebyshev response.

Solution

⇒ The characteristics of Elliptical response are:-

i) It is also known as Cauer Approximation

ii) It has sharp cut-off i.e, low transition band

iii) Equal ripple is produced in both the passband and stopband ($\alpha_1 = \alpha_2 = \alpha_{min}$)

iv) The transfer function of low pass elliptical filter is,

$$|T(j\omega)|^2 = \frac{1}{1 + \epsilon^2 R_n^2(\omega, L)}$$

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where, $R_n^2(\omega, L)$ is Chebyshev rational function.

v) Attenuation $(\alpha) = 10 \log [1 + \varepsilon^2 R_n^2(\omega)]$

vi) At the end of pass-band, $\omega_p = 1$ rad/sec

$\therefore R_n(1) = 1$

$\therefore \alpha(1) = \alpha_{max} = 10 \log [1 + \varepsilon^2]$

$\therefore \varepsilon = (10^{\alpha_{max}/10} - 1)^{1/2}$

vii) Stop band attenuation, α_{min} is when $\omega = \omega_s$,

At $\omega = \omega_s$, $R_n(s) = \pm L$

$\therefore \alpha_{min} = 10 \log (1 + \varepsilon^2 L^2)$

$$\therefore L^2 = \frac{10^{\alpha_{min}/10} - 1}{10^{\alpha_{max}/10} - 1}$$

viii) Poles of $R_n(\omega, L)$ is reciprocal of zeros of $R_n(\omega, L)$, and vice-versa

i.e $\omega_{pv} = \frac{1}{\omega_{zv}}$

→ The poles of elliptical response lies on ellipse, where as zeros are at imaginary axis.

Fig:- Elliptical filter response

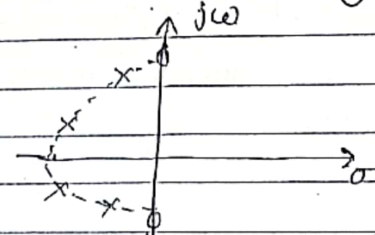

Fig:- Pole-zero plot

Second part

Comparing Elliptic response with inverse chebyshev response,

Elliptic Response	Inverse Chebyshev Response
→ Equal ripple in pass band and stop band.	→ Equal ripple in stop-band, maximally flat in pass band.
→ Order of filter is least among Butterworth, Chebyshev, Cauer filter.	→ Order of filter is greater than in Cauer filter.
→ Poles lies on ellipse whose zeros lies on imaginary axis.	→ Poles and zeros lies on ellipse. Centred at Origin.
→ $\lvert T(S) \rvert^2 = \frac{1}{1 + \varepsilon^2 R_n^2(\omega, L)}$	→ $\lvert T(S) \rvert^2 = \frac{\varepsilon^2 C_n^2(1/\omega)}{1 + \varepsilon^2 C_n^2(1/\omega)}$

Q.N.4) How can you obtain a band stop filter from given lowpass filter? Explain with a suitable example.

Solution

→ By replacing all 's' by $\frac{Bs}{s^2 + \omega_0^2}$, we can transform a low pass filter into band stop filter.

i) Resistor; $Z_R = R$

∴ $Z_{R,new} = R$ (no change)

ii) Inductor, $Z_L = Ls$

∴ $Z_{L,new} = L \cdot \frac{Bs}{s^2+\omega_0^2}$

∴ $Y_{L,new} = \frac{s^2+\omega_0^2}{LBs} = \frac{s}{LB} + \frac{\omega_0^2}{LBs}$

Inductor is replaced by parallel combination of capacitor ($C_{new} = 1/LB$) and inductor ($L_{new} = LB/\omega_0^2$).

iii) Capacitor, $Z_C = \frac{1}{Cs}$

$$\therefore Z_{C,new} = \frac{1}{C \cdot \frac{Bs}{s^2+\omega_0^2}} = \frac{s^2+\omega_0^2}{CBs}$$
$$= \frac{s}{CB} + \frac{\omega_0^2}{CBs}$$

Capacitor is replaced by series combination of inductor ($\frac{1}{CB}$) and Capacitor ($\frac{CB}{\omega_0^2}$).

Example: Transform low pass filter into band stop filter. Given $\beta = 400$ rad/s, $\omega_0 = 2000$ rad/s.

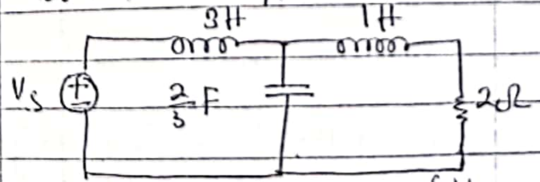

Fig:- low pass filter.

Now, $R(2) = 2\Omega$

$L(3) \equiv C_{new} = \frac{1}{3\times 400} = 833.33\,\mu F$

$L_{new} = \frac{3\times 400}{(2000)^2} = 300\,\mu H$

$L(1) \equiv C_{new} = 1/(1\times 400) = 2.5\,mF$

$L_{new} = 1\times 400/(2000)^2 = 100\,\mu H$

$C(\frac{2}{3}) \equiv L_{new} = \frac{1}{\frac{2}{3}\times 400} = 3.75\,mH$

$C_{new} = (\frac{2}{3}\times 400)/(2000)^2 = 66.67\,\mu F$

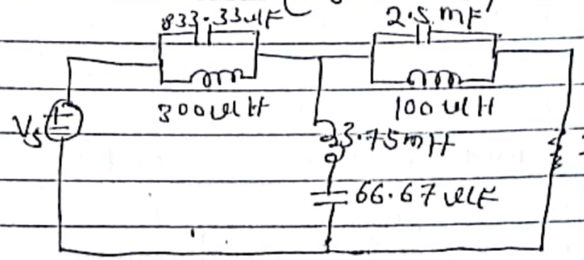

Fig:- Band-stop filter.

Q.N.5) What are the required properties of a function to be realizable? Explain the properties of lossless two port functions.

⇒ The driving properties of a function to be realizable are:-

i) If F(s) is real function, its reciprocal $\frac{1}{F(s)}$ is also real function.
ii) Sum of realizable function is a PRF but the difference may not be PRF.
iii) The poles and zeros of PRF cannot be the right half of s-plane.
iv) The only simple poles with real positive residue can exist on $j\omega$ axis.
v) The poles and zeros of PRF (Positive real function) are real or can occur in conjugate pair.
vi) The highest and lowest power of Numerator & Denominator polynomial may differ at most by Unity.

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Second Part

⇒ The properties of the lossless two port network are as follows,
For the two port network to be lossless the Z-parameters ($z_{11}, z_{12}, z_{21}, z_{22}$) and Y-parameter ($Y_{11}, Y_{12}, Y_{21}, Y_{22}$) must be lossless function.
i.e. $Re[z_{11}(j\omega)] = 0$
where,
z_{11}, z_{12} = Driving Point impedance.
z_{12}, z_{21} = Transfer impedance
Y_{11}, Y_{22} = Driving point impedance
Y_{12}, Y_{21} = Transfer admitance
⇒ Y_{11} and Y_{22} of lossless twoport network must be lossless function.
⇒ The transfer functions Y_{12}, Y_{21} have the properties that slightly different from those of the two lossless functions.

Each Transfer admittance must have the following properties:
i) It must be the ratio of an even to odd polynomials or vice-versa,
ii) All poles must lie on the $j\omega$-axis, they must be simple and their residue must be real. (may be positive and negative.

iii) Zeros may lie anywhere in the s-plane but they must occur symmetrically about both axis, they may be of any order.

iv) Driving point impedance / admittance and their transfer impedance / admittance must be ratio of even to odd or vice-versa.

Q.N6) Which of the following function is valid RC admittance function? State with reason. Realize one of the RC admittance function in Foster II and RC ladder form.

$$Y(s) = \frac{(s+1)(s+3)}{(s+2)(s+4)}, \quad Y_2(s) = \frac{(s^2+1)(s^2+3)}{s(s^2+2)(s^2+4)}$$

$$Y_3(s) = \frac{(s+2)(s+4)}{(s+1)(s+3)}, \quad Y_4(s) = \frac{(s+1)(s+3)}{s(s+2)(s+4)}$$

Solution

$\Rightarrow Y_2(s) = \frac{(s^2+1)(s^2+3)}{s(s^2+2)(s^2+4)}$ is not valid RC admittance function, because poles and zeros must lie on negative real axis,

$\Rightarrow Y_3(s) = \frac{(s+2)(s+4)}{(s+1)(s+3)}$ & $Y_4(s) = \frac{(s+1)(s+3)}{s(s+2)(s+4)}$ also not valid RC admittance function, because nearest to or at origion must be a zero whereas at $\sigma = -\infty$ must be a pole.

$$\Rightarrow Y_1(s) = \frac{(s+1)(s+3)}{(s+2)(s+4)}$$

$$Y(s) = \frac{k_1}{s+2} + \frac{k_2}{s+4} \quad \therefore k_1 = Y(s)\cdot(s+2)\Big|_{s=-2} = -\frac{1}{2}$$

$$k_2 = Y(s)\cdot(s+4)\Big|_{s=-4} = \frac{3}{2}$$

$\therefore Y_1(s)$ satisfied.

$\therefore Y(s) = \frac{(s+1)(s+3)}{(s+2)(s+4)}$ is valid RC admittance function.

a) Realization of $Y(s)$ in Foster-II form:

$$Y(s) = \frac{(s+1)(s+3)}{(s+2)(s+4)}$$

Divide both side by s, we get,

$$\frac{Y(s)}{s} = \frac{(s+1)(s+3)}{s(s+2)(s+4)} = \frac{k_1}{s} + \frac{k_2}{s+2} + \frac{k_3}{s+4}$$

$$\therefore k_1 = \frac{1*3}{2*4} = \frac{3}{8}$$

$$k_2 = \frac{(-1)*(1)}{(-2)*(2)} = \frac{1}{4}$$

$$k_3 = \frac{(-3)*(-1)}{(-4)*(-2)} = \frac{3}{8}$$

Then, $\frac{Y(s)}{s} = \frac{3}{8s} + \frac{1}{4(s+2)} + \frac{3}{8(s+4)}$

$\therefore Y(s) = \frac{3}{8} + \frac{s}{4(s+2)} + \frac{3s}{8(s+4)}$

$= Y_1(s) + Y_2(s) + Y_3(s)$

Now,

$Y_2(s) = \frac{s}{4(s+2)} \Rightarrow Z_2(s) = \frac{4(s+2)}{s} = 4 + \frac{8}{s}$

$Y_3(s) = \frac{3s}{8(s+4)} \Rightarrow Z_3(s) = \frac{8(s+4)}{3s} = \frac{8}{3} + \frac{32}{3s}$

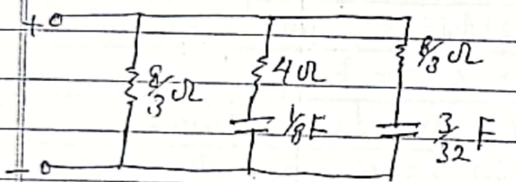


Fig: Foster-II Circuit

b) Realization of Y(s) in RC ladder form:

$Y(s) = \frac{(s+1)(s+3)}{(s+2)(s+4)} = \frac{s^2+4s+3}{s^2+6s+8}$

$\therefore Z(s) = \frac{s^2+6s+8}{s^2+4s+3}$

$s^2+4s+3 \,)\, s^2+6s+8 \,(\, 1 \to R \to Z$

$-s^2 \pm 4s \pm 3$

$2s+5 \,)\, s^2+4s+3 \,(\, \frac{s}{2} \to Y$

$-s^2 \pm \frac{5}{2}s$

$\frac{3s}{2}+3 \,)\, 2s+5 \,(\, \frac{4}{3} \to Z$

$-2s \pm 4$

$1 \,)\, \frac{3s}{2}+3 \,)\, \frac{3s}{2} \to Y$

$-\frac{3s}{2}$

$3 \,)\, 1 \,(\, \frac{1}{3} \to Z$

-1

0

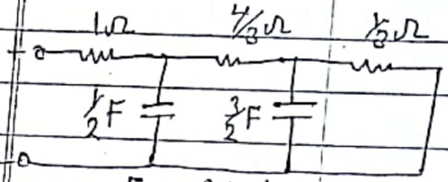
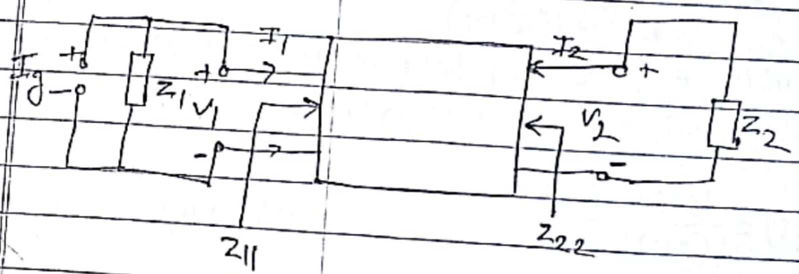


Fig: RC ladder Circuit

Q.N.7) Define transmission and reflection coefficient. Explain how resistively terminated ladder network can be realized with finite transmission zeros.

Solution



Let $z_{11}(s)$ and $z_{22}(s)$ be the impedances looking into the input and output ports when the output and input ports are terminated in $z_2(s)$ and $z_1(s)$ respectively.

The input and output reflection coefficients are defined by,

$$S_1(s) = \frac{z_{11}(s) - z_1(-s)}{z_{11}(s) + z_1(s)}$$

$$S_2(s) = \frac{z_{22}(s) - z_2(-s)}{z_{22}(s) + z_2(s)} \quad \text{respectively.}$$

Now the transducer power Gain (G) defined as the ratio of average power delivered to the load to the maximum available average power at the source is given by,

$$G(\omega^2) = |S_{21}(j\omega)|^2 = 1 - |S_1(j\omega)|^2$$

where S_{21} is known as Transmission coefficient. Also we can write,

$$|S_1(j\omega)|^2 = \text{Reflected Power} / \text{Power available}$$

$$|S_{21}(j\omega)|^2 = \text{Power to load} / \text{Power available.}$$

Second part

Resistively terminated ladder network can be realized with finite-transmission zeros as follows,

1) In Doubly terminated network, first let us consider a normalized value,

i.e $R_1 = R_2 = 1$ ohm.

2) Obtain $|S(j\omega)|$ from given $t(s)$ or $|t(j\omega)|^2$

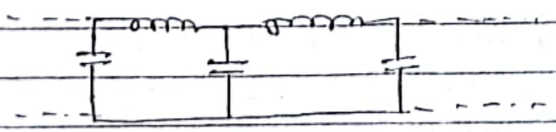
3) Calculate $z_{11}(s)$ Using,

$$z_{11}(s) = R_1 \left[\frac{1 + S(s)}{1 - S(s)}\right] \text{ or } R_1 \left[\frac{1 - S(s)}{1 + S(s)}\right]$$

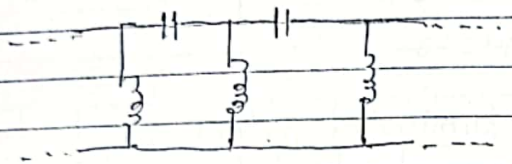
4) Finally realize $z_{11}(s)$ as a lossless two port terminated in the resistance.

Conditions

i) If $t(s)$ has all pole function, then all 'transmission zeros' will be at infinity. Then the configuration will be,



ii) If all "transmission zeros" at origin, The expected lossless two port have general configuration of figure below,



ii) If a pair of transmission zeros is present at some finite value then the expected circuit will be as shown in the figure.

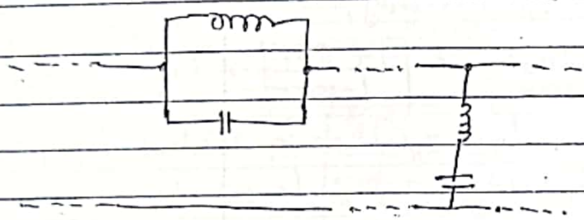


Fig: Two port LC-ladders

(F.N.8) Draw the circuit diagram of Tow Thomas biquad filter and derive its low pass transfer function. Design a second order Butterworth low pass filter having half power frequency of 5KHz using Tow Thomas biquad circuit. Your final circuit should have all capacitors of 0.001uF.

Solution

The ckt below shows the circuit diagram of Tow Thomas biquad filter,

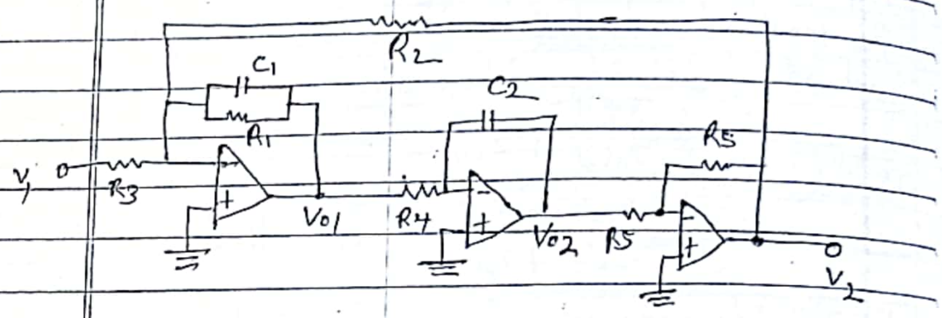


Fig: Tow thomas biquad CKT

Second part

Now, from the above circuit diagram,

$$V_{02} = -\frac{V_{01}}{R_4 C_2 s}$$

$$= +2\left[\frac{V_1}{R_3} + \frac{V_2}{R_2}\right] * \frac{1}{R_4 C_2 s}$$

$$= \frac{+R_1}{(1+R_1C_1 s)} * \frac{1}{R_4 C_2 s}\left[\frac{V_1}{R_3} + \frac{V_2}{R_2}\right]$$

Now, for unit gain follower (Inverter);

$$V_2 = -\left(\frac{R_5}{R_5}\right) V_{02} = -V_{02}$$

$$= \frac{-R_1}{(1+R_1C_1s)} * \frac{1}{R_4C_2s} * \left[\frac{V_1}{R_3} + \frac{V_2}{R_2}\right]$$

$$= \frac{-V_1R_1}{R_3R_4C_2s(1+R_1C_1s)} - \frac{V_2R_1}{R_4R_2C_2s(1+R_1C_1s)}$$

$$\text{or, } V_2\left[1 + \frac{R_1}{R_2R_4C_2s(1+R_1C_1s)}\right] = \frac{-V_1R_1}{R_3R_4C_2s(1+R_1C_1s)}$$

$$\text{or, } \left(\frac{V_2}{V_1}\right) = \frac{-R_1R_2}{R_2R_3R_4C_2s(1+R_1C_1s)+R_1R_3}$$

$$\text{or, } \left(\frac{V_2}{V_1}\right) = \frac{-R_1R_2}{R_1R_2R_3R_4C_1C_2s^2 + R_2R_3R_4 * C_2s + R_1R_3}$$

$$\text{or, } \left(\frac{V_2}{V_1}\right) = \frac{-R_1R_2/(R_1R_2R_3R_4C_1C_2)}{\left[\frac{R_2R_3R_4C_2s(1+R_1C_1s)}{R_1R_2R_3R_4C_2C_1}\right] + \left[\frac{R_1R_3}{R_1R_2R_3R_4C_1C_2}\right]}$$

$$\text{or, } \left(\frac{V_2}{V_1}\right) = \frac{-\left(\frac{1}{R_3R_4C_1C_2}\right)}{\frac{R_1R_2R_3R_4C_1C_2s^2}{R_1R_2R_3R_4C_1C_2} + \frac{R_2R_3R_4C_2s}{R_1R_2R_3R_4C_1C_2} + \frac{R_1R_3}{R_1R_2R_3R_4C_1C_2}}$$

$$\boxed{\therefore T_{LP}(s) = \frac{V_2}{V_1} = \frac{-\frac{1}{R_3R_4C_1C_2}}{s^2 + \left(\frac{1}{R_1C_1}\right)s + \frac{1}{R_2R_4C_1C_2}}}$$

Numerical Part

⇒ The transfer function function of 2nd orders low pass Butterworth filter is,

$$T_2(s) = \frac{1}{s^2 + \sqrt{2}s + 1}$$

Transfer function of low pass Tow – Thomas filter is;

$$T(s) = \frac{-1/R_3R_4C_1C_2}{s^2 + \frac{1}{R_1C_1}s + \frac{1}{R_2R_4C_1C_2}}$$

$$\therefore w_0 = \sqrt{1/R_2R_4C_1C_2} = 1 \text{ rad/sec}$$

$$\frac{w_0}{Q} = \frac{1}{R_1C_1} = \sqrt{2}$$

$$H = \frac{R_2}{R_3} = 1$$

Using Tuning algorithm,

let $C_1 = C_2 = 1F$ and $R_4 = 1\Omega$

$$\therefore w_0 = \sqrt{\frac{1}{R_2}} \Rightarrow R_2 = \frac{1}{w_0^2} = \frac{1}{1} = 1\Omega$$

$$\frac{w_0}{Q} = \frac{1}{R_1} \Rightarrow R_1 = \frac{Q}{w_0} = \frac{\sqrt{2}}{1} = \sqrt{2}\Omega$$

$$H = \frac{1}{R_3} \Rightarrow 1 = \frac{1}{R_3} \Rightarrow R_3 = 1\Omega$$

To achieve $w_0 = 2\pi * 5 * 10^3$ rad/sec, do frequency scaling, $K_f = 10\pi * 10^3$

$\therefore C_1 = C_2 = \frac{1}{10\pi * 10^3} = 31.830988$

To achieve, C_1 and C_2, 0.001 uF, do magnitude scaling, $K_m = \frac{31.830988}{0.001} = 31830$

$\therefore C_1 = C_2 = 0.001\ uF$

$R_1 = \sqrt{2} * 31830 = 45.015\ k\Omega$

$R_2 = 1 * 31830 = 31.83\ k\Omega$

$R_3 = 1 * 31830 = 31.83\ k\Omega$

$R_4 = 1 * 31830 = 31.83\ k\Omega$

Take $R_5 = 1\ k\Omega$

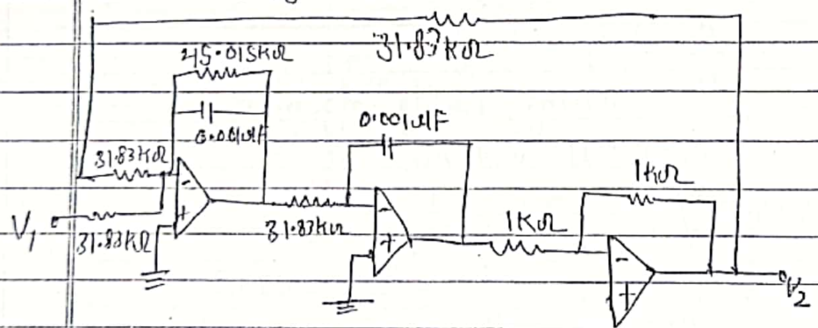


Fig:- Second Order Butterworth filter using Tow-Thomas circuit.

Q.N.9) How gain enhancement can be performed in Sallen and Key Circuit? Explain with necessary diagram.

Solution

The gain enhancement can be performed in a Sallen-key circuit by inserting resistors towards output, so that only fraction of output voltage (V_2) is sent as feedback.

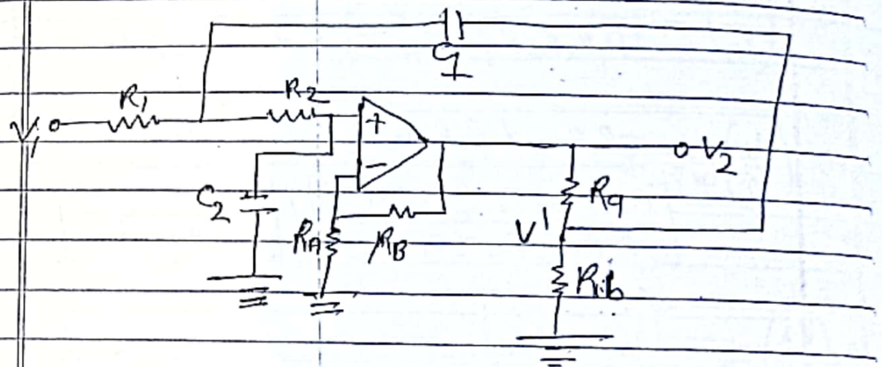


Fig:- Gain Enhancement Sallen-key Circuit

Here,

$$V' = V_2 \cdot \frac{R_b}{R_a + R_b} = V_2 \cdot k$$

$$\therefore T(s) = \frac{K/(R_1R_2C_1C_2)}{s^2+\left(\frac{1}{R_1C_1}+\frac{1}{R_2C_2}+\frac{1}{R_2C_1}-\frac{K\cdot k}{R_2C_2}\right)s+\frac{1}{R_1R_2C_1C_2}}$$

let $R_1 = R_2 = 1\Omega$
$C_1 = C_2 = 1F$

$$\therefore T(s) = \frac{K}{s^2+(3-K\cdot k)s+1}$$

By changing the value of R_a and R_b, we can control the value of k, and hence, we can control the value of gain(K). We can achieve any value of gain K.

f.N.10) What is sensitivity? What is its importance in filter design? perform the sensitivity analysis of quality factor of Tow Thomas biquad lowpass filter.

Solution
→ The case and effect relationship between Network element variation and the resulting change in Network Transfer function is known as 'sensitivity'.

Mathematically, S_x^y represents how sensitive is 'y' with respect to change in value of x and is expressed as,

$$S_x^y = \frac{\%\text{ change in } Y}{\%\text{ change in } x} = \frac{\left(\frac{\Delta y}{y}\right)}{\left(\frac{\Delta x}{x}\right)} = \frac{x}{y}\cdot\frac{dy}{dx}$$

→ gives approximately the ratio of per unit change in y to per unit change in x.

In practice, performance of filter is affected by all element changes. Thus y is function of several x values.

The sensitivity is one of the factor which determines which of the circuit to be used in the filter design.

Second Part

(See the S.N. 10, 2069 Chaitra)

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S.N.11) What is generalized impedance converter (GIC)? How Antoniou's GIC can be used to simulate grounded inductor? Explain with necessary figures and derivations.

Solution:-

(1st part see Q.N.11, 2069 Chaitra)

Second part

→ Antoniou's GIC consists of two op-Amps arranged as shown in the figure below;

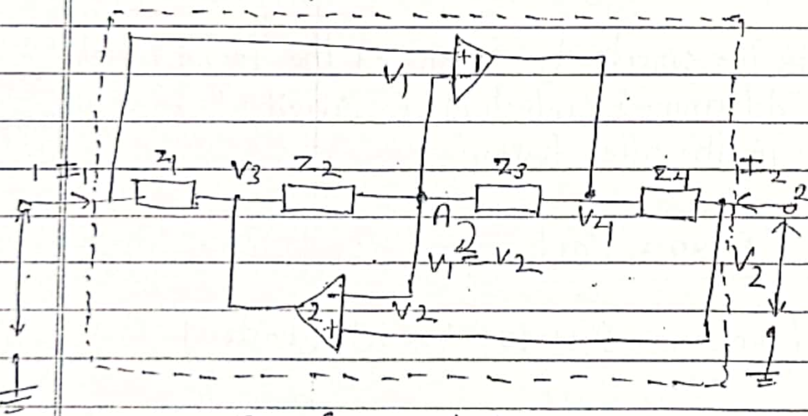


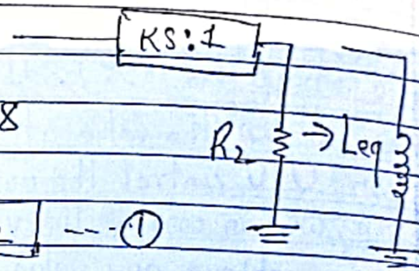
Fig:- General GIC using op-Amp

Simulation of Grounded inductor.

→ Inductor with its one terminal grounded is referred to as grounded inductor

→ If port-2 is terminated with resistance and Gyrator having impedance ratio Ks:1 then,

$Z_{in} = Ks \cdot L$



or, $Z_{in} = L_{eq} \cdot s$

where

$\boxed{L_{eq} = K \cdot R_2}$ ---(1)

Hence the input impedance is equivalent to Grounded inductor.

To realize this in Antoniou's GIC we consider either Z_2 or Z_4 as a capacitor and other as a resistors.

Then from the Antoniou's relation,

$$\frac{Z_{in}}{Z_5} = \frac{1}{f(s)}$$

$$\therefore Z_{in} = Z_{11} = \left(\frac{Z_1 Z_3}{Z_2 Z_4}\right) \cdot Z_5$$

Considering Z_2 as a Capacitor and $Z_5 = Z_2$
then i[Z_1, Z_3, Z_4 as Resistor]

$$Z_{in} = \frac{R_1 R_3 \cdot Z_2}{\left(\frac{1}{C_2 S}\right) \cdot R_4}$$

Also, $Z_1 = R_L$ so,

$$Z_{in} = \frac{R_1 R_3 C_2 * R_L * S}{R_4}$$

$$= \left(\frac{R_1 R_3 R_L C_2}{R_4}\right) \cdot S$$

$$= L_{eq} \cdot S$$

$$= (K R_L) \cdot S$$

where $L_{eq} = K \cdot R_L$ and $\boxed{K = \frac{R_1 R_3 C_2}{R_4}}$

or, $\boxed{K = \frac{R_1 R_3 C_4}{R_2}}$ [When we consider Z_4 as a Capacitor.]

Generally, we Choose, $R_1 = R_2 = R_3 = 1\Omega$
and C_2 or $C_4 = 1$;
thus giving $K = 1$ and $\boxed{L_{eq} = R_L}$

Hence the grounded inductor can be replaced by four resistors, two op-Amps and a Capacitor.

S.N.(2) What is FDNR? Explain how FDNR avoids the use of inductor. Following circuit is a low-pass filter having half power frequency of 1 rad/sec. Obtain a low pass filter having half power frequency of 5 KHz and largest Capacitor of 0.01 uF using FDNR.

Solution

→ FDNR stands for Frequency Dependent Negative Resistor. The concept of FDNR is introduced by Bruton which is used for avoiding inductor.

If we have Z_1, Z_2, Z_3 be resistor and Z_4, Z_0 be capacitor then,

$$Z_{out} = \frac{\left(\frac{1}{C_0 S}\right) * R_2 * \left(\frac{1}{C_4 S}\right)}{R_1 R_3}$$

or, $Z_{out} = \frac{R_2}{(R_1 R_3 C_0 C_4) S^2}$

or, $Z_{out} = \frac{1}{D S^2}$

or, $Z_{out}(j\omega) = -\frac{1}{D\omega^2}$ where, $D = \frac{R_1 R_3 C_0 C_4}{R_2}\ \Omega F^2$

i.e $s = j\omega$

$Z(j\omega)$ is real negative value and is function of frequency, it is called frequency Dependent Negative Resistor (FDNR).

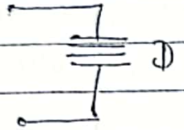


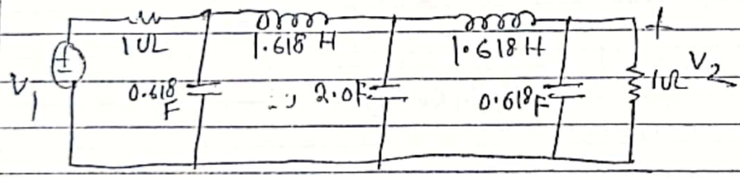
Fig:- Symbol FDNR

This avoids the use of inductor, because, by scaling factor, $K_m = 1/S$

$Z_L = LS$ and $Z_{L\,new} = \left(\frac{LS}{S}\right) = L$ gives resistor of resistance 'L' Ω. and the inductor is changed in to the resistor.

Second part

Given, Circuit diagram,



To achieve low pass filter having half power frequency, $\omega_0 = 2\pi * 5*10^3$ rad/sec

do frequency scaling, $K_f = 10\pi * 10^3$

$\therefore R(1) = 1\Omega$

$L(1.618) = \frac{1.618}{10\pi*10^3} = 51.5\ \mu H$

$C(0.618) = \frac{0.618}{10\pi*10^3} = 19.67\ \mu F$

$C(2) = \frac{2}{10\pi*10^3} = 63.66\ \mu F$

To achieve largest Capacitor of $0.01\ \mu F$

$K_m = \frac{19.67}{0.01} = 1967$

$\therefore C(19.67) = 0.01\ \mu F$

$C(63.66) = 0.03\ \mu F$

$R(1) = 1.967\ k\Omega$

$L(51.5) = 101.3\ mH$

Simulate Using FDNR,

Scale every element by $(1/S)$,

$\therefore$ Inductor is replaced by resistor (L)

Resistor is replaced by capacitor (Value $= 1/R$)

Capacitor is replaced by FDNR (value $= C$)

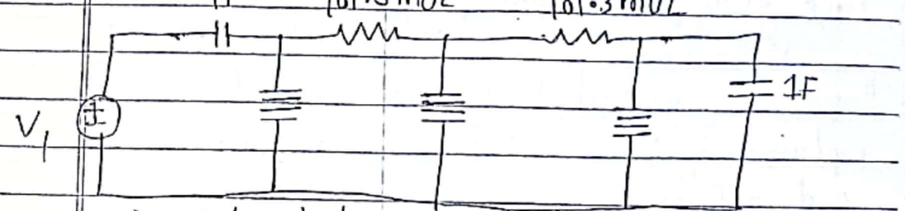


Fig:- Simulation of inductor using FDNR

8.N.13) What is Switched Capacitor filter? What are its applications? Draw the switched Capacitor equivalent circuit for inverting summer, lossy integrator and non-inverting integrator.

⇒ Solution

→ In switched Capacitor filters, a number of capacitors are switched back and forth periodically among a number of terminals which replaces the use of resistors in the circuit.

Switched Capacitor filters have many applications, they are,

i) In MOS Technology, Resistors covers large chip area which doesnot seem relevant. It is easier to fabricate Capacitor and MOS switches.

ii) It allows us to design accurate analog filters at voice-band frequencies economically in fully integrated form.

iii) For reduced power, Compact design and Compatibility with digital system, the filters had to be realized in MOS technology.

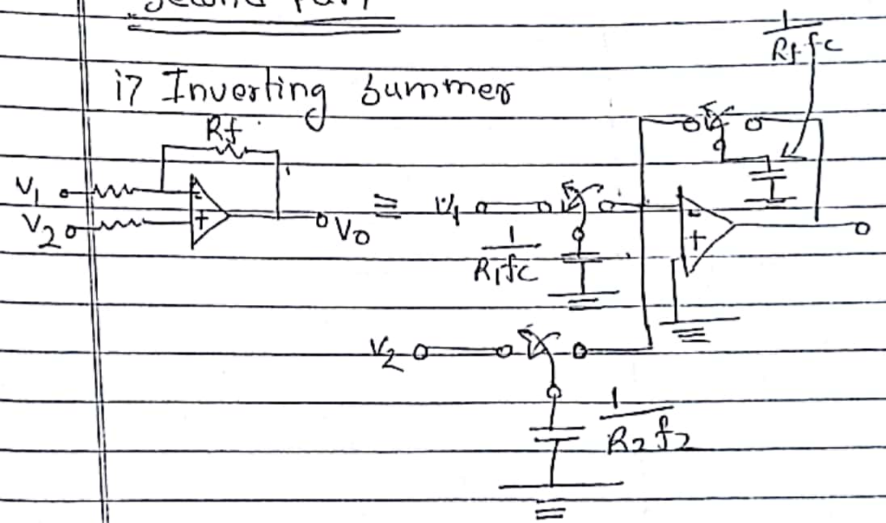
iv) Capacitors vary in the range of 0.1 pF to 100 pF

v) Clock frequency ranges from 100 KHz to 2 MHz.

vi) Best suited to IC implementation of filter with required accuracy.

Second Part

i) Inverting summer



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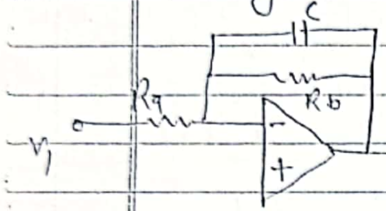
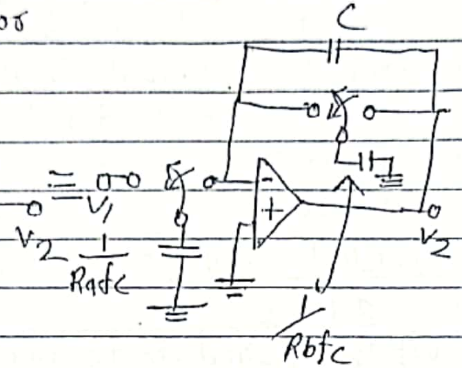
ii) lossy integrator





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1. What is the significance of scaling in filter design? Derive the necessary expression to determine the new values of circuit elements in the case of magnitude and frequency scaling.

Solution

In filter design, we first design a prototype filter, with some standard values of cut-off frequency and the circuit element. But the prototype filter may not be practically realizable. By Scaling we can achieve the filter at our required half power frequency (frequency Scaling) and circuit elements that are easily available in the market (magnitude scaling.

Second part

(See the Q.N.1 Second part 2069 Chaitra).

Q.N.2) Derive an expression to calculate the order of Inverse Chebyshev low pass filter. Use this formula to estimate the order of inverse

Chebyshev low pass filter having following specifications:

$\alpha_{max} = 0.25\ dB$, $\omega_p = 1000$ rad/s

$\alpha_{min} = 18\ dB$, $\omega_s = 1400$ rad/s

Solution

We have, transfer function of Chebyshev filter,

$$|T_{CH}(j\omega)|^2 = \frac{1}{1+\varepsilon^2 C_n^2(\omega)} \quad \text{--- (1)}$$

where, $C_n(\omega) = \begin{cases} \cos(n\cos^{-1}\omega) & \text{for } \omega \le 1 \\ \cosh(n\cosh^{-1}\omega) & \text{for } \omega > 1 \end{cases}$

Now, the Chebyshev High-pass filter TF is

$$|T_{HP-CH}(j\omega)|^2 = 1 - |T_{CH}(j\omega)|^2$$

$$= 1 - \frac{1}{1+\varepsilon^2 C_n^2(\omega)}$$

$$= \frac{\varepsilon^2 C_n^2(\omega)}{1+\varepsilon^2 C_n^2(\omega)} \quad \text{--- (2)}$$

When s is replaced by $1/s$ and ω is replaced by $1/\omega$ of high-pass filter, it will be converted into low pass filter. The resulting filter is Inverse Chebyshev low-pass filter,

$$\therefore |T_{ICH}(j\omega)|^2 = \frac{\varepsilon^2 C_n^2(1/\omega)}{1+\varepsilon^2 C_n^2(1/\omega)} \quad \text{--- (3)}$$

$C_n(1/\omega) = \begin{cases} \cos(n\cos^{-1}(1/\omega)); & \omega \ge 1 \\ \cosh(n\cosh^{-1}(1/\omega)) & \text{for } \omega < 1. \end{cases}$

Now,

Attenuation, $\alpha = -20\log[T_0]$

$$\therefore \alpha = -20\log\left[\frac{\varepsilon^2 C_n^2(1/\omega)}{1+\varepsilon^2 C_n^2(1/\omega)}\right]^{1/2}$$

$$= -10\log\left[\frac{\varepsilon^2 C_n^2(1/\omega)}{1+\varepsilon^2 C_n^2(1/\omega)}\right]$$

$$= 10\log\left[\frac{1+\varepsilon^2 C_n^2(1/\omega)}{\varepsilon^2 C_n^2(1/\omega)}\right] \quad \text{--- (4)}$$

We know, ε is always less than or equal to 1. ($\varepsilon \le 1$; taken initially)

$\therefore$ for α to be minimum, $C_n(1/\omega) = \pm 1$,

$$\therefore \alpha_{min} = 10\log\left[1 + \frac{1}{\varepsilon^2}\right]$$

$$\text{or, } \frac{\alpha_{min}}{10} = \log\left[1 + \frac{1}{\varepsilon^2}\right]$$

$$\therefore \varepsilon = \left[10^{\alpha_{min}/10} - 1\right]^{-1/2} \quad \text{--- (5)}$$

At cut-off frequency ($\omega = \omega_p$), α is maximum.

$$\therefore \alpha_{max} = 10\log\left[1 + \frac{1}{\varepsilon^2 C_n^2(1/\omega_p)}\right]$$

$$\text{or, } \frac{\alpha_{max}}{10} = \log\left[1 + \frac{1}{\varepsilon^2 C_n^2(1/\omega_p)}\right]$$

$$\text{or, } C_n^2\left(\frac{1}{\omega_p}\right) = \frac{\left(10^{\alpha_{max}/10} - 1\right)^{-1}}{\varepsilon^2} \quad \text{--- (6)}$$

from (5) and (6),

$$C_n^2\left(\frac{1}{\omega_p}\right) = \left[\frac{10^{\alpha_{max}/10}-1}{10^{\alpha_{min}/10}-1}\right]^{-1}$$

$$\therefore C_n\left(\frac{1}{\omega_p}\right) = \left[\frac{10^{\alpha_{min}/10}-1}{10^{\alpha_{max}/10}-1}\right]^{1/2}$$

$$\text{or, } \cosh\left(n\cosh^{-1}\left(\frac{1}{\omega_p}\right)\right) = \left[\frac{10^{\alpha_{min}/10}-1}{10^{\alpha_{max}/10}-1}\right]^{1/2}$$

[$\because$ for low pass, $\omega_p < 1$]

$$\text{or, } n\cosh^{-1}\left(\frac{1}{\omega_p}\right) = \cosh^{-1}\left[\frac{10^{\alpha_{min}/10}-1}{10^{\alpha_{max}/10}-1}\right]^{1/2}$$

$$\text{or, } n = \frac{\cosh^{-1}\left(\left(10^{\alpha_{min}/10}-1\right)/\left(10^{\alpha_{max}/10}-1\right)\right)}{\cosh^{-1}\left(\frac{1}{\omega_p}\right)} \quad \text{---(7)}$$

$\therefore$ Hence, eq (7) is the required eq^n of order 'n' of low pass inverse chebyshev filter.

Second Part

Now, the order of the inverse Chebyshev low pass filter is given by,

$$n = \frac{\cosh^{-1}\left(\left(10^{\alpha_{min}/10}-1\right)/\left(10^{\alpha_{max}/10}-1\right)\right)}{\cosh^{-1}\left(\omega_s/\omega_p\right)}$$

$$\text{or, } n = \frac{\cosh^{-1}\left(\left(10^{18/10}-1\right)/\left(10^{0.25/10}-1\right)\right)}{\cosh^{-1}\left(1400/1000\right)}$$

$$= 3.1806$$

Hence, the order of Inverse chebyshev low pass filter is equal to 4.

S.N.3) Explain the importance of all pass filters in delay equalization. Find the transfer function of fourth order Bessel-thomson low pass filter.

Solution

⇒ A filter is often said to be all pass, if the amplitude response is any non-zero constant, while the phase response can be arbitrary. The importance of all pass filter in delay equalization is that, it is used (in series) in addition with an uncompensated filter to achieve constant group delay.

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Second part

The transfer function of 4th order Bessel-Thomson low pass filter:

$$T_4(S) = \frac{a_0}{s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}$$

$$= \frac{1}{\cosh s + \sinh s}$$

from storch Principle,

$$\coth s = \frac{1}{s} + \cfrac{1}{\cfrac{3}{s} + \cfrac{1}{\cfrac{5}{s} + \cfrac{1}{\cfrac{7}{s}}}}$$

$$= \frac{1}{s} + \cfrac{1}{\cfrac{3}{s} + \cfrac{1}{\left(\cfrac{5}{s} + \cfrac{s}{7}\right)}}$$

$$= \frac{1}{s} + \cfrac{1}{\cfrac{3}{s} + \cfrac{7s}{s^2+35}}$$

$$= \frac{1}{s} + \frac{s^3 + 35s}{3s^2 + 105 + 7s^2}$$

$$= \frac{3s^2 + 105 + 7s^2 + s^4 + 35s^2}{3s^3 + 105 + 7s^3}$$

$$= \frac{s^4 + 45s^2 + 105}{10s^3 + 105s} = \frac{\cosh s}{\sinh s}$$

$$\therefore \cosh s = s^4 + 45s^2 + 105$$

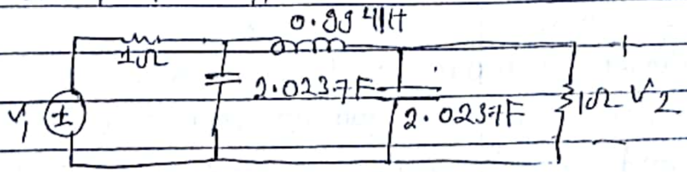
$$\sinh s = 10s^3 + 105s$$

$$\therefore T_4(S) = \frac{1}{s^4 + 10s^3 + 45s^2 + 105s + 105}$$

i.e

$$\boxed{T_4(S) = \frac{105}{s^4 + 10s^3 + 45s^2 + 105s + 105}}$$

Q.N.4) What is the importance of frequency transformation in the filter design? The circuit given in figure below is a lowpass filter having passband frequency of 1 rad/s. Obtain a bandpass filter having $\omega_0 = 2000$ rad/s and $\beta = 400$ rad/s.



Solution

⇒ Frequency transformation is the method used to transform prototype filter in to any other types of filter. In filter design, we first develop a prototype filter with cut-off frequency equals to 1 rad/sec. and then use apply the frequency transformation technique to achieve the required high pass, band pass or band stop filter at required cut-off frequency and bandwidth.

Second Part

To obtain band pass filter from prototype filter replace 's' by $\frac{s^2+\omega_0^2}{Bs}$

i) Resistor (R) : $Z_R = R$

$\therefore Z_{R,new} = R$ (no change)

ii) Inductor, (L) : $Z_L = Ls$

$$Z_{L,new} = L\left(\frac{s^2+\omega_0^2}{Bs}\right)$$

$$= \frac{Ls}{B} + \frac{L\omega_0^2}{Bs}$$

Inductor of inductance L is replaced by the series combination of inductor $\left(L_{new}=\frac{L}{B}\right)$ and capacitor, $\left(C_{new}=\frac{B}{L\omega_0^2}\right)$

iii) Capacitor (C) : $Z_C = \frac{1}{Cs}$

$$\therefore Z_{C,new} = \frac{1}{C\left(\frac{s^2+\omega_0^2}{Bs}\right)} = \frac{Bs}{Cs^2+C\omega_0^2}$$

$$\therefore Y_{C,new} = \frac{Cs^2+C\omega_0^2}{Bs} = \frac{C}{B}s + \frac{C\omega_0^2}{Bs}$$

Capacitor is replaced by parallel combination of capacitor $\left(C_{new}=\frac{C}{B}\right)$ and inductor $\left(L_{new}=\frac{B}{C\omega_0^2}\right)$

$$\therefore L(0.9941) \equiv L_{new} = \frac{0.99411}{400} = 2.485\ mH$$

$$C_{new} = \frac{400}{0.9941 * (2000)^2} = 100\mu F$$

$$\text{And, } C(2.0237) \equiv C_{new} = \frac{2.0237}{400} = 5.059\ mF$$

$$L_{new} = \frac{400}{2.0237 * (2000)^2} = 49.41\mu H$$

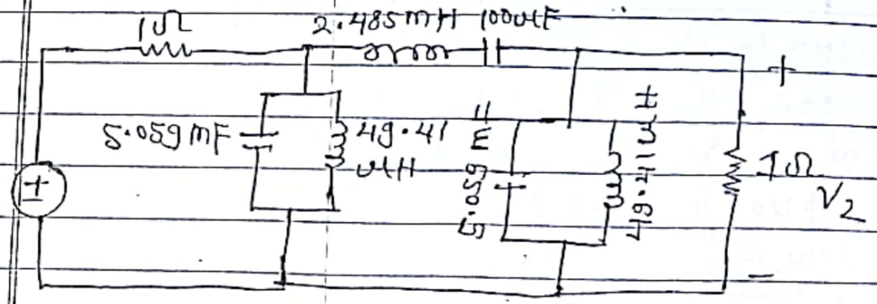


Fig:- Band Pass filter.

Q.5) Which of the following function are LC driving point impedance function and why?

$$z(s) = \frac{2s(s^2+4)(s^2+16)}{(s^2+1)(s^2+9)}$$

$$z(s) = \frac{4(s+2)(s+5)}{(s+1)(s+4)}$$

Solution

$$z_1(s) = \frac{4(s+2)(s+5)}{(s+1)(s+4)}$$, is not LC driving point impedance function, because poles and zeros aren't lie on imaginary axis, and the ratio of numerator and denominator are even/even.

$$z_2(s) = \frac{2s(s^2+4)(s^2+16)}{(s^2+1)(s^2+9)}$$; is LC driving point impedance function, because it satisfies all the required condition.

Second part,

a) Foster series realization,

$$z(s) = \frac{2s(s^2+4)(s^2+16)}{(s^2+1)(s^2+9)} = \frac{2s^5+40s^3+128s}{s^4+10s^2+9}$$

$$s^4+10s^2+9 \,)\, 2s^5+40s^3+128s \,(\, 2s$$
$$\underline{2s^5+20s^3+18s}$$
$$20s^3+110s$$

$$\therefore z(s) = 2s + \frac{20s^3+110s}{s^4+10s^2+9} = z_1(s) + z_2(s)$$

Now,

$$z_2(s) = \frac{20s^3+110s}{(s^2+1)(s^2+9)} = \frac{k_1 s}{(s^2+1)} + \frac{k_2 s}{s^2+9}$$

$$\therefore k_1 = z_2(s)\cdot\frac{(s^2+1)}{s}\Big|_{s^2=-1} = \frac{45}{4}$$

$$k_2 = z_2(s)\frac{(s^2+9)}{s}\Big|_{s^2=-9} = \frac{35}{4}$$

$$\therefore z_2(s) = \frac{45}{4}\cdot\frac{s}{s^2+1} + \frac{35}{4}\cdot\frac{s}{s^2+9}$$
$$= z_{21}(s) + z_{22}(s),$$

Now,

$$z_{21}(s) = \frac{45}{4}\cdot\frac{s}{s^2+1} \Rightarrow Y_{21}(s) = \frac{4}{45}\cdot\frac{s^2+1}{s}$$
$$= \frac{4}{45}s + \frac{4}{45s}$$

$$z_{22}(s) = \frac{35}{4}\cdot\frac{s}{s^2+9} \Rightarrow Y_{22}(s) = \frac{4}{35}\cdot\frac{s^2+9}{s}$$
$$= \frac{4}{35}s + \frac{36}{35s}$$

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∴ The foster series circuit is,

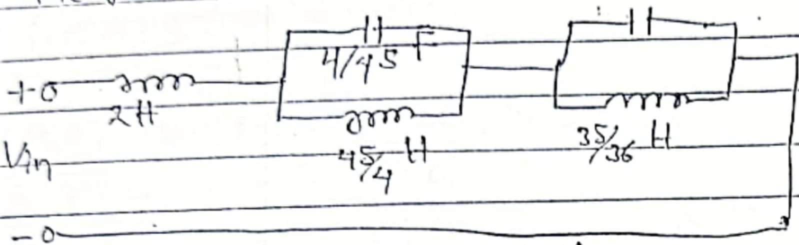

Fig:- Foster series circuit

b) Cauer-II realization:-

$$z(S) = \frac{128s + 40s^3 + 2s^5}{9 + 10s^2 + s^4}$$

looking in to first term of both numerator and denominator, numerator polynomial (only first term) is greater than denominator.

$$\therefore Y(s) = \frac{9 + 10s^2 + s^4}{128s + 40s^3 + 2s^5}$$

$\therefore 128s + 40s^3 + 2s^5 \,)\, 9 + 10s^2 + s^4 \,(\frac{9}{128s} \to Y$

$\quad -9 \; - \frac{45}{16}s^2 - \frac{9}{64}s^4$

$\quad \frac{115}{16}s^2 + \frac{55}{64}s^4$

$\frac{115}{16}s^2 + \frac{55}{64}s^4 \,)\, 128s + 40s^3 + 2s^5 \,(\frac{2048}{115s} \to Z$

$\quad -128s - \frac{352}{23}s^3$

$\quad \frac{568}{23}s^3 + 2s^5 \,)\, \frac{115}{16}s^2 + \frac{55}{64}s^4 \,(\frac{2645}{9088} \cdot \frac{1}{s} \to Y$

$\quad -\frac{115}{16}s^2 - \frac{2645}{4544}s^4$

$\quad \frac{315}{1136}s^4 \,)\, \frac{568}{23}s^3 + 2s^5 \,(\frac{89.06}{s} \to Z$

$\quad -\frac{568}{23}s^3$

$\quad 2s^5 \,)\, \frac{315}{1136}s^4 \,(\frac{315}{2272s} \to Y$

$\quad -\frac{315}{1136}s^4$

$\quad 0$

∴ The Circuit is,

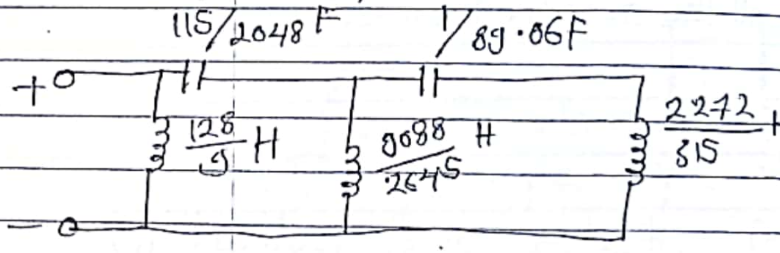

Fig:- Cauer-II circuit

Q.N.6) What is transmission zeros? Explain "zero shifting by partial removal of pole" with example.

Solution

⇒ In two port network, for input there exists an output. When zero output occurs for finite input, the network is said to have 'Zero-Transmission'.

Second Part

(See the Q.N.6 of 2069 Chaitra, first part).

Q.N.7) What is transmission coefficient? What information do we get from it? Derive the expression for reflection coefficient for a resistively terminated RC ladder Circuit

Solution

First Part

(See the Q.N.7 (first part) 2069 Chaitra)

Second part

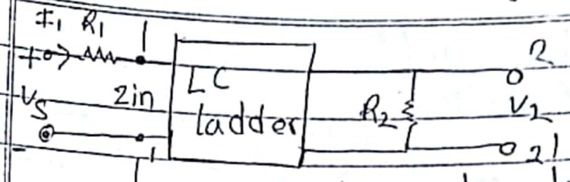

Fig:- Two port network.

We know,

$$|\rho(j\omega)|^2 = 1 - |H(j\omega)|^2 \quad \text{--- (1)}$$

Also,

$\rho(s) \rightarrow$ Reflection Coefficient

$H(s) \rightarrow$ Transmission Coefficient

But,

$$|H(j\omega)|^2 = |T(j\omega)|^2 \cdot \frac{4R_1}{R_2} \quad \text{--- (2)}$$

And,

$$|T(j\omega)|^2 = \frac{R_2 \cdot Rin(\omega)}{|R_1 + Zin|^2} \quad \text{--- (3)}$$

∴ from (1), (2) & (3),

$$|\rho(j\omega)|^2 = 1 - \frac{R_2 \cdot Zin}{|R_1 + Zin|^2} \cdot \frac{4R_1}{R_2}$$

$$= \frac{|R_1 + Zin|^2 - 4R_1 Zin}{|R_1 + Zin|^2}$$

$$= \left| \frac{R_1 - Z_{in}}{R_1 + Z_{in}} \right|^2$$

$$\therefore \rho(s) = \pm \left(\frac{R_1 - Z_{in}}{R_1 + Z_{in}} \right) \quad \text{--- (4)}$$

Hence, eqⁿ (4) is the expression for reflection coefficient for a resistively terminate LC ladder circuit.

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Q.N.8) Realize a System using inverting op-amp configuration with zero at $s = -2$ and pole at $s = -5$ and having high frequency gain of 2.

Solution

From given,

zero at $s = -2$

poles at $s = -5$

and Gain(H) = 2,

Then, the transfer function of an Inverting op-amp, is given by,

$$T(s) = \frac{2(s+2)}{(s+5)}$$

or, $T(s) = \frac{Z_2}{Z_1} = \frac{2s+4}{s+5} = \frac{2 + 4/s}{1 + 5/s}$

$\therefore Z_2 = 2 + 4/s$

$Z_1 = 1 + 5/s$

Then the Circuit is drawn as,

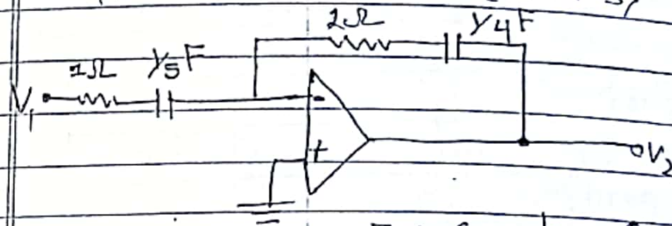


Fig:- Inverting Op-amp.

Q.N.9) Perform Sensitivity analysis for Center frequency (ω_0) and quality factor (Q) of the Tow Thomas low pass filter with respect to all the resistors and Capacitors present in the circuit.

Solution

(See Q.N.10 of 2069 Chaitra, Second part)

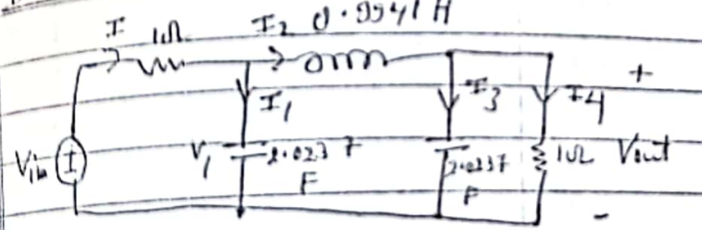
Q. What is frequency Dependent Negative Resistor? How can it be used to avoid bulky inductor in the design of your circuits? Explain with suitable examples.

Solution

(See Q.N. 12, first part and example with second part which removes the inductor in the circuit.)

Q. Using leapfrog method simulate the LC ladder circuit given in question number 4 to obtain a low pass filter having pass band of 6KHz and suitable element values.

Solution



$$I = \frac{V_i - V_1}{R}$$

$$V_1 = I_1 \cdot Z_1 = (I - I_2)\cdot Z_1$$

$$V_3 = I_3 \cdot Z_3 = (I_2 - I_4)\cdot Z_3$$

$$V_I = \frac{(V_i - V_1)}{R_1}$$

$$-V_1 = (V_I - V_{I_2})(-T_{z1})$$

$$-V_{I_2} = (-V_1 + V_3)\cdot(T_{y2})$$

$$V_3 = (-V_{I_2})\cdot(-T_{z3})$$

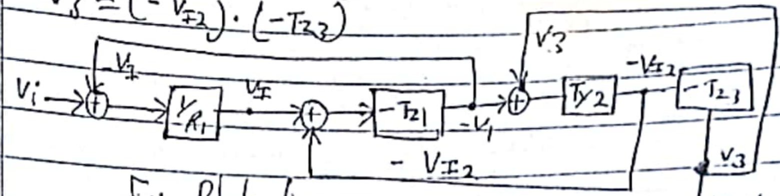


Fig:- Block diagram

$$-T_{z1} = \frac{-1}{C_1 s} \Rightarrow \text{Integrator,} \quad T(s) = \frac{-1}{RCs}$$

$$\therefore C_1 = RC \Rightarrow C = \frac{C_1}{R}$$

let $\boxed{R = 1}$, $\therefore \boxed{C = 2.0237}$

$$T_{y2} = \frac{1}{L_2 s} \Rightarrow \text{Integrator + Unit gain amplifier}$$

$$T(s) = \frac{-1}{RCs} \times (-1)$$

$$\therefore RC = L_2 \Rightarrow C = \frac{L_2}{R}$$

let $\boxed{R = 1}$ $\therefore \boxed{C = 0.9941}$

$$-T_{z3} = \frac{-1}{C_3 s + \frac{1}{R_2}} = \frac{-1/C_3}{s + \frac{1}{R_2 C_3}}$$

⇒ Lossy Integrator,

$$T(s) = \frac{-\,{}^{1}/_{R_a C}}{s + {}^{1}/_{R_b \cdot C}}$$

$\therefore R_a \cdot C = C_3$

let $\boxed{C=1}$, $\therefore R_a = C_3$

i.e. $R_a = 2.0237$

$R_b \cdot C = R_2 R_3$

$\therefore R_b = R_2 \cdot C_3 = 1 * 2.0237 = 2.0237 = R_L$

To achieve $\omega_o = 2\pi * 6 * 10^3$ rad/s and suitable element value i.e. scaling,

$K_f = 12\pi * 10^3$, $K_m = 10000$

$\therefore R_1 = 10\,k\Omega$, $R_a = 20.237\,k\Omega$

$R_2 = 10\,k\Omega$ $\quad R_b = 20.237\,k\Omega$

$R = 10\,k\Omega$

$$C(-T_{z1}) = \frac{2.0237}{12\pi * 10^3 * 10 * 10^3} = 5.368\,nF$$

$$C(T_{y2}) = \frac{0.9941}{12\pi * 10^3 * 10 * 10^3} = 2.6369\,nF$$

$$C(-T_{z3}) = \frac{1}{12\pi * 10^3 * 10 * 10^3} = 2.65\,nF$$

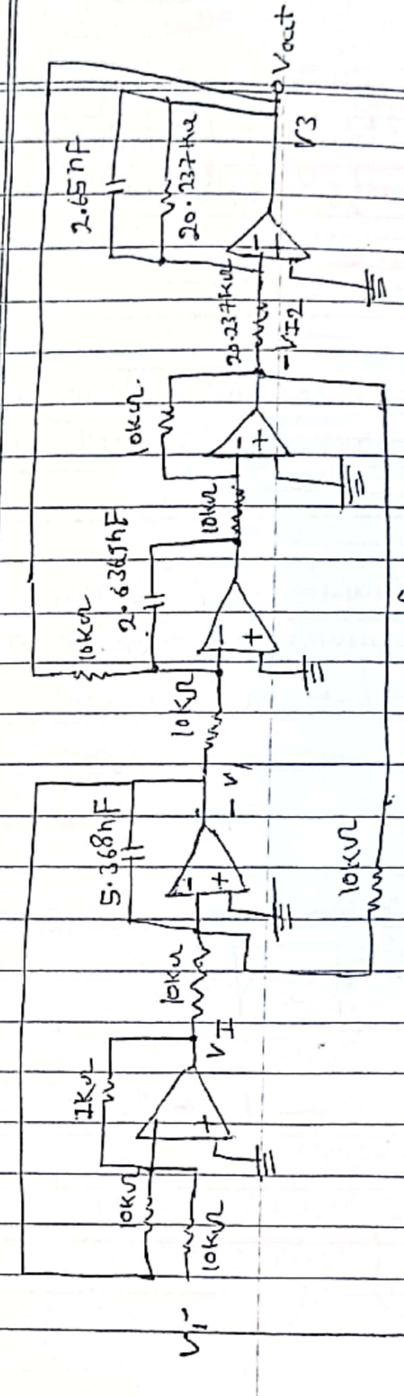


Fig: Leapfrog Simulation.

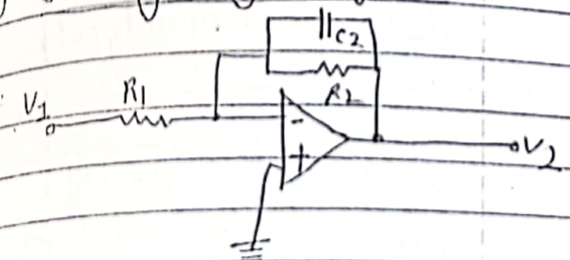
(iii) What is switched capacitor filter? How inverting lossy integrator, integrator and non-inverting integrator can be realized using switched capacitor? Explain with necessary diagram and transfer functions.

⇒ Solution

(First part, see Q.N. 12 of 2069 Chaitra)

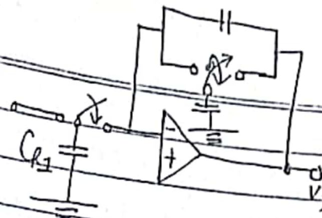
Second Part

i) Inverting lossy integrator:



$$\therefore \left(\frac{V_2}{V_1}\right) = -\frac{1/(C_2 s + 1/R_3)}{R_1} = -\frac{1}{R_1(C_2 s + 1/R_3)}$$

Here both the resistors are replaced by the switched capacitor then,

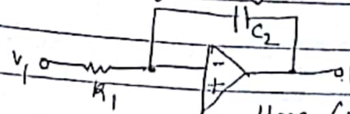


Here, $R_{eq1} = \frac{1}{f_c \cdot C_{R1}}$

$R_{eq2} = \frac{1}{f_c \cdot C_{R2}}$

$$\therefore \boxed{\left(\frac{V_2}{V_1}\right) = \frac{-f_c \cdot C_{R1}}{[C_2 s + f_c \cdot C_{R2}]}}$$

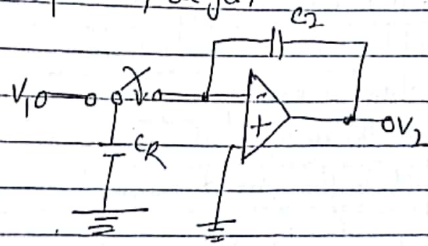
ii) Inverting integrator



Here, $\left(\frac{V_2}{V_1}\right) = -\frac{(1/C_2 s)}{R_1}$

$$\therefore \boxed{\frac{V_2}{V_1} = \frac{-1}{R_1 C_2 s}}$$

Replacing the input resistor by switched capacitor we get,



Here, $R_{eq} = \frac{1}{f_c \cdot C_R}$

$$\text{So, } \boxed{\left(\frac{V_2}{V_1}\right) = -\frac{f_c \cdot C_R}{C_2 \cdot s}}$$

iii) Non-Inverting Integrator

⇒ In continious time operations, it is comparatively difficult to realize a non-inverting integrator. A non-inverting integrator is obtained simply by cascading an inverter and an inverting integrator. Also in switched capacitor circuits, Negative valued resistors are available so that non-inverting integrator is easy to Realize.

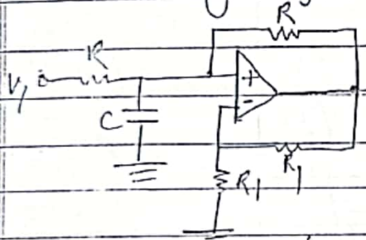


fig:- Non-inverting integrator

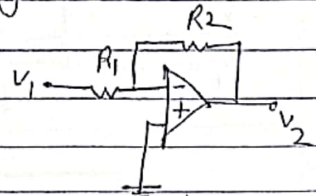


Fig:- inverting Voltage amplifier

In switched-Capacitor Circuits, negative valued Resistors are available. So we simply replace the input resistor of an inverting amplifier R_1 of the above figure (In the right) with the negative resistor. The result is the circuit shown below,

$$\left(\frac{V_2}{V_1}\right) = -\left(\frac{R_2}{R_1}\right)$$ for inverting voltage amplifier

When we replace by switched capacitor then,

$$Req\,1 = \frac{1}{f_c \cdot C_{R_1}} \quad \text{and} \quad Req_2 = \frac{1}{f_c \cdot C_{R_2}}$$

$$\text{So, } \left(\frac{V_2}{V_1}\right) = -\frac{R_2}{R_1} = \frac{Req_2}{Req\,1} = \frac{C_{R1}}{C_{R2}}$$

and the output of the circuit becomes,

$$\boxed{\frac{V_2}{V_1} = \frac{f_c \cdot C_{R1}}{C} \cdot \frac{1}{S}}$$

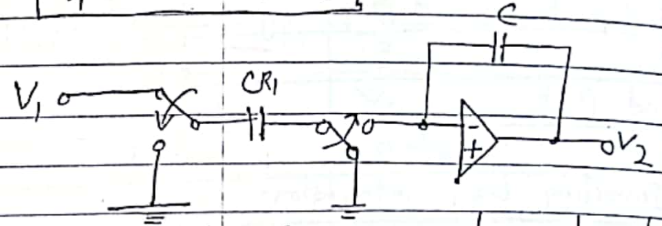


Fig:- A non-inverting integrator

Q.N.B) Draw a neat and clean circuit diagram of Tow-Thomas Low pass Biquad filter and derive it's transfer function. Design a low pass filter using Tow-Thomas Biquad circuit which has poles at $1000 \pm 8994.03j$ and DC gain of 1.89. Use 0.01 μF Capacitor in your design.

Solution

First part

(See the Q.N.8 (first part) 2071 Shawan)

Second Part

$\Rightarrow$ The transfer function is,

$$T(s) = \frac{1}{(s+\alpha+j\beta)(s+\alpha-j\beta)} = \frac{-1/R_3R_4C_1C_2}{s^2+\frac{s}{R_1C_1}+\frac{1}{R_2R_4C_1C_2}}$$

$$= \frac{1}{s^2+2\alpha s+(\alpha^2+\beta^2)}$$

Here, $\alpha = -1000$, $\beta = \pm 8994.03j$

$\therefore \omega_0^2 = \alpha^2+\beta^2 \Rightarrow \omega_0 = \sqrt{\alpha^2+\beta^2} = 9049.45 \text{ rad/s}$

$\frac{\omega_0}{g} = 2\alpha \Rightarrow g = \frac{\omega_0}{2\alpha} = 4.5247$

$H = 1.39$

Using Tuning algorithm at normalized frequency,

$\omega_0 = 1$ rad/sec

let $C_1 = C_2 = 1F$ and $R_4 = 1\Omega$

$\therefore \omega_0 = \sqrt{\frac{1}{R_2R_4C_1C_2}} \Rightarrow \omega_0 = \sqrt{\frac{1}{R_2}} \Rightarrow R_2 = \frac{1}{\omega_0^2}$

$\boxed{\therefore R_2 = 1\Omega}$

$\frac{\omega_0}{g} = \frac{1}{R_1C_1} \Rightarrow \frac{\omega_0}{g} = \frac{1}{R_1} \Rightarrow R_1 = \frac{g}{\omega_0}$

$\boxed{\therefore R_1 = 4.5247\Omega}$

To get $\omega_0 = 9049.45$ rad/s, do frequency Scaling, $k_f = 9049.45$

$\therefore R_1 = 4.5247\Omega$, $R_2 = 1\Omega$, $R_3 = 0.5291\Omega$

$C_1 = C_2 = \frac{1}{9049.45} = 110.5\Omega$

To get capacitor value 0.01 uF, do magnitude Scaling, $k_m = \frac{110.5}{0.01} = 11050$

$\therefore R_1 = 5.6k\Omega$, $R_2 = 11k\Omega$, $R_3 = 6k\Omega$

$R_4 = 11k\Omega$

$C_1 = C_2 = 0.01 \mu F$

let $R_5 = 1k\Omega$.

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1) Define α_{max}, α_{min} and half power bandwidth with necessary diagram. At frequency $f = 20kHz$ and $f = 30kHz$ a filter is designed to attenuate the input signal by 78 dB and 90 dB respectively. Find the amplitude of the output signal if the 30 kHz input signal has amplitude of 1V.

Solution:-

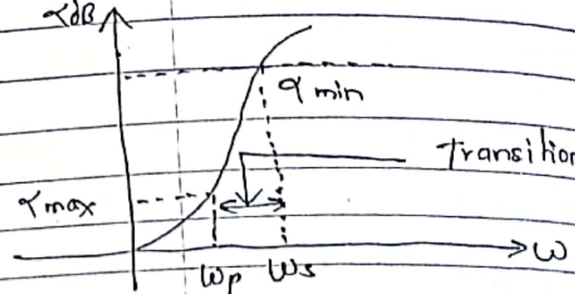
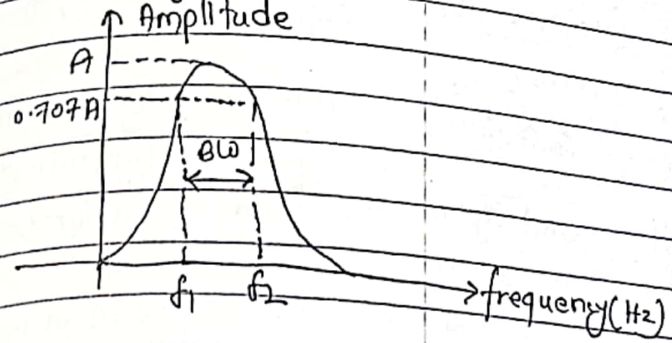


Fig:- Attenuation Curve for LPF

α_{max} is the maximum value of attenuation present in the pass band. It's value is maximum at ω_p frequency. Attenuation less than α_{max} is desirable in pass band.

α_{min} is the minimum possible value of attenuation available in the stop band of filter. It is present at ω_s in stopband. Attenuation greater than α_{min} is desirable.

The two edges of transition band gives α_{max} and α_{min} respectively.



Half power bandwidth is the range of frequencies where output power becomes half of the input power i.e $\frac{P_{out}}{P_{in}} = \frac{1}{2}$, In this range, output voltage/Current amplitude is larger than 0.707 times of its maximum magnitude.

Second part

We know,

$$\text{Attenuation}(\alpha) = -20\log\left(\frac{V_2}{V_1}\right)$$

for 30 kHz input signal, $\alpha = 90$ dB and and $V_1 = 1V$ then,

$$90 = -20\log\left(\frac{V_2}{V_1}\right)$$

$$\text{or, } V_2 = 10^{-90/20} = 3.162\times10^{-5} V$$

Now, for 20 kHz frequency, the V_2 of 30 kHz input signal becomes V_1 and attenuation is given as 78 dB then,

$$78 = 20\log\left(\frac{V_1}{10^{-90/20}}\right)$$

$$\therefore V_1 = 10^{78/20} \times 10^{-90/20}$$

$$\text{or, } V_1 = 0.251 V$$

$\therefore$ Amplitude of output signal is 0.251 V.

OR Alternatively,

$$90 = 20\log\left(\frac{1}{V_2}\right) \quad \text{.... (i)}$$

$$78 = 20\log\left(\frac{V_1}{V_2}\right) \quad \text{.... (ii)}$$

Subtracting eqn (ii) from eqn (i) gives, $V_1 = 0.251V$.

S.N.2) Derive an expression to calculate the order of Chebyshev low pass filter. Use this formula to estimate the order of Chebyshev low pass filter having following specification.

$\alpha_{max} = 0.1$ dB , $\omega_p = 1000$ rad/s

$\alpha_{min} = 20$ dB , $\omega_s = 2500$ rad/s

Solution:

First part (See S.N.2 (first part) 2071 Shawan)

Second part, according to the given Specification,

$$\alpha_{max} = 0.1 \, dB \quad \omega_p = 1000 \text{ rad/s}$$
$$\alpha_{min} = 20 \, dB \quad \omega_s = 2500 \text{ rad/s}$$

Now,

$$n = \frac{\cosh^{-1}\left[\left(10^{20/10} - 1\right)/\left(10^{0.1/10} - 1\right)\right]^{1/2}}{\cosh^{-1}(2500/1000)}$$

$$= 3.1085$$

∴ The required order is 4.

- 92 - (N.3) What is constant delay filter? What are the steps involved in designing constant delay filter? Explain with necessary example.

Solution

First part (see first part (Q.N.3) of 2069 chaitra).

Second Part

General steps involved in designing constant delay filter are:

i) Assume a form of transfer function.

ii) Compute the corresponding delay, $D = -\frac{d\theta}{d\omega}$ where,

$$\theta = \tan^{-1}\left(\frac{\text{Imaginary part of denominator}}{\text{real part of denominator}}\right)$$

iii) Expand D in the form of Taylor Series about $\omega = 0$.

iv) Find the conditions that will cause as many as coefficient in the series expansion to vanish as possible.

Example: let all pole second order transfer function is,

$$T_2(s) = \frac{a_0}{s^2 + a_1 s + a_0}$$

or, $$T_2(j\omega) = \frac{a_0}{-\omega^2 + a_1 j\omega + a_0}$$

$$\therefore \theta = \tan^{-1}\left(\frac{a_1\omega}{a_0 - \omega^2}\right)$$

Hence,

$$D = -\frac{d\theta}{d\omega} = \frac{d}{d\omega}\left[\tan^{-1}\left(\frac{a_1\omega}{a_0 - \omega^2}\right)\right]$$

$$= \frac{d\left[\tan^{-1}\left(\frac{a_1\omega}{a_0 - \omega^2}\right)\right]}{d\left[\frac{a_1\omega}{a_0 - \omega^2}\right]} * \frac{d\left[\frac{a_1\omega}{a_0 - \omega^2}\right]}{d\omega}$$

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$$\frac{1}{1+\left(\frac{a_1\omega}{a_0-\omega^2}\right)^2} * \frac{(a_0-\omega^2)\frac{d(a_1\omega)}{d\omega} - a_1\omega\frac{d(a_0-\omega^2)}{d\omega}}{(a_0-\omega^2)^2}$$

$$= \frac{(a_0-\omega^2)^2}{(a_0-\omega^2)^2+(a_1\omega)^2} * \left[\frac{a_1(a_0-\omega^2) - a_1\omega*(-2\omega)}{(a_0-\omega^2)^2}\right]$$

$$= \frac{a_1(a_0-\omega^2)+2a_1\omega^2}{(a_0-\omega^2)^2+(a_1\omega)^2} = \frac{a_1a_0+a_1\omega^2}{a_0^2+\omega^4-2a_0\omega^2+a_1^2\omega^2}$$

$$= \frac{a_1(a_0+\omega^2)}{a_0^2+(a_1^2-2a_0)\omega^2+\omega^4}$$

$$= \left(\frac{a_1}{a_0^2}\right) * \frac{a_0\left(1+\frac{\omega^2}{a_0}\right)}{\left[1+\left(\frac{a_1^2-2a_0}{a_0^2}\right)\omega^2+\frac{\omega^4}{a_0^2}\right]}$$

$$= \left(\frac{a_1}{a_0}\right) * \frac{\left(1+\omega^2/a_0\right)}{1+\left(\frac{a_1^2}{a_0^2}-\frac{2}{a_0}\right)\omega^2+\frac{\omega^4}{a_0^2}}$$

Now Using Taylor Series,

$$① = \left(\frac{a_1}{a_0}\right)\left[1+\left(\frac{1}{a_0}-\frac{a_1^2}{a_0^2}+\frac{2}{a_0}\right)\omega^2+\dots\dots\right]$$

For Second term to vanish,

$$3/a_0 = a_1^2/a_0^2$$

$$3a_0 = a_1^2$$

For ① to normalize; set $a_1 = a_0 =$ then,

$\therefore a_1 = a_0 = 3$

Hence, $T_2(s) = \frac{a_0}{s^2+a_1s+a_0} = \frac{3}{s^2+3s+3}$

Q.N.4) What is the Significance of frequency transformation in filter design? How band pass filter can be obtained from prototype low pass filter? Explain with example.

Solution

First part (See Q.N.4 (first part), 2071 Chaitra)

Second part

To obtain band pass filter from prototype low pass filter, replace 's' by $\frac{s^2+\omega_0^2}{Bs}$ and change the elements of filter as,

i) Resistor (R): $Z_R = R$

$\therefore Z_{R,new} = R$ (no Change)

ii) Inductor (L), $Z_L = Ls$

$$\therefore Z_{L,new} = L\left(\frac{s^2+\omega_0^2}{Bs}\right) = \frac{Ls}{B} + \frac{L\omega_0^2}{Bs}$$

Inductor of inductance L is replaced by the series combination of Inductor $\left(L_{new} = \frac{L}{B}\right)$ and Capacitor $\left(C_{new} = \frac{B}{L\omega_0^2}\right)$

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iii) Capacitor 'C': $Z_c = \frac{1}{Cs}$

$$\therefore Z_{c,new} = \frac{1}{C\left((s^2+\omega_0^2)/Bs\right)}$$

$$= \frac{Bs}{Cs^2 + C\omega_0^2}$$

$$\therefore Y_{c,new} = \frac{Cs^2 + C\omega_0^2}{Bs} = \frac{C}{B}s + \frac{C\omega_0^2}{Bs}$$

Capacitor is replaced by parallel combination of Capacitor ($C_{new} = C/B$) and inductor ($L_{new} = B/C\omega_0^2$).

Example:

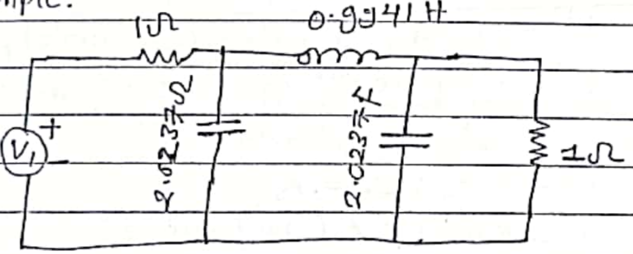


Fig: LPF at $\omega_p = 1$ rad/s

To obtain Band pass filter at $\omega_0 = 2000$ rad/s & $B = 400$ rad/s.

For resistors, no change.

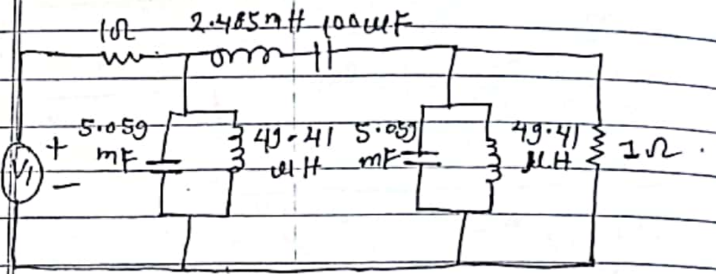
for inductor: $L(0.9941) \equiv L_{new} = \frac{0.9941}{400}$
$= 2.485$ mH

$$C_{new} = \frac{400}{0.9941 * 2000^2} = 100\,\mu F$$

For Capacitor: $C(2.0237) \equiv C_{new} = \frac{2.0237}{400}$
$= 5.059$ mF

$$L_{new} = \frac{400}{2.0237 * 2000^2}$$
$= 49.41\,\mu H$

$\therefore$ Required band pass filter is;



S.N.S) Which of the following functions are LC driving point impedance function and why? Pick one of the valid LC driving point impedance and synthesize it in Foster-I and Cauer-I form,

$$Z_1(s) = \frac{(s^2+1)(s^2+5)}{(s^2+2)(s^2+10)}, \quad Z_2(s) = \frac{5s(s^2+4)}{(s^2+1)(s^2+3)}$$

$$Z_3(s) = \frac{2(s^2+1)(s^2+9)}{s(s^2+4)}, \quad Z_4(s) = \frac{4(s+2)(s+5)}{(s+1)(s+4)}$$

Solution

To be LC driving point impedance function, $z(s)$ must have following properties:

i) $z(s) = \dfrac{\text{even polynomial}}{\text{Odd polynomial}}$ only,

ii) The highest power of numerator and denominator differ at most by unity, so in the case for lowest power as well.

iii) Succeding powers of numerator and denominator polynomial differ by two all the way through.

Only $Z_3(s)$ satisfies all the above conditions. Hence, only $Z_3(s)$ is LC driving point impedance function.

$$z(s) = \frac{2(s^2+1)(s^2+9)}{s(s^2+4)} = \frac{2s^4+20s^2+18}{s^3+4s}$$

Using Foster-I method,

$$
\begin{array}{r|l}
s^3+4s\,) & 2s^4+20s^2+18\,(\,2s \\
 & \underline{-2s^4 \pm 8s^2} \\
 & 12s^2+18
\end{array}
$$

$$\therefore z(s) = 2s + \frac{12s^2+18}{s^3+4s} = \frac{K_1}{s} + \frac{K_2 s}{s^2+4}$$

or, $12s^2+18 = (K_1+K_2)s^2 + 4K_1$

Comparing coefficients,

$$K_1 = 18/4 = 9/2$$

$$K_2 = 15/2$$

$$\therefore z(s) = 2s + \frac{9}{2s} + \frac{15}{2}\cdot\frac{s}{s^2+4}$$

$$= 2s + \frac{1}{\left(\frac{2}{9}\right)s} + \frac{1}{\left(\frac{2}{15}\right)s + \frac{1}{\left(\frac{15}{8}\right)s}}$$

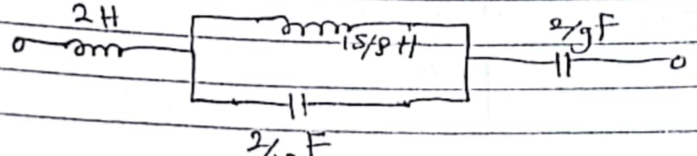


Fig: Foster-I network.

Using Cauer-I method,

Numerator power $= 4 = m$

Denominator Power $= 3 = n$

$\therefore m > n$, Then,

$$
\begin{array}{r|l}
s^3+4s\,) & 2s^4+20s^2+18\,(\,2s \leftarrow Z_1(s) \\
 & \underline{-2s^4 \pm 8s^2} \\
12s^2+18\,) & s^3+4s\,(\,s/12 \leftarrow Y_2(s) \\
 & \underline{-s^3 \pm \tfrac{3}{2}s} \\
 & \tfrac{5}{2}s
\end{array}
$$

$$\tfrac{5}{2}s \;\big)\; 12s^2+18 \;\big(\; \tfrac{24}{5}s \leftarrow Z_3(s)$$
$$-12s^2$$
$$18 \;\big)\; \tfrac{5}{2}s \;\big(\; \tfrac{5}{36}s \leftarrow Y_4(s)$$
$$-\tfrac{5}{2}s$$
$$\times$$

$$\therefore Z(s) = Z_1(s) + \cfrac{1}{Y_2(s) + \cfrac{1}{Z_3(s) + \cfrac{1}{Y_4(s)}}}$$

$$\therefore Z(s) = 2s + \cfrac{1}{\left(\tfrac{1}{12}\right)s + \cfrac{1}{\left(\tfrac{24}{5}\right)s + \cfrac{1}{\left(\tfrac{5}{36}\right)s}}}$$

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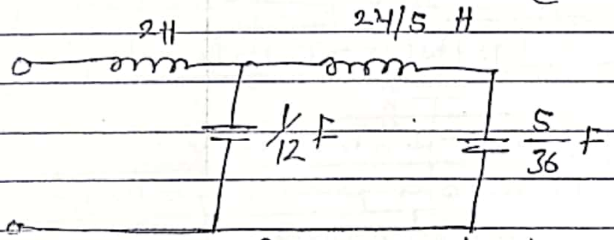


Fig:- Cauer-I network.

Q.N.6) What is transmission zeros? What are the steps involved in realizing transmission zeros of a lossless two port network? Explain with suitable example.

Solution:

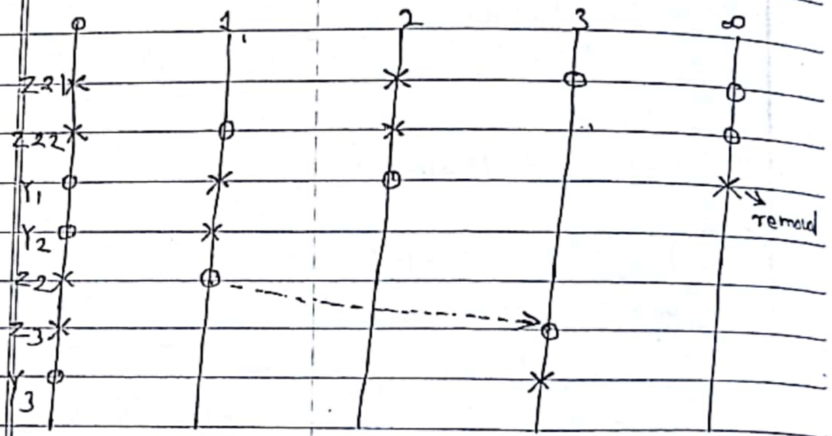
The frequencies at which a two port network yields a zero output for a finite input are referred as transmission zeros.

Example: $z_{21}(s) = \frac{K(s^2+9)}{s(s^2+4)}$; $z_{22}(s) = \frac{s^2+1}{s(s^2+4)}$

We have, $z_{21} = \frac{K(s^2+9)}{s(s^2+4)}$; Zeros $= \pm 3j$, poles $= 0, \pm 2j$

$z_{22} = \frac{s^2+1}{s(s^2+4)}$; Zeros $= \pm 1j$, poles $= 0, \pm 2j$

Poles-zeros location:



Here, Zeros of transmission at $s=\infty$

→ No need of Zero shifting

→ Need Complete removal of pole.

Now, $Y_1 = \frac{1}{z_{22}} = \frac{s(s^2+4)}{(s^2+1)}$; zeros $= 0, \pm 2j$; poles $= \pm 1j$

for Complete removal of pole,

$Y_2 = Y_1 - K_\infty s$ where, $K_\infty = \left.\frac{Y_1(s)}{s}\right|_{s=\infty}$

$= \left.\frac{\cancel{s}(s^2+4)}{\cancel{s}(s^2+1)}\right|_{s=\infty}$

$= \left.\frac{s^2(1+4/s^2)}{s^2(1+1/s^2)}\right|_{s=\infty}$

$= 1$

$\therefore Y_2 = \frac{s(s^2+4)}{s^2+1} - s = \frac{s^3+4s-s^3-s}{s^2+1}$

$= \frac{3s}{s^2+1}$; zeros $= 0$; poles $= \pm 1j$

Zeros of transmission at $s = \pm 3j$;

→ Requires Zero shifting (because zero of Y_2 and zero of z_{21} are not at same position). A pole $s = \pm 1j$ lies between zero of Y_2 and zero of z_{21}.

Reciprocating Y_2 ;

$z_2 = \frac{1}{Y_2} = \frac{s^2+1}{3s}$; zeros $= \pm 1j$; poles $= 0$;

For zero shifting,

$z_3 = z_2 - K_p H s$; $H = 1/3$

To find K_p, $\left[z_3 = 0\right]_{s=\pm 3j} \Rightarrow s^2 = -9$

$\left[\frac{s^2+1}{3s} - K_p \cdot \frac{1}{3} \cdot s = 0\right]_{s^2=-9}$

or, $K_p = \left.\frac{3(s^2+1)}{3s^2}\right|_{s^2=-9}$

or, $K_p = 8/9$

$\therefore z_3 = \frac{s^2+1}{3s} - \frac{8}{27}s = \frac{9s^2+9-8s^2}{27s}$

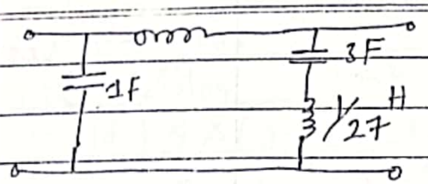
$= \frac{s^2+9}{27s}$; zero $= \pm 3j$; pole $= 0$

Now, removing pole Completely,

$Y_3 = \frac{1}{z_3} = \frac{27s}{s^2+9}$; zero $= 0$; pole $= \pm 3j$

Here, $z_3 = \frac{s^2+9}{27s} = \frac{s}{27} + \frac{1}{3s}$

The Circuit is,



S.11.7) What is reflection Coefficient? Design a third Order Butterworth high pass filter using resistively terminated lossless ladder with equal termination of 1Ω. (Refer following Table).

Pole location for Butterworth Responses,

n=2	n=3	n=4	n=5
-0.7071068	-0.50	-0.3826834	-0.809017
±j0.7071068	±j0.86603	±j0.9238795	±j0.5877852
	-1.0	-0.9238795	-0.309017
		±j0.3826834	±j0.9510565
			-1.0

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Solution

⇒ Reflection Coefficient is the ratio of power reflected back by the two port network to the available power at the network. It denotes how much power is reflected from network without passing through it.

From the table, the third Order Butterworth low pass filter has the transfer function as,

$$T_3(S) = \frac{1}{(s+1)(s+0.5+j0.86603)(s+0.5-j0.86603)}$$

$$= \frac{1}{(s+1)(s^2+s+1)} = \frac{1}{s^3+2s^2+2s+1}$$

We know,

$$D(s) = \frac{1}{T(s)} = (s+1)(s^2+s+1)$$

Also, $$\rho(s) = \frac{s^n}{D(s)} = \frac{s^3}{s^3+2s^2+2s+1}$$

Now, $$z_{11}(s) = R_1 \frac{[1 \pm \rho(s)]}{[1 \mp \rho(s)]}$$

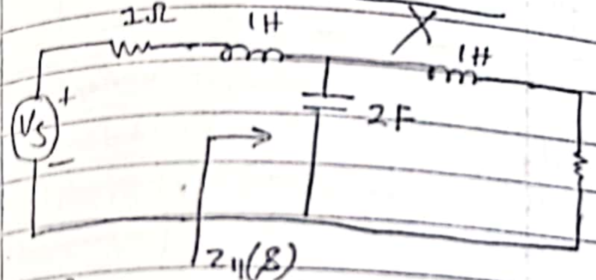
Here, $R_1 = 1\Omega$ (Given)

So, $$z_{11}(s) = \frac{1 \pm s^3/(s^3+2s^2+2s+1)}{1 \mp s^3/(s^3+2s^2+2s+1)}$$

$$= \frac{2s^3+2s^2+2s+1}{2s^2+2s+1} \text{ and } \frac{2s^2+2s+1}{2s^3+2s^2+2s+1}$$

For $z_{11}(s)$, $$\frac{2s^3+2s^2+2s+1}{2s^2+2s+1}$$

$$2s^2+2s+1 \,)\, 2s^3+2s^2+2s+1 \,(\, s$$
$$\underline{-2s^3+2s^2+s}$$
$$s+1 \,)\, 2s^2+2s+1 \,(\, 2s$$
$$\underline{-2s^2 \pm 2s}$$
$$1 \,)\, s+1 \,(\, s$$
$$\underline{-s}$$
$$1 \,)\, 1 \,(\, 1 \leftarrow \boxed{R_2}$$
$$\underline{-1}$$
$$\times$$



Again,

For $Z_{11}(s) = \dfrac{2s^2+2s+1}{2s^3+2s^2+2s+1}$

$$Y_{11}(s) = \frac{2s^3+2s^2+2s+1}{2s^2+2s+1}$$

$$2s^2+2s+1 \,)\, 2s^3+2s^2+2s+1 \,(\, s$$
$$\underline{-2s^3+2s^2 \pm s}$$
$$s+1 \,)\, 2s^2+2s+1 \,(\, 2s$$
$$\underline{-2s \pm 2s}$$
$$1 \,)\, s+1 \,(\, s$$
$$\underline{-s}$$

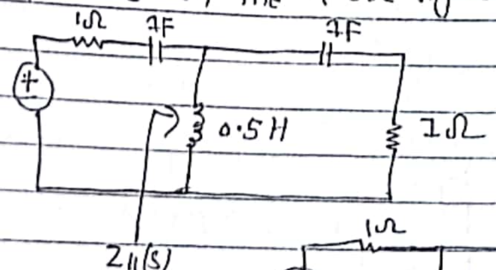
$$1 \,)\, s+1 \,(\, s$$
$$\underline{-s}$$
$$1 \,)\, 1 \,(\, 1 \leftarrow \boxed{R_L}$$
$$\underline{-1}$$
$$\times$$

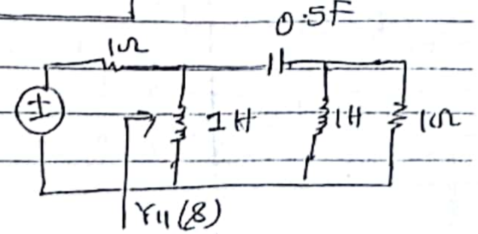
Above figure are for Butterworth low pass filter. Using frequency transformation we can change them to high pass filter using $\left(\frac{w_c}{s}\right)$ instead of s.

Here, Capacitor of Capacitance 'C' is replaced by inductor of inductance equivalent to $\frac{1}{Cw_c}$.

Inductor of inductance (L) is replaced by capacitor of Capacitance equivalent to $\frac{1}{Lw_c}$. No change for resistor.

for, $w_c = 1$, the above figure are,





Q.N.8) Draw the circuit diagram and derive transfer function of Tow Thomas Biquad circuit.
Design a low pass filter using Tow-Thomas Biquad circuit with poles at $-500 \pm j2449.49$ and dc gain of 2. The final circuit should consists of capacitor of value $0.1\mu F$.

Solution

First Part (see Q.N.8 (first part) 2071 Shrawan)

Second part

Given,

$$-\alpha \pm j\beta = -500 \pm j2449.49$$

so, $\beta = 2449.49$

$$\omega_0 = \sqrt{\alpha^2+\beta^2} = \sqrt{500^2 + 2449.49^2}$$
$$= 2500 \text{ rad/sec}$$

Also, $2\alpha = \omega_0/Q$

or, $Q = \frac{\omega_0}{2\alpha} = \frac{2500}{2\times500} = 2.5$

DC Gain (H) = 2

The standard eqn for low pass biquad is,

$$T_{LP}(s) = \frac{-H\omega_0^2}{s^2+\left(\frac{\omega_0}{Q}\right)s+\omega_0^2}$$

and $$T_{LP}(s) = \frac{-\left(1/R_3R_4C_1C_2\right)}{s^2+\left(\frac{1}{R_1C_1}\right)s+\frac{1}{R_2R_4C_1C_2}}$$

Comparing we get; $H = R_2/R_3$

Now, Using Tunning algorithm,

We choose $\omega_0 = 1$ rad/sec, $R_4 = 1\Omega$ and $C_1 = C_2 = 1F$ then,

$$\omega_0^2 = \frac{1}{R_2R_4C_1C_2} \quad \text{or, } R_2 = \frac{1}{\omega_0^2R_4C_1C_2}$$
$$= 1\Omega$$

Similarly,

$$\frac{\omega_0}{Q} = \frac{1}{R_1C_1}$$

or, $Q = \omega_0R_1C_1$ or, $R_1 = 2.5\Omega$

so, $H = R_2/R_3$

or, $2 = 1/R_3$ or, $R_3 = 1/2 = 0.5\Omega$

To achieve $\omega_0 = 2500$ rad/sec, do frequency scaling,

$$K_f = \frac{2500}{1} = 2500$$

To achieve, $C_1 = C_2 = 0.1\mu F$,

$$C_{new} = \frac{C_{old}}{K_m \cdot K_f}$$

or, $0.1\times10^{-6} = \frac{1}{K_m * 2500}$

or, $K_m = 4000$

$\therefore R_1 = 2.5 * 4000 = 10K\Omega$, $R_2 = 4K\Omega$, $R_3 = 2K\Omega$

$R_4 = 4K\Omega$

$C_1 = C_2 = 0.1\mu F$

Take $R_5 = 10K\Omega$

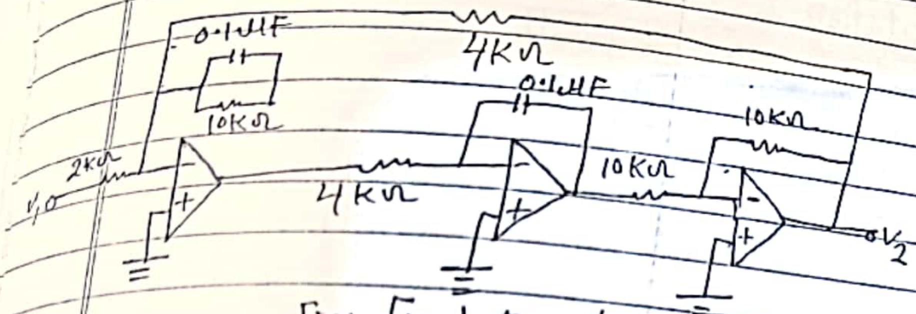


Fig:- Final Tow thomas Circuit.

P.N.9) What is RC-CR transformation? Draw the circuit diagram of high pass sallen-key biquad obtained by RC-CR transformation of its low pass counterpart.

Solution

→ We can design highpass filters by a simple change of the kind of element (element transformation) of sallen-key circuit, the process is known as RC-CR transformation. In RC-CR transformation R_i is replaced by C_i of value $\frac{1}{R_i}$ and C_j is replaced by R_j of value $1/C_j$. Gain elements are not transformed in this case,

We have,

$$T_{LP}(s) = \frac{K/R_1R_2C_1C_2}{s^2 + \left(\frac{1}{R_1C_1} + \frac{1}{R_2C_1} + \frac{1-K}{R_2C_2}\right)s + \frac{1}{R_2R_1C_1C_2}} \quad \text{---(1)}$$

Using design - IV,

$K = 1$, $R_1 = R_2 = 1\Omega$

$$\therefore T_{LP}(s) = \frac{1/C_1C_2}{s^2 + \left(\frac{2}{C_1}\right)s + \frac{1}{C_1C_2}} \quad \text{---(2)}$$

At normalized frequency, $\omega_0 = 1$ rad/sec

$\omega_0 = \sqrt{\frac{1}{C_1C_2}} \Rightarrow C_1C_2 = 1$,

$\frac{\omega_0}{Q} = \frac{2}{C_1} \Rightarrow \frac{1}{Q} = \frac{2}{C_1}$

$\therefore C_1 = 2Q$, $C_2 = \frac{1}{2Q}$

Now, reduce C_1 and R_1 ($R_1 = 1/2Q$) & C_2 by R_2 ($R_2 = 2Q$).

To obtain, high pass filter, replace s by $1/s$.

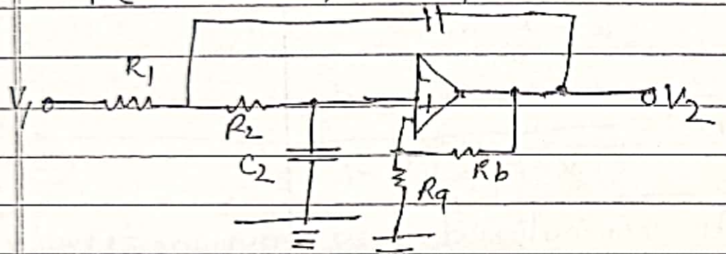
$\therefore$ eq^n (2) becomes,

$$T_{HP}(s) = \frac{1/R_1R_2}{(1/s)^2 + \left(\frac{2}{R_1}\right)\cdot \frac{1}{s} + \frac{1}{R_1R_2}}$$

$$= \frac{s^2}{s^2 + \left(\frac{2}{R_2}\right)s + \frac{1}{R_1R_2}}$$

$$= \frac{s^2}{s^2 + \left(\frac{1}{Q}\right)s + 1}$$

The circuit is,



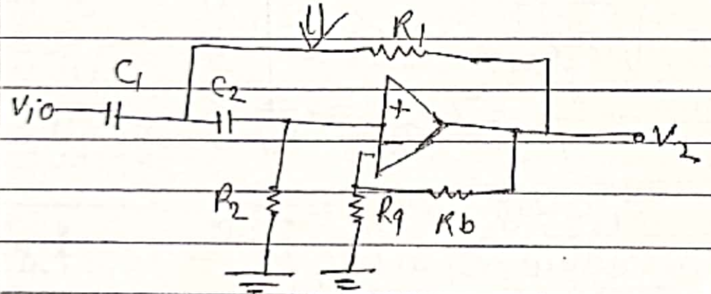


Fig:- high pass filter.

Q.N.19) What is Sensitivity? Perform Sensitivity analysis for center frequency (ω_0) of sallen-key biquad with respect to all resistors and capacitors present in the circuit.

Solution

The cause and effect relationship between network element variation and the resulting changes in Network transfer function is known as sensitivity. If x is changed and output reasult is y, then sensitivity is denoted by S_x^y.

and given as,

$$S_x^y = \frac{\%\text{ change in } y}{\%\text{ Change in } x} = \frac{\Delta y/y}{\Delta x/x}$$

$$= \left(\frac{x}{y}\right)\cdot\frac{\partial y}{\partial x}$$

The transfer function of lowpass sallen-key Circuit is;

$$T_{LP}(s) = \frac{K/R_1R_2C_1C_2}{s^2 + \left[\frac{1}{R_1C_1} + \frac{1}{R_2C_1} + \frac{1-K}{R_2C_2}\right]s + \frac{1}{R_1R_2C_1C_2}}$$

Comparing with the standard equation of LPF;

$$T_{LP}(s) = \frac{-H\omega_0^2}{s^2 + \left(\frac{\omega_0}{Q}\right)s + \omega_0^2}$$ gives,

$$\omega_0 = \frac{1}{\sqrt{R_1R_2C_1C_2}} = R_1^{-1/2}\cdot R_2^{-1/2}\cdot C_1^{-1/2}\cdot C_2^{-1/2}$$

Now,

$$S_{R_1}^{\omega_0} = \frac{R_1}{\omega_0}\frac{d\omega_0}{dR_1} = \frac{R_1}{R_1^{-1/2}\cdot R_2^{-1/2}\cdot C_1^{-1/2}\cdot C_2^{-1/2}} \times \left(-\tfrac{1}{2}\right)\cdot R_1^{-3/2}\cdot R_2^{-1/2}\cdot C_1^{-1/2}\cdot C_2^{-1/2}$$

$$= -0.5$$

$$S_{R_2}^{\omega_0} = \frac{R_2}{\omega_0}\frac{d\omega_0}{dR_2} = \frac{R_2}{R_1^{-1/2}\cdot R_2^{-1/2}\cdot C_1^{-1/2}\cdot C_2^{-1/2}}\cdot\left(-\tfrac{1}{2}\right)\cdot R_1^{-1/2}\cdot R_2^{-3/2}\cdot C_1^{-1/2}\cdot C_2^{-1/2}$$

$$= -0.5$$

$$S_{C_1}^{\omega_0} = \frac{C_1}{\omega_0}\frac{d\omega_0}{dC_1} = \frac{C_1}{\omega_0}\cdot\left(-\tfrac{1}{2}\right) R_1^{-1/2}\cdot R_2^{-1/2}\cdot C_1^{-1/2}\cdot C_2^{-1/2}$$

$$= -0.5$$

$$S_{C_2}^{\omega_0} = -0.5$$

$$\& \ S_{R_3}^{\omega_0} = S_{R_4}^{\omega_0} = 0 \ ///$$

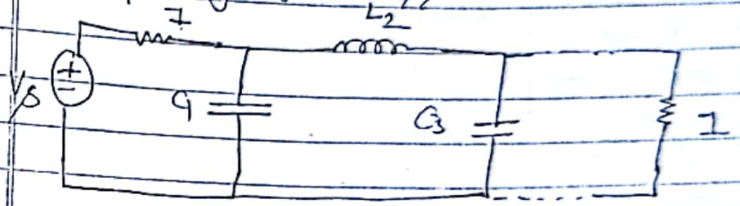
Q.N.11) What is GIC? How a GIC can be used to simulate grounded inductor? Explain with necessary figures and expression.

Solution

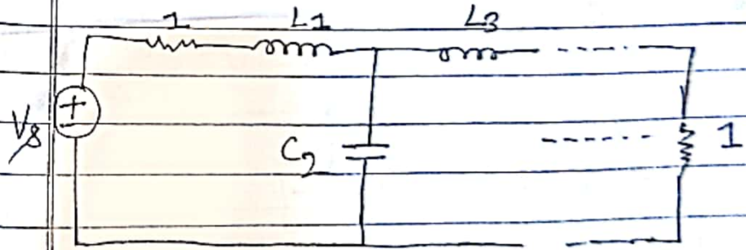
See (Q.N.11 (first part) 2069 Chaitra)

Q.N.12) Simulate third order Butterworth lowpass filter using Leapfrog simulation. (Refer following table)

Elements values for doubly terminated Butterworth filter normalized to half power frequency of 1 rad/s.



n	C_1	L_2	C_3	L_4	C_5
2	1.414	1.414			
3	1	2	1		
4	0.7654	1.848	1.848	0.7654	
5	0.618	1.618	2	1.618	0.618
n	L_1	C_2	L_3	C_4	L_5



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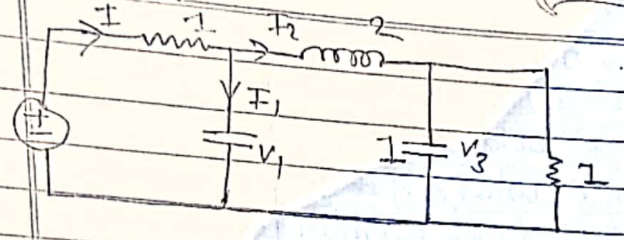
Solution



$$I_1 = (V_S - V_1)/R_1$$
$$-V_1 = (V_I - V_{I2})(-T_{Z1})$$
$$-V_{I2} = (V_3 - V_1)(T_{Y2})$$
$$V_3 = (-V_{I2})(-T_{Z3})$$

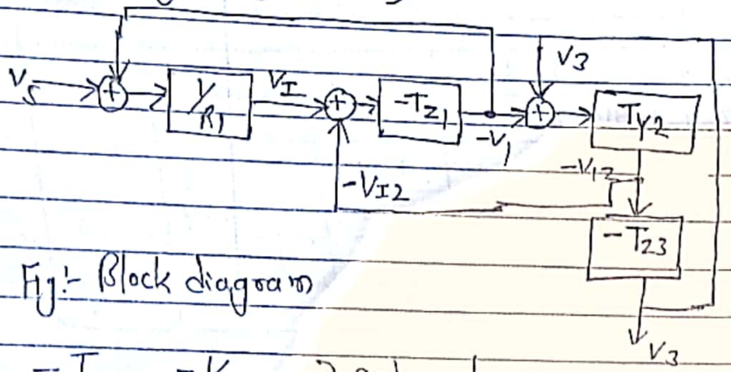


Fig:- Block diagram

$-T_{Z1} = -1/C_1 s \Rightarrow$ Integrator, $T(s) = -\frac{1}{RCs}$

$\therefore RC = C_1$

let $R = 1$ $\therefore$ $\boxed{C = C_1 = 1}$

$T_{Y2} = \frac{1}{L_2 s} \Rightarrow$ Integrator + Unity gain amplifier

$$T(s) = \frac{1}{RCs}$$

$\therefore RC = L_2$

let $R = 1$ $\therefore$ $\boxed{C = L_2 = 2}$

$$-T_{Z3} = -\frac{1}{C_3 s + R_0} = \frac{-1/C_3}{s + R_0/C_3}$$
$$= \frac{-1/C_3}{s + 1/C_3}$$

$\Rightarrow$ lossy integrator,

$$T(s) = \frac{-1/R_a C}{s + 1/R_b C}$$

$\therefore R_a C = C_3$

let $C = 1$ $\therefore$ $\boxed{R_a = C_3 = 1}$

& $R_b C = C_3$ $\therefore$ $\boxed{R_b = C_3 = 1}$

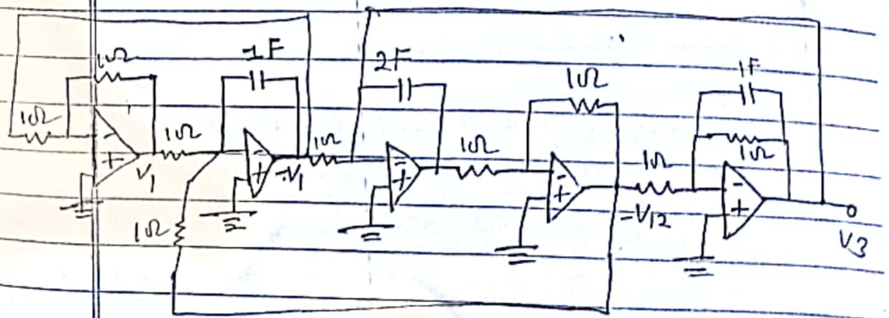
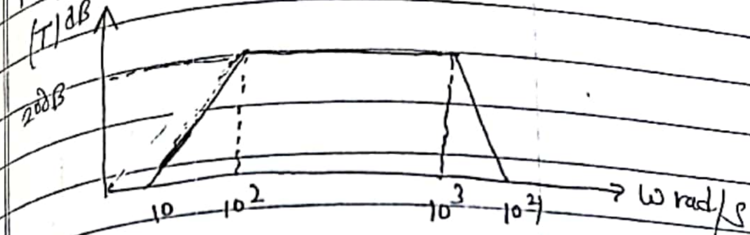


Fig:- Leapfrog simulation.

What is Switched Capacitor filter? What are its applications? Design a Switched Capacitor filter to realize the magnitude response given below:



Solution

Any filter contain resistor. But, resistor takes large space on IC when fabricated. Therefore, the resistor is replaced by the circuit combination of MOSFET and capacitor. Those filters are called Switched Capacitor filters.

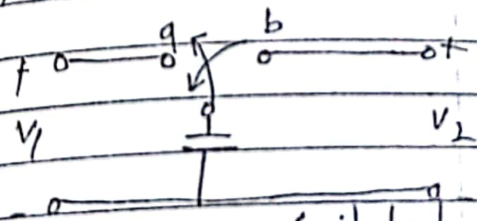


Fig: Switched - Capacitor

Applications

→ Easier to fabricate than resistor because of small area than resistors.
→ Reduce power, compatibility with digital systems.
→ For accuracy in IC implementation of filters.

Second Part

From figure,

Zeros : $10, 10^4$
Poles : $10^2, 10^3$

$$\therefore T(s) = \frac{K\left(1+\frac{s}{10}\right)\left(1+\frac{s}{10^4}\right)}{\left(1+\frac{s}{10^2}\right)\left(1+\frac{s}{10^3}\right)}$$

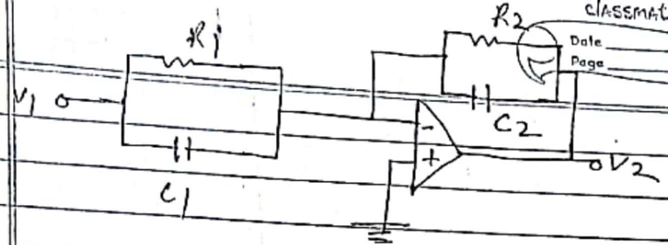
$$\text{or, } T(s) = \frac{K(s+10)(s+10^4)}{(s+10^2)(s+10^3)}$$

For $\omega=0$ rad/sec, $T(j\omega)=0$ dB gives $K=1$

$$\therefore T(s) = \left[-\frac{(s+10)}{(s+100)}\right] * \left[\frac{-(s+10^4)}{(s+10^3)}\right]$$

$$= T_1(s) * T_2(s) \quad \cdots \text{eq}^n (1)$$

lets take the filter circuit as a first order active section,



Then, $T(s) = -\dfrac{1/(C_2 s + 1/R_2)}{1/(C_1 s + 1/R_1)} = \dfrac{-(C_1 s + 1/R_1)}{(C_2 s + 1/R_2)}$ ---- (2)

By Using Switched Capacitor filter,
We have,

$R_1 = \dfrac{1}{f_c \cdot C_{R_1}}$ & $R_2 = \dfrac{1}{f_c C_{R_2}}$

Now, let us assume, $f_c = 10\,KHz$ and $C_1 = C_2 = 10pF$
then eqⁿ (2) becomes,

$$T(s) = \frac{-C_1 s + f_c C_{R_1}}{C_2 s + f_c C_{R_2}}$$

$$= -\left[\frac{10 * 10^{-12} s + 10^4 C_{R_1}}{10 * 10^{-12} s + 10^4 C_{R_2}}\right]$$

$$= -\left[\frac{s + 10^{15} C_{R_1}}{s + 10^{15} C_{R_2}}\right] \quad \text{--- eq}^n (3)$$

Comparing $T_1(s)$ at eqⁿ (1) with eqⁿ (3) gives

$10^{15} C_{R_1} = 10$

$C_{R_1} = 0.01 PF$

$10^{15} C_{R_2} = 100$ $\boxed{\therefore C_{R_2} = 0.1\,PF}$

Similarly Comparing $T_2(s)$ with eqⁿ (3) we get,

$10^{15} C_{R_3} = 10^4$

$\therefore C_{R_3} = 10\,PF$

$10^{15} \cdot C_{R_4} = 10^3$

$\therefore C_{R_4} = 1\,PF$

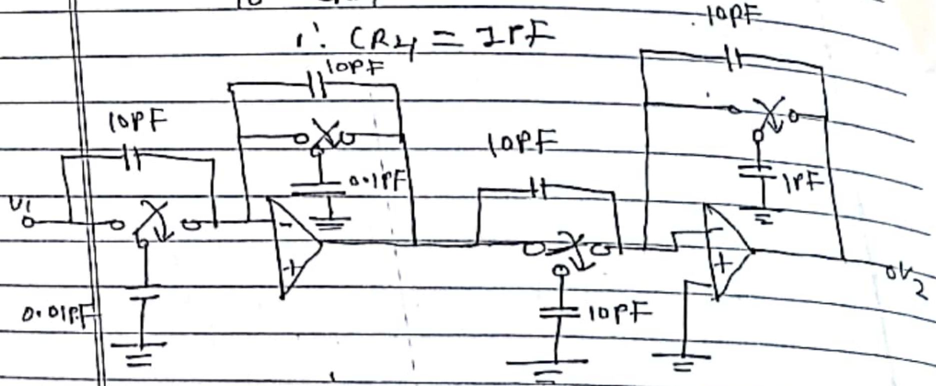


Fig:- Switched Capacitor filter.

S.N.	Title	Page No.	Teacher's Sign/ Remarks

2069 Chaitra

S.N.1. What is the importance of Normalization and Denormalization in filter design? Derive element Scaling equations.

Solutions:

A normalized filter is one in which the BP cut-off point is at $\omega = 1$ rad/sec This is $(1/2\pi)$ Hz or about 0.159 Hz

At $\omega = 1$ rad/sec

$2\pi f = 1$ rad/sec or $f = \frac{1}{2\pi} = 0.159$ Hz

At this condition impedance of reactive components is simply X_L and X_C.

$X_L = L$ and $X_C = 1/C$

This makes calculation simpler.

Normalization to 1 Hz would introduces 2π factors in to the equations.

$X_L = 2\pi f L$ and $X_C = \frac{1}{2\pi f C}$

The Reason for normalization is to make the calculation of values simple, which in turn makes the filter design simple.

Denormalization, in other word scalling, means to convert the circuit into required specification. Circuit elements are also scaled to get values that are easily available in the market. Two types of the scaling are frequency scaling and magnitude scaling.

Second part

Element Scaling equations,

There are two types of the scaling,

(i) Magnitude Scaling
(ii) Frequency Scaling

(i) Magnitude Scaling

→ It scales magnitude of the elements in order to maintain desired physical properties.

→ An impedance $|Z(j\omega)|$ is said to be magnitude scaled by a factor of 'K_m' if it is multiplied by a real positive constant 'K_m'.

Magnitude is saled up if 'K_m' > 1 and scaled down if 'K_m' < 1

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→ To achieve the impedance change, we alter the magnitude of impedance for every element in the network.

→ For passive element (R, L, C) the impedance is given as,

- For Resistor:

$$|Z_R| = R_{old}$$

After scaling by scale factor 'K_m' we get,

$$K_m |Z_R| = K_m \cdot R_{old}$$

$$\boxed{\therefore R_{new} = K_m \cdot R_{old}}$$

- For inductor:

$$|Z_L| = \omega \cdot L_{old}$$

After scaling by scale factor 'K_m' we get,

$$K_m \cdot |Z_L| = K_m \cdot \omega \cdot L_{old}$$

$$|Z_L|_{new} = \omega \cdot (K_m \cdot L_{old}) = \omega \cdot L_{new}$$

$$\boxed{\therefore L_{new} = K_m \cdot L_{old}}$$

- For Capacitor:

$$|Z_C| = \frac{1}{\omega \cdot C_{old}}$$

Multiplying by scale factor 'K_m',

$$K_m \cdot |Z_C| = \frac{K_m}{\omega \cdot C_{old}} = \frac{1}{\omega \cdot \left(\frac{C_{old}}{K_m}\right)}$$

$$\text{or } |Z_{new}| = \frac{1}{\omega \cdot C_{new}}$$

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$\because C_{new} = \dfrac{1}{\omega \cdot |Z_C|_{new}} = \dfrac{1}{\omega\left(\dfrac{1}{\omega \cdot C_{old}}\right) \cdot K_m}$

$= \dfrac{1}{\left(\dfrac{K_m}{C_{old}}\right)}$

$\boxed{\therefore C_{new} = \dfrac{C_{old}}{K_m}}$

ii) Frequency Scaling

⇒ It changes or effects the frequency dependent components such as 'C' and 'L' but not 'R'. With frequency scaling, the transfer function also changes and quantities are represented in new frequency domain.

let ω = old frequency domain and

Ω = new frequency domain

then, $\boxed{\therefore \Omega = K_f \cdot \omega}$

Now to scale frequency in passive components.

- For inductor:

$|Z_L| = \omega \cdot L_{old}$

$= K_f \cdot \omega \cdot \dfrac{L_{old}}{K_f}$

$= \Omega \cdot L_{new}$

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$\boxed{\therefore L_{new} = \dfrac{L}{K_f}}$

- For capacitor:

$|Z_C| = \dfrac{1}{\omega \cdot C} = \dfrac{1}{K_f \cdot \omega \cdot C} \times K_f = \dfrac{1}{K_f \cdot \omega \cdot \left(\dfrac{C}{K_f}\right)}$

$= \dfrac{1}{\Omega \cdot C_{new}}$

$\boxed{\therefore C_{new} = \dfrac{C}{K_f}}$

- For Resistor:

$|Z_R| = R_{old}$

$\boxed{\therefore R_{new} = R_{old}}$

All these are the element scaling equations.

Q.N.2) Derive the expression to calculate the order of Butterworth approximation for given low pass filter specifications. Calculate the order of Butterworth low pass filter having following specification,

(i) Passband extends from $\omega = 0$ to $\omega = 1000$ rad/s and the attenuation in the passband should no exceed to 0.1 dB.

(ii) Stop band extends from $\omega = 2000$ rad/s to $\omega = \infty$ and the attenuation in stop band should not less than 30 dB.

Solution:

For Order (n):

Given Specifications are:

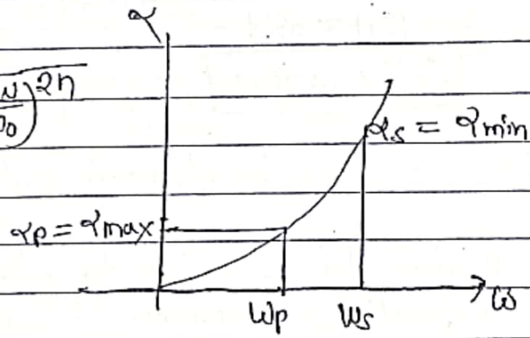
Passband (PB): $\omega = 0$ to $\omega = \omega_p$ and attenuation $\alpha = \alpha_{max}$

Stop band (SB): $\omega = \omega_s$ to ∞ and attenuation $\alpha = \alpha_{min}$

From ω_p to ω_s is a transition Band.

We have,

$$|T_n(j\omega)|^2 = \frac{1}{1+\left(\frac{\omega}{\omega_0}\right)^{2n}}$$



We know,

Attenuation (α),

$$\therefore \alpha = -20\log|T(j\omega)|$$

$$\text{or, } \alpha = -20\log\left[1+\left(\frac{\omega}{\omega_0}\right)^{2n}\right]^{-1/2}$$

$$\text{or, } \alpha = 10\log\left[1+\left(\frac{\omega}{\omega_0}\right)^{2n}\right]$$

$$\text{or, } \frac{\alpha}{10} = \log\left[1+\left(\frac{\omega}{\omega_0}\right)^{2n}\right]$$

$$\text{and, } \left[1+\left(\frac{\omega}{\omega_0}\right)^{2n}\right] = (10)^{\alpha/10}$$

$$\therefore \left(\frac{\omega}{\omega_0}\right)^{2n} = 10^{\alpha/10} - 1$$

At $\omega = \omega_p$; $\alpha = \alpha_{max}$

$$\text{then, } \left(\frac{\omega_p}{\omega_0}\right)^{2n} = 10^{\frac{\alpha_{max}}{10}} - 1 \quad \text{--- (I)}$$

At $\omega = \omega_s$; $\alpha = \alpha_{min}$

then,

$$\left(\frac{\omega_s}{\omega_0}\right)^{2n} = 10^{\frac{\alpha_{min}}{10}} - 1 \quad \text{--- (II)}$$

Dividing eq^n (I) by eq^n (II) we get,

$$\frac{\left(\frac{\omega_p}{\omega_0}\right)^{2n}}{\left(\frac{\omega_s}{\omega_0}\right)^{2n}} = \frac{10^{\frac{\alpha_{max}}{10}} - 1}{10^{\alpha_{min}/10} - 1}$$

$$\text{or, } \left(\frac{\omega_s}{\omega_p}\right)^{2n} = \frac{10^{\frac{\alpha_{max}}{10}} - 1}{10^{\alpha_{min}/10} - 1}$$

$$\therefore \left(\frac{\omega_s}{\omega_p}\right)^{2n} = \frac{10^{\frac{\alpha_{min}}{10}} - 1}{10^{\frac{\alpha_{min}}{10}} - 1}$$

Taking log on both sides, we get,

$$\log\left(\frac{\omega_s}{\omega_p}\right)^{2n} = \log\left[\frac{10^{\alpha_{min}/10} - 1}{10^{\alpha_{max}/10} - 1}\right]$$

$$2n \log\left(\frac{\omega_s}{\omega_p}\right) = \log\left[\frac{10^{\alpha_{min}/10} - 1}{10^{\alpha_{max}/10} - 1}\right]$$

$$\therefore n = \boxed{\frac{\log\left[\frac{10^{\alpha_{min}/10} - 1}{10^{\alpha_{max}} - 1}\right]}{2 \log\left(\frac{\omega_s}{\omega_p}\right)}} \quad \text{--- (iii)}$$

Hence eqn (iii) is the required equation to find the order of the butterworth low pass filter.

i) Given,

Passband frequency $(\omega_p) = 200$ rad/s

Attenuation in the passband $(\alpha_{max}) = 0.1$ dB

Also stopband frequency $(\omega_s) = 2000$ rad/s

Attenuation in stopband $(\alpha_{min}) = 30$ dB

Now, We have,

the transfer function of Butterworth filter,

$$T_n(s) = \frac{1}{\sqrt{1+\omega^{2n}}} \quad \text{--- (1)}$$

at normalized frequency of $\omega_0 = 1$ rad/s

We have,

Attenuation, $\alpha = -20 \log|T|$

$$\therefore \alpha = 10 \log[1+\omega^{2n}] \quad \text{--- (2)}$$

Then the order of butterworth filter is given by,

$$\therefore n = \frac{\log\left[\frac{10^{\alpha_{min}/10} - 1}{10^{\alpha_{max}/10} - 1}\right]}{2 \log(\omega_s/\omega_p)}$$

$$\text{or, } n = \frac{\log\left[\left(10^{30/10} - 1\right) / \left(10^{0.1/10} - 1\right)\right]}{2 * \log\left(\frac{2000}{200}\right)}$$

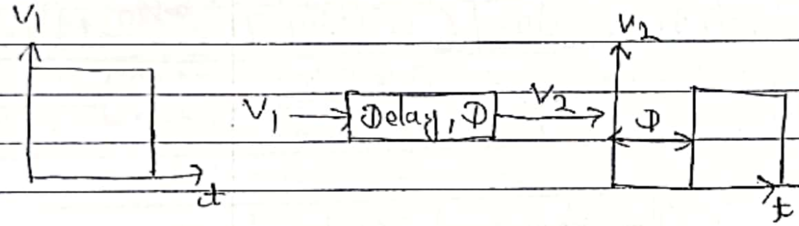
$$= 2.316 \approx 3$$

$\therefore$ The required order is 3.

Q.N.3) What is constant delay filter? Find the transfer function of a third order Bessel Thomson response having constant delay.

Solution

→ If the phase is linear with negative slope, the magnitude is constant and the delay will be constant. Then the obtained filter is called as constant delay filter.



The relation between o/p and i/p becomes,

$v_2(t) = v_1(t - D)$

let $v_1(t) = A\sin(\omega t + \phi)$ --- ① then

$v_2(t) = A\sin(\omega t - \omega D + \phi)$

i.e output and input signal differ by phase angle, $\theta = -\omega D$

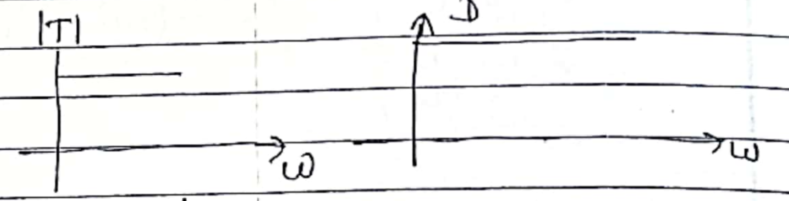
i.e $\frac{V_2}{V_1} = 1\angle -\omega D$

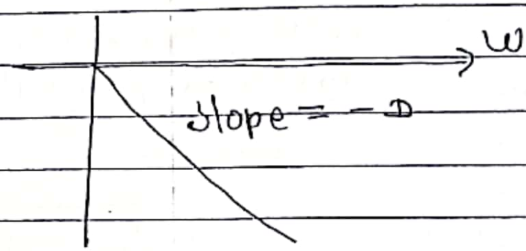
or, $|T| = \frac{V_2(s)}{V_1(s)} = e^{-j\omega D} = e^{-sD}$

for normalized delay $D = 1$, so we get,

$|T| = \frac{V_2(s)}{V_1(s)} = e^{-s}$

We have,





2nd part.

⇒ Third order transfer function is,

$$T_3(s) = \frac{a_0}{s^3 + a_2 s^2 + a_1 s + a_0} = \frac{1}{\cosh s + \sinh s}$$

Now from the storch principle,

$$\coth s = \frac{1}{s} + \cfrac{1}{\frac{3}{s} + \cfrac{1}{5/s}}$$

$$= \frac{1}{s} + \cfrac{1}{\frac{3}{s} + \frac{s}{5}}$$

$$= \frac{1}{s} + \frac{5s}{s^2+15}$$

$$= \frac{s^2+5s^2+15}{s^3+15s} = \frac{\cosh s}{\sinh s}$$

$\therefore \cosh s = 6s^2+15$

$\sinh s = s^3+15s$

$$T_3(s) = \frac{1}{s^3+6s^2+15s+15}$$

$$\boxed{\therefore \text{Actual, } T_3(s) = \frac{15}{s^3+6s^2+15s+15}}$$

Which is the required transfer function.

Q.N.4) What is frequency transformation? Describe the frequency transformation from low pass to band stop filter with example.

⟹ 1st part

Frequency transformation is the method used to transform prototype filter into any other types of filter. In filter design, we must first develop a prototype filter with cut-off frequency equals 1 rad/sec and then we apply the frequency transformation technique to achieve the required high pass, band pass or band-stop filter at required cut-off frequency and bandwidth.

Second Part

⟹ By replacing all (s) by $\frac{Bs}{s^2+\omega_0^2}$, we can transform a low pass filter to the band stop filter,

i) Resistor, $Z_R = R$

$Z_{R,new} = R$ (no change)

ii) Inductor, $Z_L = Ls$

$\therefore Z_{L,new} = L \cdot \frac{Bs}{s^2+\omega_0^2}$

$\therefore Y_{L,new} = \frac{s^2+\omega_0^2}{LBs} = \frac{s}{LB} + \frac{\omega_0^2}{LBs}$

Inductor is replaced by parallel combination of capacitor ($C_{new} = \frac{1}{LB}$) and Inductor. ($L_{new} = LB/\omega_0^2$)

iii) Capacitor, $Z_c = \frac{1}{Cs}$

$$\therefore Z_{c,new} = \frac{1}{C \cdot \frac{BS}{S^2+\omega_0^2}} = \frac{S^2+\omega_0^2}{CBS}$$

$$= \frac{S}{CB} + \frac{\omega_0^2}{CBS}$$

Capacitor is replaced by series combination of inductor $\left(\frac{1}{CB}\right)$ and Capacitor $\left(CB/\omega_0^2\right)$.

Example :
Transform low pass filter in to bandstop filter. Given, $B = 400$ rad/s, $\omega_0 = 2000$ rad/s

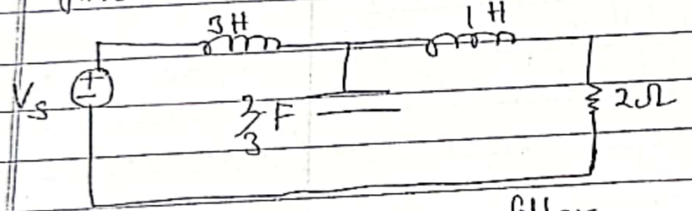


Fig:- low pass filter

Now,

$R(2) = 2\Omega$

$L(3) \equiv C_{new} = \frac{1}{LB} = \frac{1}{3 \times 400} = 833.33\mu F$

$L_{new} = \frac{3 \times 400}{(2000)^2} = 300\mu H$

$L(1) \equiv C_{new} = \frac{1}{(1 \times 400)} = 2.5\,mF$

$L_{new} = 1 \times 400 / (2000)^2 = 100\mu H$

And $C(2/3) \equiv L_{new} = \frac{1}{2/3 \times 400} = 3.75\,mH$

$C_{new} = \left(2/3 \times 400\right) / (2000)^2 = 66.67\mu F$

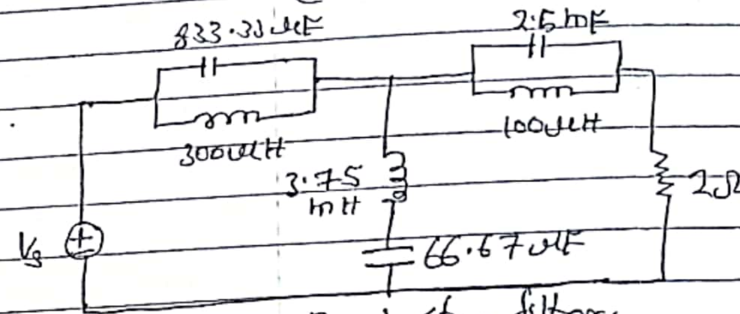


Fig:- Band stop filter.

S.N.5) Which of the following functions are LC driving point impedance function and why?

$Z_1(S) = \frac{S(S^2+4)}{(S^2+9)(S^2+16)}$, $Z_2(S) = \frac{S(S^2+1)(S^2+9)}{(S^2+4)(S^2+16)}$

$Z_3(S) = \frac{S(S^2+4)}{2(S^2+1)(S^2+9)}$, $Z_4(S) = \frac{2(S+1)(S+3)}{(S+2)(S+4)}$

Solution :-

- $Z_1(S) = \frac{S(S^2+4)}{(S^2+9)(S^2+16)}$, not LC driving point impedance function, i.e two consequitive zeros i.e 0 and $\pm 2i$.

• $Z_2(s) = \frac{s(s^2+1)(s^2+9)}{(s^2+4)(s^2+16)}$, not LC driving point impedance function, because of two consequitive zeros i.e. $0, \pm 1i$,

• $Z_4(s) = \frac{2(s+1)(s+3)}{(s+2)(s+4)}$, not LC driving point impedance function because of, $\frac{\text{even}}{\text{even}}$,

• $Z_3(s) = \frac{s(s^2+4)}{2(s^2+1)(s^2+9)}$, is LC driving point impedance function which satisfies all the condition.

2nd part

And, The Cauer II realization of the valid LC driving point impedance function $Z_3(s)$,

Then,

$$Z_3(s) = \frac{4s+s^3}{18+20s^2+2s^4}$$

$$\therefore\ Y_3(s) = \frac{18+20s^2+2s^4}{4s+s^3}$$

Now, $4s+s^3 \,\big)\, 18+20s^2+2s^4 \,\big(\, \frac{18}{4s} \longrightarrow Y$

$-\ 18+\frac{9}{2}s^2$

$\frac{31}{2}s^2+2s^4 \,\big)\, 4s+s^3 \,\big(\, \frac{8}{31s} \longrightarrow Z$

$-4s+\frac{16}{13}s^3$

$\frac{15}{31}s^3$

Also, $\frac{15}{31}s^3 \,\big)\, \frac{31}{2}s^2+2s^4 \,\big(\, \frac{961}{30s} \longrightarrow Y$

$-\frac{31}{2}s^2$

$2s^4 \,\big)\, \frac{15}{31}s^3 \,\big(\, \frac{15}{62s} \longrightarrow Z$

$-\frac{15}{31}s^3$

0

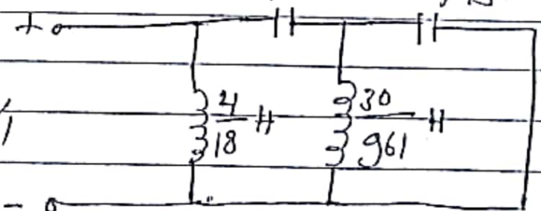


Fig: Cauer – II Circuit.

S.N 6) What is "Zero shifting by partial removal of pole"? Explain with example. Also mention its importance in two port network synthesis.

Solution

⇒ "Zero shifting by partial removal of pole" is that which shifts the zero towards that value of Kp and proximity of zero to that pole.

⇒ The partial removal of pole at origion does not affect zero at infinity, nor the partial removal of pole at infinity affect a zero at origion.

Example

The extraction of a pole at a specified frequency with the aid of zero shifting or partial pole removal will now be illustrated by way of an example,

$$Y(s) = \frac{s^4 + 10s^2 + 9}{s(s^2+4)}$$

This admittance has a pole at infinity but has no pole or zero at $s^2 = (-10)$ say.

A pole may, however, be extracted from it by first placing a zero there by a partial removal of the nearest pole which happens in this case to lie at infinity such that, $Y(s) - Ks\big|_{s^2=-10} = 0$.

This equation may now be solved for the residue K of the pole which must be partially extracted,

$$K = \frac{s^4+10s^2+9}{s^2(s^2+4)}\bigg|_{s^2=-10} = \frac{3}{20}$$

Extracting this partial pole at infinity but in the form of a shunt capacitor from $Y(s)$ gives,

$$Y_1(s) = \frac{17s^4 + 188s^2 + 180}{20s(s^2+4)}$$

$Y_1(s)$ has still a pole at s equal to infinity but it now also has a zero at $s^2 = -10$, as is readily verified. from $Z_1(s)$ gives a pole there which may be removed.

$$Z_1(s) = \frac{1}{Y_1(s)} = \frac{20s(s^2+4)}{17s^4+188s^2+180}$$

The pole of $Z_1(s)$ at $s^2 = -10$ may now be extracted by expanding it in partial functions:

$$Z_1(s) = \frac{15s/19}{s^2+10} + \frac{250s/38}{17s^2+18}$$

A pair of conjugate poles may now be removed at $s^2 = -10$ in the form of a parallel LC circuit in series with the network giving a remainder reactance,

$$Z_2(s) = \frac{250s/38}{17s^2+18}$$

~~This reactance~~ A pair of conjugate poles may now be removed at $s^2=-10$ in the form of a parallel LC circuit in series with the network giving a remainder reactance,

$$Z_2(s) = \frac{250s/38}{17s^2+18}$$

This reactance has a zero at infinity but its reciprocal has a pole there.

$$Y_2(s) = \frac{1}{Z_2(s)} = \frac{646s}{250} + \frac{684}{250s}$$

The remaining susceptance may now be realized by inspection by removing a pole at both the origin and at infinity in the form of a shunt Capacitor and series inductor.

$$Y_1(s) = Y(s) - \frac{K}{s}\Big|_{s^2=-1/2} = 0$$

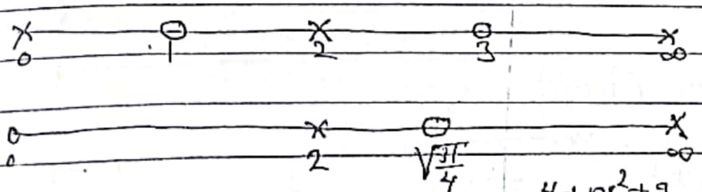


Fig:- Pole zero diagram for $Y(s) = \frac{s^4+10s^2+9}{s(s^2+4)}$

Shifting zero to remove pole at $s^2=-10$.

3rd part

Importance of "zero shifting by partial removal of pole" in two port network synthesis.

S.N. 7) What is transmission coefficient? What information do you get from the transmission coefficient? Design a second order Butterworth low pass filter and using lossless ladder with equal termination of 1Ω i.e $R_1 = 1\,\Omega$ and $R_2 = 1\Omega$ (Refer Table 7)

Solution

Now, we have from the relation of the transducer power gain 'G',

The Transducer power gain 'G' defined as the ratio of average power delivered to the load to the maximum available average power at the Source is given by,

$$G(\omega^2) = |S_{21}(j\omega)|^2 = 1 - |S_{11}(j\omega)|^2$$

where S_{21} is known as Transmission Coefficients.

Also, we can write,

$$|S_1(j\omega)|^2 = \frac{\text{Reflected power}}{\text{Power Available}}$$

$$|S_{21}(j\omega)|^2 = \text{Power to load} / \text{Power available}$$

⇒ From the value of the transmission coefficient, we can conclude that how much power is lost in load, from the power available.

Second Part

From table 1, the transfer function of 2nd order Butterworth low pass filter is,

$$T_2(S) = \frac{1}{(S+0.7071068+j0.7071068)(S+0.7071068-j0.7071068)}$$

$$= \frac{1}{(S+0.7071068)^2 - (j0.7071068)^2}$$

$$= \frac{1}{s^2+1.414s+0.5+0.5} = \frac{1}{s^2+\sqrt{2}s+1}$$

$$= \frac{1}{D(S)}$$

$$\therefore D(S) = s^2+\sqrt{2}s+1$$

Also,

$$S(s) = \frac{s^n}{D(S)} = \frac{s^2}{s^2+\sqrt{2}s+1}$$

$$\therefore Z_{in} = R_1\left[\frac{1-S(s)}{1+S(s)}\right]^{\pm 1}$$

$$\text{or, } Z_{in} = 1\left[\frac{1-s^2/(s^2+\sqrt{2}s+1)}{1+s^2/(s^2+\sqrt{2}s+1)}\right]^{\pm 1}$$

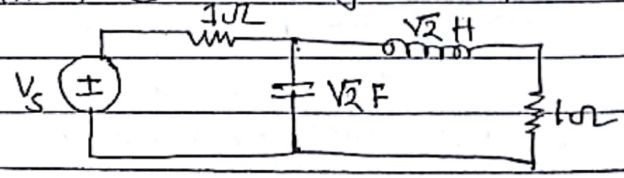
Taking +ve and -ve sign respectively,

$$\therefore Z_{in} = \frac{\sqrt{2}s+1}{2s^2+\sqrt{2}s+1} \quad \& \quad Z_{in} = \frac{2s^2+\sqrt{2}s+1}{\sqrt{2}s+1}$$

Taking $Z_{in} = \frac{\sqrt{2}s+1}{2s^2+\sqrt{2}s+1} \Rightarrow Y_{in} = \frac{2s^2+\sqrt{2}s+1}{\sqrt{2}s+1}$

$$\begin{array}{l}
\sqrt{2}s+1 \,\big)\, 2s^2+\sqrt{2}s+1 \,\big(\, \sqrt{2}s \\
\qquad\quad -2s^2 \pm \sqrt{2}s \\
\qquad\qquad\qquad 1 \,\big)\, \sqrt{2}s+1 \,\big(\, \sqrt{2}s \\
\qquad\qquad\qquad\quad -\sqrt{2}s \\
\qquad\qquad\qquad\qquad 1 \,\big)\, 1 \,\big(\, 1 \to Y \\
\qquad\qquad\qquad\qquad\quad -1 \\
\qquad\qquad\qquad\qquad\quad\ 0
\end{array}$$

Then, Circuit diagram is,



Now again taking, $Z_{in} = \dfrac{2s^2 + \sqrt{2}s + 1}{\sqrt{2}s + 1}$

(Same as above), then the circuit diagram is,

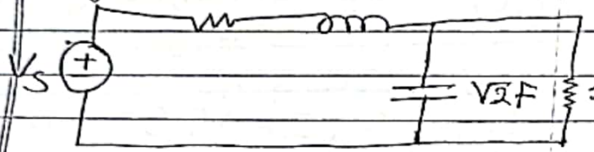


Fig:- Two port resistively ladder network.

s.v.8) Draw the circuit diagram of Tow thomas biquad low pass filter and derive its transfer function. Design a second order low pass filter using Tow Thomas biquad circuit having poles at $-750 \pm j\,661.44$ and dc gain of 2. Use capacitor of value $0.01\mu F$ in your design.

Solution

1st part,

Consider the circuit diagram of Tow thomas biquad low pass filter;

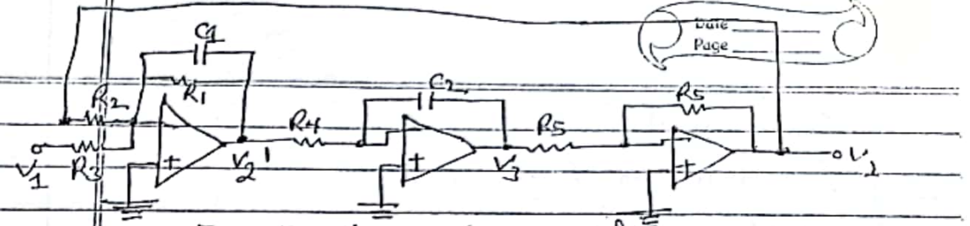


Fig:- Tow Thomas low pass filter

Now, from the Circuit diagram,

$$T(S) = \frac{V_2}{V_1} = \frac{V_2}{V_3} \cdot \frac{V_3}{V_2'} \cdot \frac{V_2'}{V_1}$$

or, $T(S) = (-1)\left(\dfrac{-1}{R_4 C_2 s}\right)\left(\dfrac{-R_1/R_3}{R_1C_1 s + 1} - \dfrac{T(S)\cdot R_1/R_2}{R_1C_1 s + 1}\right)$

or, $T(S) = \dfrac{-R_1/(R_3R_4C_2)}{R_1C_1 s^2 + s} - T(S) \cdot \dfrac{R_1/(R_2R_4C_2)}{R_1C_1 s^2 + s}$

or, $T(S)\left[1 + \dfrac{R_1/(R_2R_4C_2)}{R_1C_1 s^2 + s}\right] = \left[\dfrac{-R_1/(R_3R_4C_2)}{R_1C_1 s^2 + s}\right]$

or, $T(S)\left[1 + \dfrac{1/(R_2R_4C_1C_2)}{s^2 + s/(R_1C_1)}\right] = \left[\dfrac{-1/(R_3R_4C_1C_2)}{s^2 + s/(R_1C_1)}\right]$

or, $T(S)\left[s^2 + \dfrac{1}{R_1C_1}\cdot s + \dfrac{1}{R_2R_4C_1C_2}\right] = \left[\dfrac{-1}{(R_3R_4C_1C_2)}\right]$

$$\boxed{\therefore T(S) = \frac{-1/(R_3R_4C_1C_2)}{s^2 + \dfrac{s}{R_1C_1} + \dfrac{1}{R_2R_4C_1C_2}}} \quad \text{--- (1)}$$

eqn (1) is the required transfer function.

Second part

The transfer function is,

$$T(S) = \frac{1}{(s+\alpha+j\beta)(s+\alpha-j\beta)} = \frac{1/R_3R_4C_1C_2}{s^2+\frac{s}{R_1C_1}+\frac{1}{R_2R_4C_1C_2}}$$

$$= \frac{1}{s^2+2\alpha s+(\alpha^2+\beta^2)} \quad \text{---(1)}$$

From given,

$$-\alpha \pm j(\beta) = -750 \pm j661.44$$

$$\text{So, } \alpha = 750 \;\&\; \beta = 661.44$$

$$\text{So, } \omega_0 = \sqrt{\alpha^2+\beta^2} = \sqrt{(750)^2+(661.44)^2}$$

$$= 967.6563 \text{ rad/sec}$$

$$\text{And, } Q = \frac{\omega_0}{2\alpha} = \frac{967.6563}{2\times 750} = 0.6451$$

And Gain(H) = 2

Also, the standard equation of low pass Biquad is,

$$T_{LP}(S) = \frac{-H\omega_0^2}{s^2+\left(\frac{1}{R_1C_1}\right)s+\frac{1}{R_2R_4C_1C_2}} \quad \text{---(1)}$$

So, $H = R_2/R_3$ [Comparing we get]

Now, using TUNING Algorithm;

We choose, $\omega_0 = 1$ rad/sec (normalized frequency), $R_4 = 1\Omega$ and $C_1 = C_2 = 1F$ then,

$$\omega_0^2 = \frac{1}{R_2R_4C_1C_2} \quad \text{and} \quad R_2 = \frac{1}{\omega_0^2R_4C_1C_2} = 1\Omega$$

Similarly,

$$Q = \sqrt{\frac{R_1^2C_1}{R_2R_4C_2}} = \sqrt{R_1^2} = R_1$$

$$\therefore R_1 = Q = 0.6451\Omega$$

then, $H = R_2/R_3$

$$\text{or, } 2 = \frac{1}{R_3} \quad \text{or, } R_3 = \frac{1}{2} = 0.5\Omega$$

then, the final value of the capacitance is,

$$C = 0.01\mu F$$

$$\text{So, } K_m = C_{old}/C_{new} = \frac{0.001033 F}{0.01\mu F} \quad 103342.5$$

$$C_{old} = \frac{1}{\omega_0} = \frac{1}{K_f}$$

$$C_1 = C_2 = \frac{1}{K_f} = \frac{1}{\omega_0} = \frac{1}{967.6563} = 0.001033 F$$

To achieve C_1 and C_2, $0.01\mu F$, do magnitude scaling, $K_m = 103342.5$.

$$\therefore R_1 = 0.6451\times 103342.5 = 66.666 \text{ k}\Omega$$

$$R_2 = 1\times 103342.5 = 103.3425 \text{ k}\Omega$$

$$R_3 = 0.5\times 103342.5 = 51.671 \text{ k}\Omega$$

$$C_1 = C_2 = 0.01\mu F \;\&\; \text{Take } R_5 = 10 \text{ k}\Omega$$

$$R_4 = 103.34 \text{ k}\Omega$$

Then the circuit diagram can be drawn as,

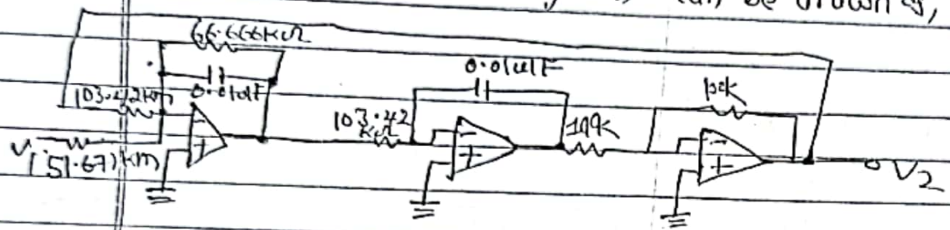


Fig:- Required Circuit

[For suitable market value of resistor, use $\omega_0 = 1000$ rad/sec and continue the same process.] i.e $k_f = 1000$

Q.N.9) Design the following transfer function using inverting op-amp configuration.

$$T(s) = 7 \cdot \left(\frac{s+400}{s+200}\right)$$

You are not allowed to use inductor in the design.

Solution

Transfer function of inverting op-amp is,

$$T(s) = \frac{Z_2}{Z_1}$$

$$\therefore \frac{Z_2}{Z_1} = \frac{7s + 2800}{s + 200}$$

or $$\frac{Z_2}{Z_1} = \frac{7 + 2800/s}{1 + 200/s}$$

$\therefore Z_2 = 7 + 2800/s$

$Z_1 = 1 + 200/s$

Then the Circuit is drawn as,

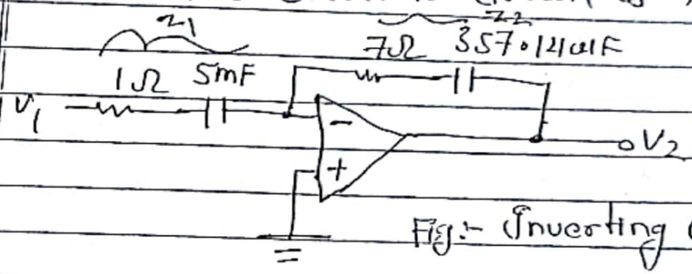


Fig:- Inverting Op-amp

Where,

$$\left.\begin{aligned} Z_2 &= 7 + \frac{1}{s/2800} \\ Z_1 &= 1 + \frac{1}{s/200} \end{aligned}\right\}$$

Q.N.10) What do you understand when the sensitivity of y with respect to x is equal to -3? Perform the sensitivity analysis for quality factor Q of the Tow Thomas low pass filter with respect to all the resistors and capacitors present in the circuit.

Solution

When the sensitivity of y with respect to x is equal to -3, from this we understand that ($S_x^y = -3\%$) i.e 1% change in x reasults 3% decrease in y or negative change in y.

Second Part

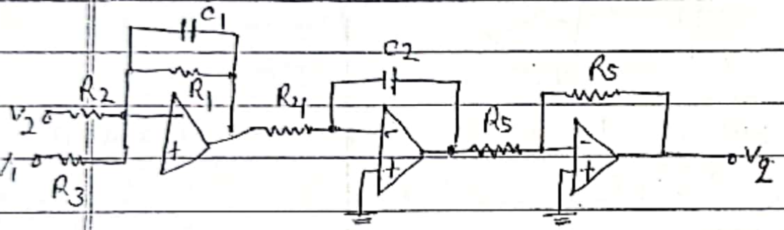


Fig:- Tow - Thomas low pass filter

Transfer function of Tow - Thomas low pass filter is

$$T(s) = \frac{-1/(R_3R_4C_1C_2)}{s^2 + \frac{1}{R_1C_1}s + \frac{1}{R_2R_4C_1C_2}}$$

$$\therefore \omega_0 = R_2^{-1/2} \cdot R_4^{-1/2} \cdot C_1^{-1/2} \cdot C_2^{-1/2}$$

And $\frac{\omega_0}{Q} = \frac{1}{R_1C_1}$ or $Q = \omega_0 R_1 C_1$

$$= R_1 \cdot R_2^{-1/2} \cdot R_4^{-1/2} \cdot C_1^{1/2} \cdot C_2^{-1/2}$$

∴ Centre frequency sensitivity: $S_x^y = \frac{x}{y}\frac{\partial y}{\partial x}$

$$\therefore S_{R_2}^{\omega_0} = \frac{R_2}{\omega_0} \cdot \frac{d(\omega_0)}{dR_2}$$

$$= R_2 \cdot R_2^{1/2} \cdot R_4^{1/2} \cdot C_1^{1/2} \cdot C_2^{1/2} \cdot \left(-\frac{1}{2}\right) \cdot R_2^{-3/2} \cdot R_4^{-1/2} \cdot C_1^{-1/2} \cdot C_2^{-1/2}$$

$$= -0.5$$

$$\therefore S_{R_4}^{\omega_0} = S_{C_1}^{\omega_0} = S_{C_2}^{\omega_0} = -0.5$$

$\boxed{S = \text{Sensitivity}}$

And, $S_{R_1}^{\omega_0} = S_{R_3}^{\omega_0} = S_{R_5}^{\omega_0} = 0$

Now,

Quality factor sensitivity :-

$$S_{R_1}^{Q} = \frac{R_1}{Q} \cdot \frac{d(Q)}{\partial R_1}$$

$$= R_1 \cdot \left(R_1^{-1} \cdot R_2^{1/2} \cdot R_4^{1/2} \cdot C_1^{-1/2} \cdot C_2^{1/2}\right) \cdot \left(R_2^{-1/2} \cdot R_4^{-1/2} \cdot C_1^{1/2} \cdot C_2^{-1/2}\right)$$

$$= 1$$

$$S_{R_2}^{Q} = \frac{R_2}{Q} \cdot \frac{dQ}{dR_2}$$

$$= R_2 \cdot \left(R_1^{-1} \cdot R_2^{1/2} \cdot R_4^{1/2} \cdot C_1^{-1/2} C_2^{1/2}\right) \cdot \left(-\frac{1}{2}\right) \cdot R_2^{-3/2} \cdot R_1 \cdot R_4^{-1/2} \cdot C_1^{1/2} \cdot C_2^{-1/2}$$

$$= -0.5$$

$$S_{R_4}^{Q} = S_{C_2}^{Q} = -0.5$$

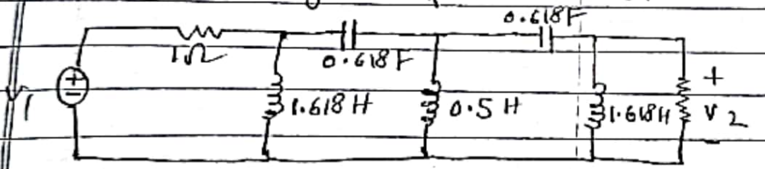
$$S_{C_1}^{Q} = \frac{C_1}{Q} \cdot \frac{dQ}{dC_1}$$

$$= C_1\left(R_1^{-1} R_2^{1/2} R_4^{1/2} C_1^{-1/2} C_2^{1/2}\right) \cdot \left(\frac{1}{2}\right) \cdot C_1^{-1/2} \cdot R_1 \cdot R_2^{-1/2} \cdot R_4^{-1/2} \cdot C_2^{-1/2} = 0.5$$

And, $S_{R_3}^{Q} = S_{R_5}^{Q} = 0$

(This question needs Quality factor Sensitivity only)

S.N.11. What is generalized impedance converter (GIC)? How can you simulate the grounded inductor in the passive filter using GIC? Explain the following circuit is a high pass filter having half power frequency of 1 rad/sec. Design a high pass filter having half power frequency of 4.5 KHz by active simulation of inductor. In your final Circuit the largest Capacitance should be 0.1 uF.



Solution

GIC is the impedance convertor circuit that removes all the inductor in the circuit.

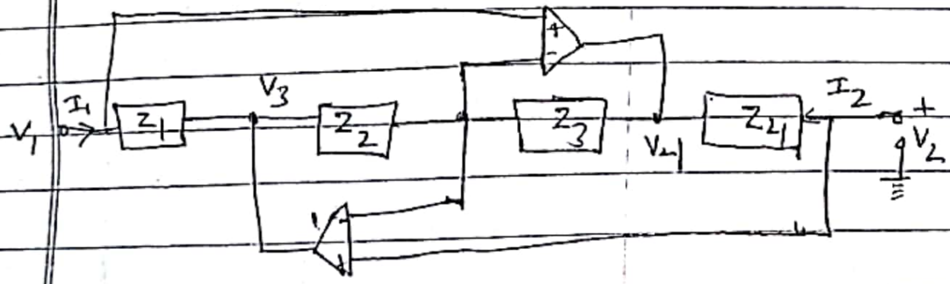
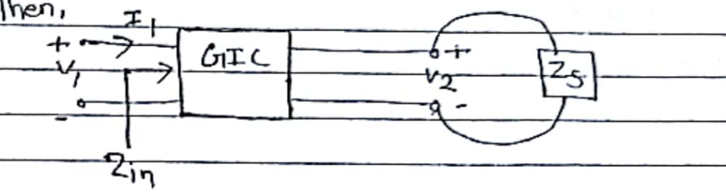


Fig: GIC

We know, $I_1 = -I_2 \dfrac{Z_2 \cdot Z_4}{Z_1 \cdot Z_3}$

If port-2 is terminated with impedance, Z_5

Then,



$Z_{in} = \dfrac{Z_1 Z_3 Z_5}{Z_2 Z_4}$, let $Z_5 = R_L$

let Z_2 or Z_4 be Capacitor and rest be resistor

$\therefore Z_1 = R_1,\ Z_2 = \frac{1}{C_2 S},\ Z_3 = R_3,\ Z_4 = R_4$

$\therefore Z_{in} = R_1 R_3 R_L \Big/ \frac{1}{C_2 S} \cdot R_4$

or, $Z_{in} = \dfrac{R_1 C_2 R_3 \cdot R_L \cdot S}{R_4} = K \cdot R_L \cdot S$

$\boxed{\therefore Z_{in} = L_{eq} \cdot S}$ --- (1)

Hence, eq^n (1) is the required expression to simulate the grounded inductor in the passive filter using GIC.

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3rd part

To achieve high pass filter having half power frequency, $\omega_0 = 2\pi \times 4.5 \times 10^3$ rad/sec, so, do frequency scaling, at $K_f = 9\pi \times 10^3$

$\therefore R(1) = 1\Omega$

$R(2) = 1\Omega$

$L(1.618) = \frac{1.618}{9\pi \times 10^3} = 57.25\ \mu H$

$L(0.5H) = \frac{0.5}{9\pi \times 10^3} = 17.69\ \mu H$

$C(0.618) = \frac{0.618}{9\pi \times 10^3} = 21.86\ \mu F$

To achieve the largest capacitor $0.1\mu F$, do magnitude scaling,

$K_m = \frac{21.86\mu F}{0.1\mu F} = 218.6$

$\therefore C(21.86\mu F) = 0.1\mu F$

$R(1) = 1 \times 218.6 = 0.2186\ k\Omega$

$L(57.25) = 57.25 \times 218.6 = 12.514\ mH$

$L(17.69) = 17.69 \times 218.6 = 3.867\ mH$

Now, simulate by using FDNR, scale every element by $1/S$,

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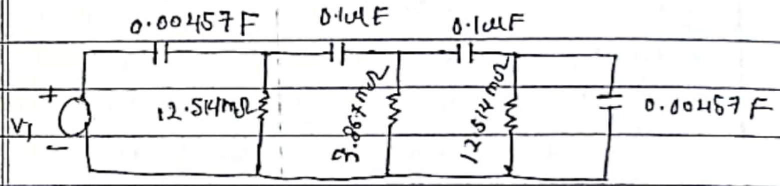
∴ Inductor is replaced by resistor (Value = L)

Resistor is replaced by capacitor (value = $1/R$)

Capacitor is replaced by FDNR (Value = C)

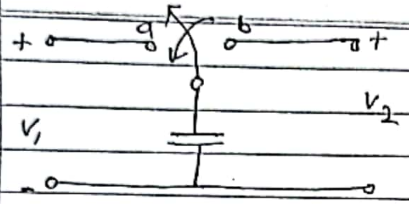
Then, the circuit diagram is,



S.N.12) What is the switched capacitor filter? What are its applications? How can you simulate a resistor using switched capacitor? Explain with necessary derivation.

Solutions

⇒ Any filter (active or passive) contain resistor. But, resistor takes larger space on IC when fabricated. Therefore, the resistor is replaced by the circuit combination of MOSFET and capacitor. Those filters are called switched capacitor filter.



Fig(a) Switched - Capacitor.

Second Part

From the figure(a),

let switch is at position 'a',

⇒ Capacitor is fully charged $[V_c = V_1]$

∴ Charge on Capacitor, $q_c = C \cdot V_c$

Now,

Charge switch from 'a' to 'b' ; Discharging.

$i = \frac{q_c}{T_c}$, T_c ⇒ discharging time

or, $i = \frac{(V_c - V_2) \cdot C}{T_c} = \frac{(V_1 - V_2) \cdot C}{T_c}$

or, $\frac{V_1 - V_2}{i} = \frac{T_c}{C}$

or, $R = \frac{1}{C \cdot f_c}$

$\boxed{\therefore\ C = \frac{1}{R \cdot f_c}}$ Hence, the resistor is replaced by the above circuit, where the Capacitor's value equals $1/R \cdot f_c$.