2069 CHAITRA

→ REGULAR

1. What are the advantages of digital communication system as compared to analog communication system? Eleborate the importance of source and channel encoders in Digital communication system.

⇒ The advantages of digital communication system compared to analog c. system are :-

i) Higher noise immunity
ii) long distance transmission with greater accuracy
iii) Multiplexing
iv) Data encryption.
v) DCS is inherently more efficient in realizing the exchange of SNR for bandwidth.
vi) By channel coding error may be detected and corrected at receiver.

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Source encoder :-

It converts the sequence of symbols at its input into binary sequence of 0's and 1's by assigning code word to each symbol. An optimum source encoder is that which has output data rate nearly or equal to input data rate.

channel encoder :-

It is used to enhance reliability and efficiency of high speed digital signal transmission. It adds some error controlling bits to its input binary sequence but doesnot carry any information. These error control bits make possible for the receiver to detect and correct errors in message bearing bits.

2. What do you mean by aperture effect in aliasing? sampling? How can it be corrected? A band pass signal with the spectrum in the range of 80-115 KHz is to be digitized, calculate the minimum sampling frequency required for the signal.

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⇒ The real sampling pulse is flat topped with finite duration z. The net result is;

$$x_s(t) = \sum_{k=-\infty}^{\infty} x(kT_s)\, P(t - kT_s) \quad \text{-------(i)}.$$

where;

$P(t)$ = sampling pulse duration of z

In other word, Finding convolution of flat topped pulse and ideally sampled signal can derive the real sampled signal.

$$x_s(t) = P(t) * \left[\sum_{k=-\infty}^{\infty} x(kT_s)\,\delta(t - kT_s)\right] \quad \text{-----(ii)}$$

& the spectrum of the sampled signal will be

$$x_s(f) = P(f) \cdot x_\delta(f)$$

$$= P(f) \left\{ f_s \sum_{n=-\infty}^{\infty} x(f - nf_s) \right\} \quad \text{-----(iii)}$$

where $P(f)$ = sinc function

By using the flat top samples, an amplitude distortion is introduced in the reconstructed signal $x(t)$ from $g(t)$. In fact the high frequency roll off of $H(f)$ acts like the LPF and thus attenuates the upper portion of message signal spectrum. These high frequency of $x(t)$ are effected & this type of effect is known as aperture effect.

$$H(f) = z\, \text{sinc}(f \cdot t)\, e^{-j\pi f z} \quad \text{-------(a)}$$

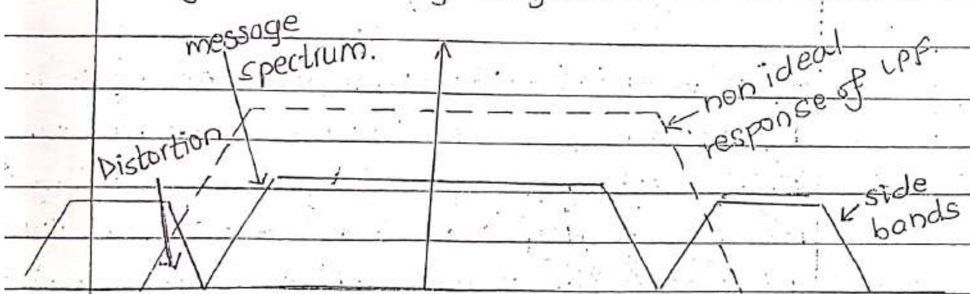
Solution of aperture effect.

i) Using the sampling pulse as narrow as possible. If $\tau << fs$ then $g(f)$ is more or less constant over message frequency band and aperture effect can be neglected.

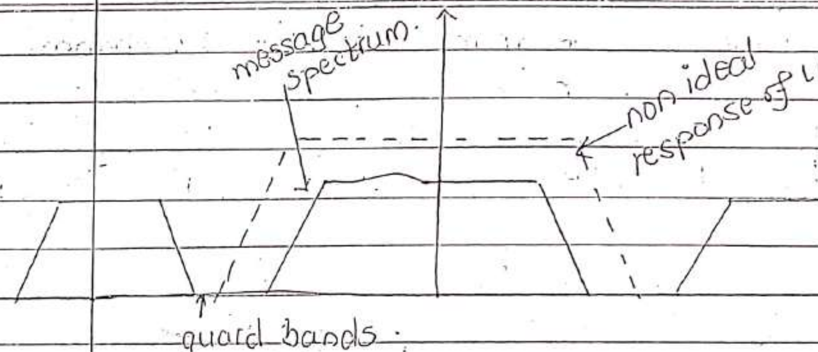
ii) By the use of equalizer during reconstruction, having the transfer function of

$$H_{eq} = \frac{1}{P(f)}$$

iii) Effect of non-ideal reconstruction filter is that portions of side band will also be filtered out along with message signal.



Good filter design may eliminate this problem. other way to minimize this effect is to introduce the guard band by selecting Fs slightly higher than $2f_x$.



for numerical portion:-

$$m = \left\lfloor \frac{f_{mu}}{B} \right\rfloor = \frac{115}{35} = 3.285 = 3 \text{ (flooring)}$$ -5-

$$F_{S} = \frac{2f_m}{m} = \frac{2*115}{3} = 76.66 \text{ KHZ.}$$

3. Explain the E_1 digital hierarchy. A speech signal with maximum frequency of 4 KHZ and maximum amplitude of ±1.1 V is applied to a PCM system with its bit rate of 32 kbps. Calculate the SQNR and no of bits per sample.

⇒ E_1 Digital hierarchy:-

It multiplex 30 voice channels. One frame contains a signalling channel and a synchronization channel so one frame contains 32 channel each of 8 bit.

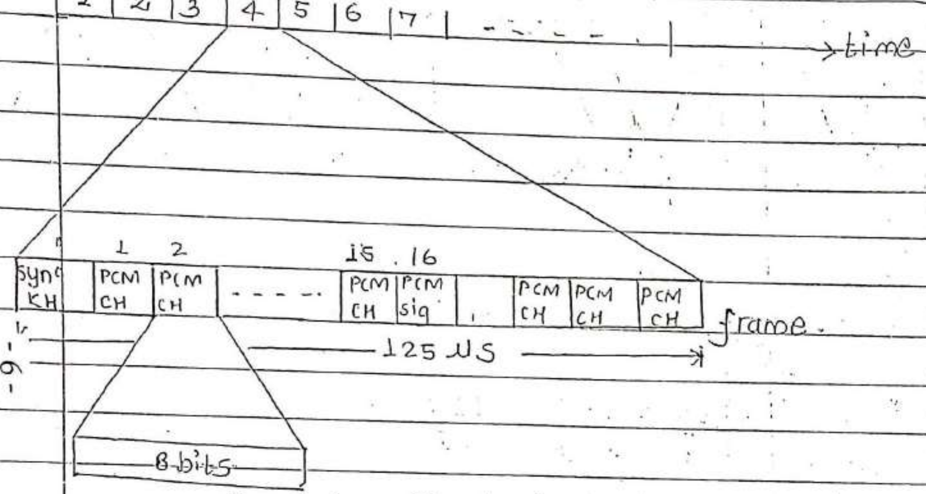


fig:- Frame structure of E_1 system

Time slot :- 1-15 and 17-30 } voice channels

Time slot 0 :- synchronization channel.

Time slot 16 :- signalling channel.

Total no of bits in a frame = 32 * 8 = 256 bits/frame

ie in 125 µs → 256 bit Tx

so bit rate = $\frac{256}{125 * 10^{-6}}$ $\frac{bits}{sec}$

= 2.048 Mbps

Bit rate (R) = 8000 frames/sec * 256 bits/frame
= 2.048 mbps

Maximum duration of each bit

$T_{bmax} = \frac{125 \mu s}{256} = 0.488 \mu s$.

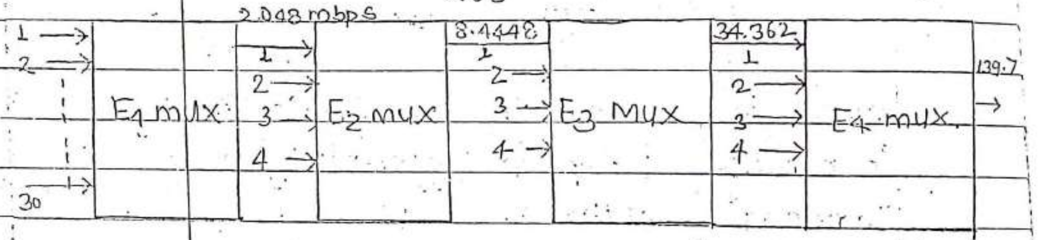


fig:- E_1 Digital hierarchy

Groups	No of voice channels	bit rate (mbps)
E_1	30	2.048
E_2 ($4E_1$)	120	8.448
E_3 ($4E_2$)	480	34.362
E_4 ($4E_3$)	1920	139.764

Soln;

$f_s = 8$ KHZ (2×4)

$\pm A = \pm 1.1$ V

Bit rate = 32 kbps = f_s sample/sec

or, $32 = 8n$

or, $n = 4$ bits/sample.

$\therefore$ SQNR = $4.8 + 6n$

$= 4.8 + 6 * 4$

$= 28.8$ dB.

4. What do you mean by companding? Why is it necessary? Explain different types of companding methods.

⇒ In companding process weak signals are amplified and strong signals are attenuated before applying them to a uniform quantizer. This process is called compression. At the receiver exactly opposite process is followed called expansion.

i/p → [compressor] → [uniform quantization] → [expander] → o/p

Fig:- companding modes

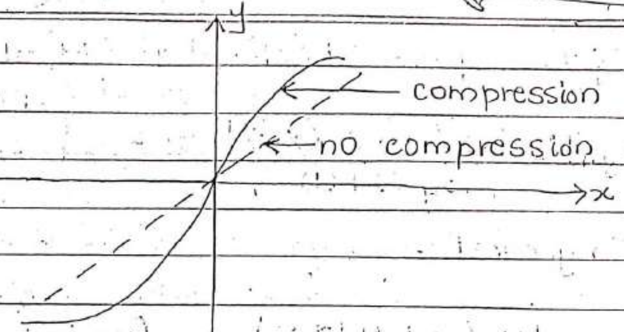


Fig:- Transfer characteristics of compressor

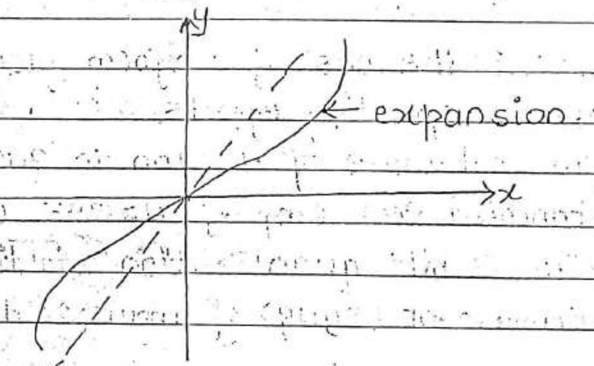


fig:- expander characteristics

Two most commonly used compression laws are:-

i) μ-law :- American standard :- US, Canada, Japan

⇒ In μ law the output signal 'y' is related to the normalized input signal level as:-

$$|y| = \frac{\log(1+\mu|x/x_{max}|)}{\log(1+\mu)} \quad \text{---- (i)}.$$

If x and y are normalized to (+1, -1) then

$$|y_n| = \frac{\log(1+\mu(x_n))}{\log(1+\mu)} \quad \text{-------(ia)}$$

In general :-

$$y_n = \frac{\log(1+\mu|x_n|)}{\log(1+\mu)} \, \text{sgn}(x_n) \quad \text{-----(ii)}$$

→ $\mu = 0$ is the case of uniform quantization with no corresponding effects.
→ Practical value of μ = 100 to 300
→ standard PCM employ $\mu = 255$ compander with 7 bits quantization yielding system improvement (SQNR) of about 24 dB.

ii) A-law
· European standard ; Europe, Nepal.
→ A law compander utilizes the following relationship between input and output signal levels.

$$|y_n| = \begin{cases} \dfrac{A|x_n|}{1+\log A} \; ; \text{ for } 0 \le |x_n| \le 1/A \end{cases}$$

&

$$y|n| = \begin{cases} \dfrac{1+\log(A|x_n|)}{1+\log A} \; ; \text{ for } 1/A \le |x_n| < 1 \end{cases}$$

→ A law compressor characteristics is piece-wise made of linear segments for low level inputs and logarithmic segment for higher level inputs.
→ For practical purpose the value of A is choosen around 100 and improvement is about 25 dB.

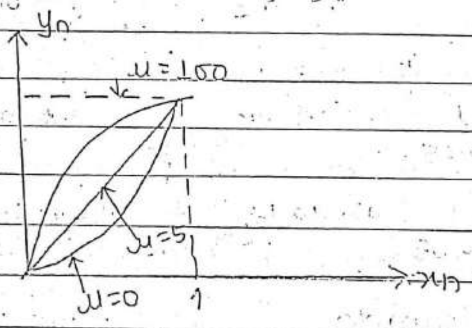


Fig:- μ-law

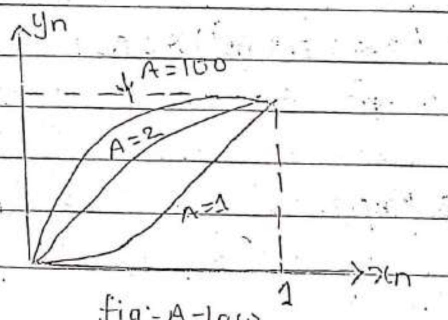


fig:- A-law

5. A signal of bandwidth 4.5 KHZ is sampled at double rate given by Nyquist, the signal is quantized in 8 levels, the probability of occurance of level are 0.1, 0.15, 0.15, 0.05, 0.2, 0.05, 0.18, 0.12. Find the minimum no of bits per sample and information rate.

⇒ sol^n,

Entropy $= \sum P_i \log_2 \frac{1}{P_i}$

$= 0.1 \log_2 \frac{1}{0.1} + 0.15 \log_2 \frac{1}{0.15} +$

$0.15 \log_2 \frac{1}{0.15} + 0.05 \log_2 \frac{1}{0.05} + 0.2 \log_2 \frac{1}{0.2} +$

$0.05 \log_2 \frac{1}{0.05} + 0.18 \log_2 \frac{1}{0.18} + 0.12 \log_2 \frac{1}{0.12}$

$=$ 2.86 bits/sample.

$R_{info} = 2 * (2 * 4.5 * 10^3) * 2.86$

$= 51480$ bps

$= 51.48$ Kbps.

6. What is ISI? explain any 2 practical methods of minimizing ISI

⇒ The residual effect of all the unnecessary transmitted bits on the m^{th} bit being decoded is known as ISI.

In PAM;

$$y = \mu A_n + \underbrace{\mu \sum_{\substack{k=-\infty \\ k \neq n}}^{\infty} A_k P(nT_b - kT_b)}_{\text{residual effect.}}$$

ISI carries due to the dispersion of pulse shape by filters and channels. Therefore one of the major task of system designer is to optimally design transmitting/receiving filters and the shape of the basic pulse to minimize ISI.

Methods of minimizing ISI.

i) The function which produces zero ISI is the sinc function hence instead of rectangular pulses if we transmit sinc pulse then ISI can be reduced to zero. This is known as Nyquist pulse shaping.

we have;

$$y(nT_b) = \mu A_n + \sum_{\substack{k=-\infty \\ k \neq n}}^{\infty} \mu A_k \, p(nT_b - kT_b)$$

$$= \mu \sum_{k=-\infty}^{\infty} A_k \, p(nT_b - kT_b)$$

for zero ISI.

$$p(nT_b - kT_b) = \begin{cases} 1 & ; k = n \\ 0 & ; k \neq n \end{cases}$$

ie $$p(mT_b) = \begin{cases} 1 & ; m = 0 \\ 0 & ; m \neq 0 \end{cases}$$

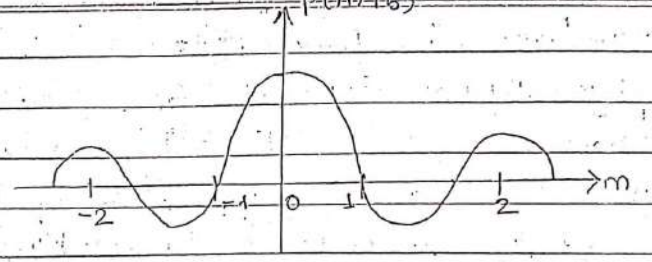
The pulse $p(mT_b)$ with above condition is nothing but a sinc function.

$$p(t) = \text{sinc}\,(2\pi B_o t)$$

$$= \frac{\sin(2\pi B_o t)}{2\pi B_o t}$$

bit rate.

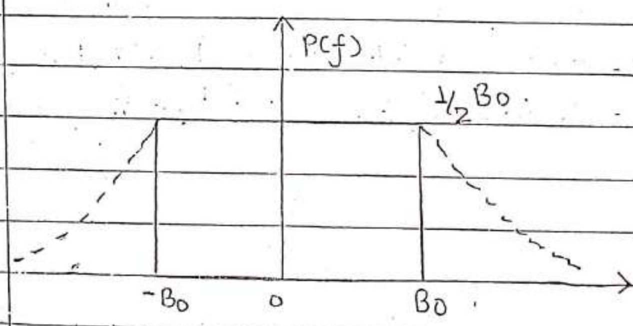
where, $B_o = \frac{1}{2T_b} = \frac{R_b}{2}$



ii) The fourier transform of sinc pulse is rectangular function hence to preserve all the frequency components the frequency response of filter must be exactly flat in the pass band and zero in the attenuation band.

The fourier transform of $p(t)$ is:

$$P(f) = \begin{cases} 1/2B_o & ; \quad 0 \le |f| \le B_o \\ 0 & ; \quad B_o < |f|. \end{cases}$$



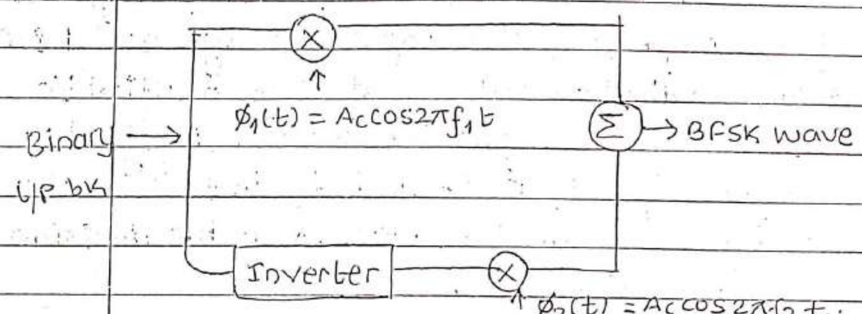
The ideal sinc function is non-causal and physically non realizable. But we can approximate p(t) so that the shape of the pulse is near to sinc function. The idea is to make P(f) to roll off at the ends gradually then abruptly and have maximum span of flat top one of the such approximation is called raised cosine frequency characteristics.

7. Explain FSK modulation with its modulator demodulator and signal space diagram.

⇒ In FSK, the frequency of the carrier is shifted according to the binary symbol. Amplitude & phase of the carrier is kept constant so we can have two different frequency carrier signal.

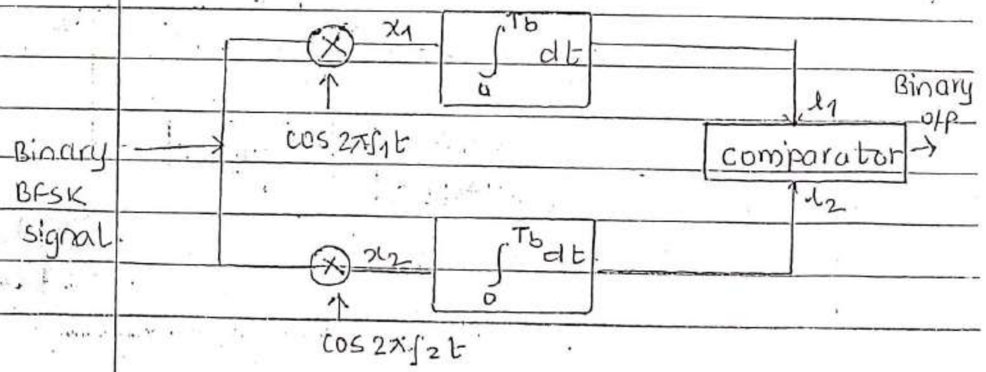
$$s(t) = A_c \cos 2\pi f_1 t \quad \text{for } b_k = 1$$
$$= A_c \cos 2\pi f_2 t \quad \text{for } b_k = 0.$$

Generation of FSK:-



when symbol 1 is at the input, output of lower product modulator is zero. whereas the upper product modulator is switched 'ON' transmitting f_1 frequency signal. But for symbol '0' lower product modulator is activated and transmit f_2 frequency signal.

Coherent Detection of BFSK.



→ The detector consists of two correlators that are individually termed to two different carrier frequencies to represent symbols 1 & 0
→ The multiplied output of each multiplier is subsequently passed through integrators generating output l_1 and l_2 in two paths which is compared by the comparator decision making device.

Decision making rule

$$\hat{b}_k = 1 \quad \text{if } l_1 > l_2$$
$$0 \quad \text{if } l_1 < l_2$$

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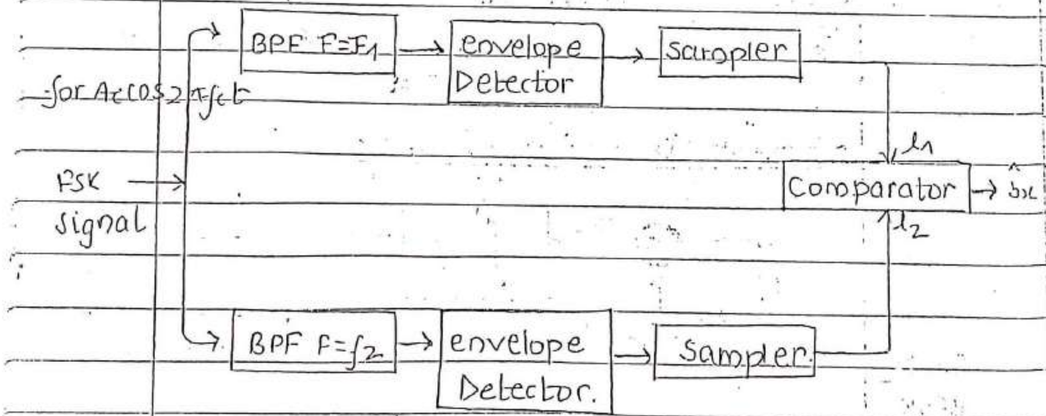
Non-coherent detection of FSK.



Binary FSK may be demodulated non-coherently using envelope detector. The received FSK signal is applied to the bank of 2 band pass filters tuned to frequencies f_1 and f_2. Each filter is followed by an envelope detector. The resulting of the two envelope detectors are sampled and then compared by the comparator and decision is made as a rule stated above.

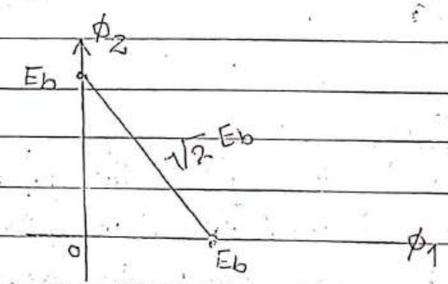
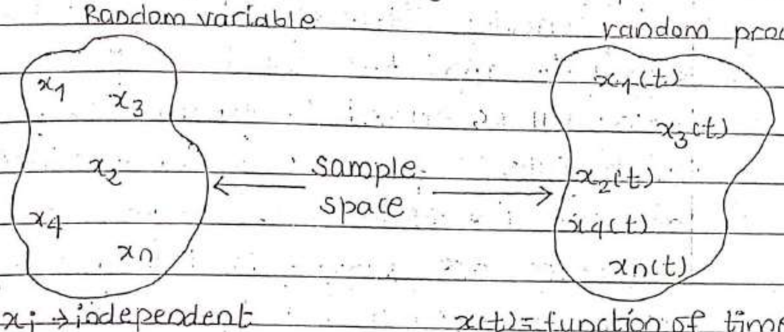


Fig:- space diagram.

8. What do you mean by random process? Explain white noise with its psdf and auto correlation function

⇒ The notion of the random process is an extension of random variables. A random variable which is a function of time is called random process. The collection of all the possible waveform is called ensemble (correspo to the sample space) of random process.



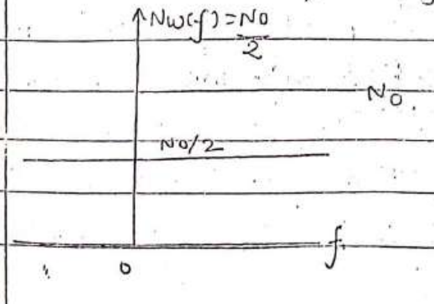
$x_i \to$ independent of time $\qquad$ $x(t)$ = function of time

white noise.

In communication system the noise analysis is based on ideal / white noise. Power spectrum density of white noise is independent of the frequency. This means that white noise has flat spectrum density $N_w(f)$ over $-\infty < f < \infty$.

$N_w(f) = N_0/2 \;;\; -\infty < f < \infty \leftarrow$ bilateral case.

$= N_0 \;;\; 0 \le f \le \infty \leftarrow$ unilateral case.



→ The average or mean value of noise is 0.

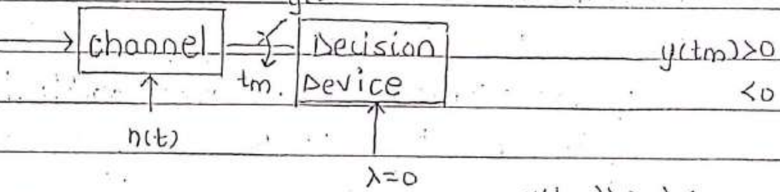
→ If the probability of occurance of white noise is specified by a Gaussian distribution factor (function) it is called white Gaussian noise.

→ Autocorrelation function of white noise may be obtained by simply taking the inverse fourier transform

$$R_{WN}(\tau) = \frac{N_0}{2}\,\delta(\tau) \quad ----(*).$$

$R_{WN}(\tau) = 0$ for $\tau \neq 0$, two different samples of WN no matters how close they are in time shift ($\tau \to 0$) are uncorrelated ie extremely random.

9. Derive the expression for error probability for binary PAM system and extend it to M-ary system.



$y(t_m) > 0 \to 1$

$< 0 \to 0.$

In baseband no carrier modulation is carried out (PAM, PWM, PPM, PCM). PAM is most commonly used so lets analyse PAM system.

The signal at the output of the decision makes at $t = t_m$ is

$y(t_m) = A_m + n_0(t_m) + ISI$

where, $A_m = \begin{cases} +A & \text{if } b_m = 1 \\ -A & \text{if } b_m = 0 \end{cases}$

The decision making device compares $y(t_m)$ against threshold level $\lambda = 0$.

If $y(t_m) > 0 \rightarrow y(t_m)$ is decoded to 0

& $y(t_m) < 0 \Rightarrow y(t_m)$ is decoded to 1.

The presence of noise and ISI may introduce errors in decision making

Error conditions are :-

$y(t_m) > 0$ when bit 0 transmitted.

$y(t_m) < 0$ " " 1 "

Let us assume noise is present but ISI = 0 in the system. consider bit sequence 0 and 1 are equiprobable and statistically independent

$P(b_m = 0) = P(b_m = 1) = \frac{1}{2}$.

and $P[y(t_m) > 0 / b_m = 0]$ is probability of $y(t_m) > 0$ when binary 0 is transmitted.

$P(y(t_m) < 0 / b_m = 1)$ is probability of $y(t_m) < 0$ when binary 1 is transmitted.

∴ Total error probability is :-

$$P_e = P[y(t_m) > 0 \cap b_m = 0] + P[y(t_m) < 0 \cap b_m = 1]$$

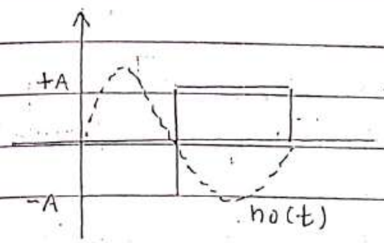
$$= P[y(t_m) > 0 / b_m = 0] * P(b_m = 0) + P[y(t_m) < 0 / b_m = 1] * P(b_m = 1)$$

or, $$P_e = \frac{1}{2} \{ P[y(t_m) > 0 / b_m = 0] + P[y(t_m) < 0 / b_m =$$

in terms of noise above expression can be written as;

$$Pe = \frac{1}{2}\left\{P[n_o(t_m) > A] + P[n_o(t_m) < -A]\right\}$$

$$= \frac{1}{2}\left\{P[|n_o(t_m)| > A]\right\}$$



As input noise n(t) is zero.
mean Gaussian process with variance No so,
output noise $n_o(t)$ is also Gaussian process
with variance N_o

So,

$$Pe = \frac{1}{2}\int_{|x|>A}^{\infty} \frac{1}{\sqrt{2\pi N_o}} \exp\left(-\frac{x^2}{2N_o}\right)dx$$

let,

$$z^2 = \frac{x^2}{2N_o}$$

or, $x = \sqrt{2N_o z}$

$$\therefore Pe = 2 * \frac{1}{2}\int_{A/\sqrt{2N_o}}^{\infty} \frac{1}{\sqrt{\pi}} \exp(-z^2)\,dz$$

$$= \frac{1}{2}\int_{A/\sqrt{2N_o}}^{\infty} \frac{2}{\sqrt{\pi}} \exp(-z^2)\,dz$$

$$= \frac{1}{2}\,\mathrm{erfc}\left(\frac{A}{\sqrt{2N_o}}\right)$$

$$= \frac{1}{2}\,\mathrm{erfc}(u)$$

where: $\mathrm{erfc}(u) = \frac{2}{\sqrt{\pi}}\int_{u}^{\infty} \exp(-z^2)\,dz$.

In M-ary,

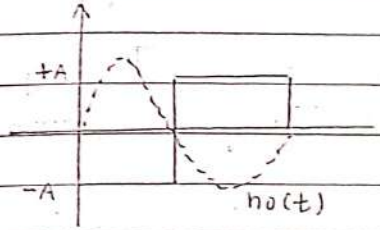
$$Pe = \frac{m-1}{m}\,\mathrm{erfc}\left(\frac{A}{\sqrt{2N_o}}\right)$$

10. Explain the threshold effect in non-coherent detection of FM signal. How can it be corrected?

⇒ Detection gain ($\gamma = \frac{SNR_o}{SNR_i}$)
is proportional to the β^2 for given SNR.
Rise in β increases γ or SNR_o but with increase in β the system bandwidth increases
$B = 2(\beta+1)f_m$ ← carson's rule.

$$P_e = \frac{1}{2}\left\{P[n_o(t_m) > A] + P[n_o(t_m) < -A]\right\}$$

$$= \frac{1}{2}\left\{P[|n_o(t_m)| > A]\right\}$$



As input noise $n(t)$ is zero mean Gaussian process with variance N_0 so, output noise $n_o(t)$ is also Gaussian process with variance N_0

So,

$$P_e = \frac{1}{2}\int_{|x|>A}^{\infty} \frac{1}{\sqrt{2\pi N_0}} \exp\left(-\frac{x^2}{2N_0}\right)dx$$

let,

$$z^2 = \frac{x^2}{2N_0}$$

or, $x = \sqrt{2N_0 z}$

$$\therefore P_e = 2 * \frac{1}{2}\int_{A/\sqrt{2N_0}}^{\infty} \frac{1}{\sqrt{\pi}} \exp(-z^2)\,dz$$

$$= \frac{1}{2}\int_{A/\sqrt{2N_0}}^{\infty} \frac{2}{\sqrt{\pi}} \exp(-z^2)\,dz$$

$$= \frac{1}{2}\operatorname{erfc}\left(\frac{A}{\sqrt{2N_0}}\right)$$

$$= \frac{1}{2}\operatorname{erfc}(u)$$

where: $\operatorname{erfc}(u) = \frac{2}{\sqrt{\pi}}\int_u^{\infty} \exp(-z^2)\,dz.$

In M-ary,

$$P_e = \frac{M-1}{M}\operatorname{erfc}\left(\frac{A}{\sqrt{2N_0}}\right)$$

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⇒ Detection gain ($\gamma = \frac{SNR_o}{SNR_i}$) is proportional to the β^2 for given SNR. Rise in β increases γ or SNR_o but with increase in β the system bandwidth increases

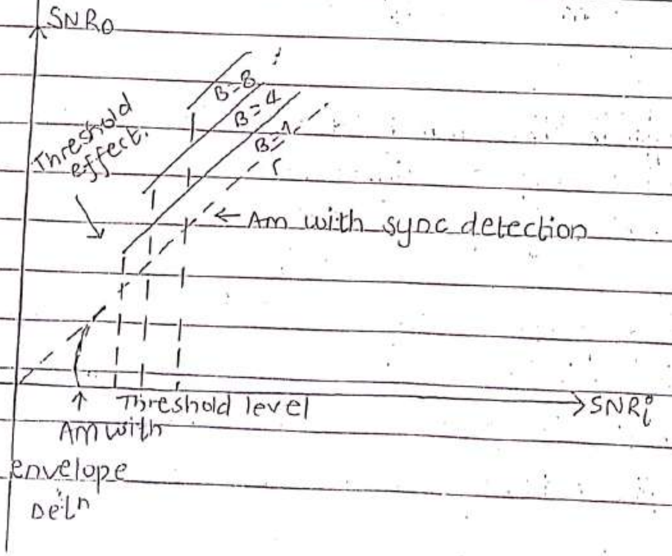
$B = 2(\beta+1)f_m$ ← carson's rule.

($B\uparrow$ $\gamma\uparrow$ $SNR_o\uparrow$ $B\uparrow$ noise at input $(2B \times N_0)\uparrow$

Increase in BW increases the input noise power and decreases SNR_i so the assumption $A_c >> n(t)$ is not more valid.

ie when the noise power at the demodulator input is comparable to the carrier power, the threshold phenomena appears. In FM this effect is much more pronounced then in AM.

In otherword, For SNR_i less than threshold level FM system will no more detect the message signal. This effect is called threshold effect.



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The threshold effect in FM can be reduced by the following techniques.

i) use of PLL can reduce the threshold level by the order of 5-7 dB.

ii) use of Pre-emphasis, de-emphasis network improves system performance by about 6 dB.

11. Derive the expression of error probability for coherent detection of ASK.

⇒ For ASK,

$x_1(t) = A\cos 2\pi f_c t$ for $b_k = 1$ $0 \le t \le T$

$x_2(t) = 0$.

now,

Probability of error

$$P_e = \frac{1}{2}\, \text{erfc}\left\{ \frac{n_{01}(T) - x_{02}}{2\sqrt{2}\,\sigma} \right\}$$

$$= \frac{1}{2}\, \text{erfc}\left\{ \frac{\gamma}{2\sqrt{2}} \right\}$$

$$\gamma^2_{max} = \int_{-\infty}^{\infty} \frac{|x(f)|^2}{S_N(f)}\, df$$

$$S_N(f) = N_0/2$$

$$Y^2_{max} = \frac{2}{N_0}\int_{-\infty}^{\infty} |x(f)|^2 df$$

By Parseval's theorem;

$$Y^2_{max} = \frac{2}{N_0}\int_{-\infty}^{\infty} x^2(t)\, dt.$$

& $x(t) = x_1(t) - x_2(t)$

$= A\cos 2\pi f_c t - 0$

$= A\cos 2\pi f_c t$

$$Y^2_{max} = \frac{2}{N_0}\int_0^T A^2\cos^2 2\pi f_c t\, dt$$

$$= \frac{2}{N_0} A^2 \frac{T}{2}$$

$$= \frac{TA^2}{N_0}$$

$$\therefore P_e = \frac{1}{2}\,\mathrm{erfc}\left\{\sqrt{\frac{A^2T}{N_0}} \cdot \frac{1}{2\sqrt{2}}\right\}$$

Power of signal $= A\cos 2\pi f_c t = \frac{A^2}{2}$ and

energy$(E) = P\cdot T = \frac{A^2T}{2}$

$$\therefore P_e = \frac{1}{2}\,\mathrm{erfc}\left\{\sqrt{\frac{E}{4N_0}}\right\}.$$

now, average energy per bit

$E_b = (0 - E)/2$

$$\therefore P_e = \frac{1}{2}\,\mathrm{erfc}\left\{\sqrt{\frac{E_b}{2N_0}}\right\}$$

or, $P_e = \mathrm{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right).$

12) write short notes on

a) eye diagram.

b) syndrome calculation in linear block codes.

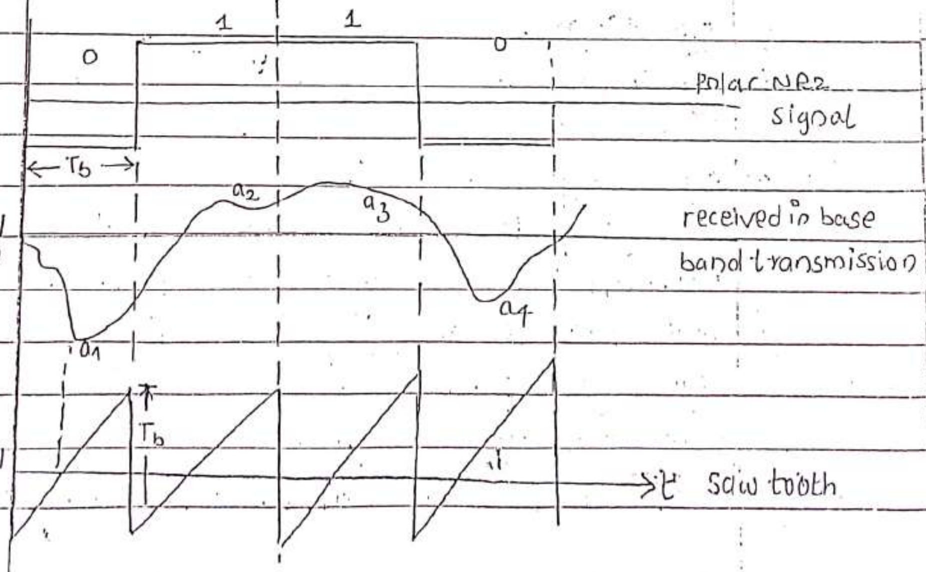
a) eye diagram.

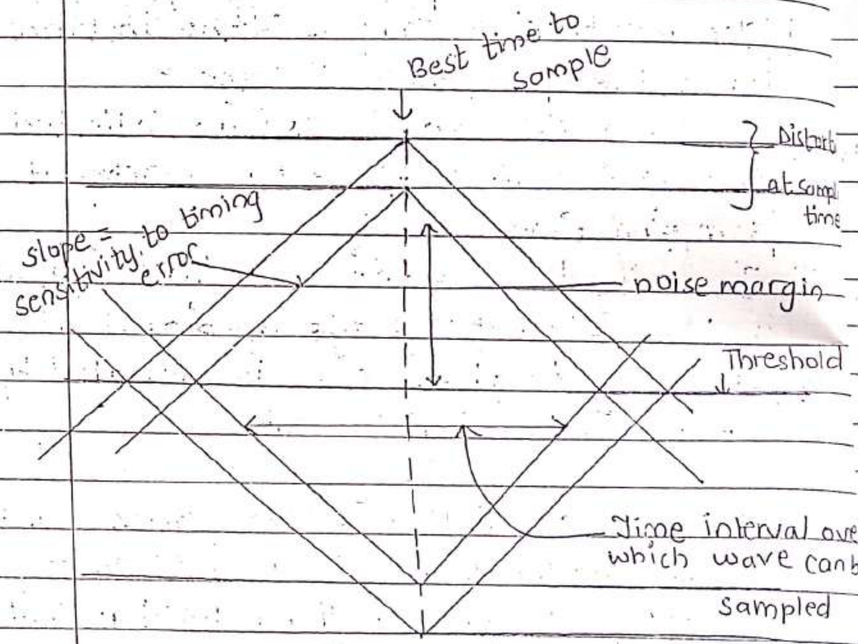
An eye pattern plot is the simple and convenient tool to study the effect of ISI and other channel impairments for digital transmission. It is a multiple overlaid trace plot of the received signal used to determine the optimal demapping rate, by finding where the eye is most widely open.

Eye pattern is the pattern displayed on the screen of the cathode Ray oscilloscope (CRO). The eye pattern is obtained by

→ applying received wave (duration $T = T_b$) to to the vertical input (deflection plate) and saw both wave with duration $T = T_b$ or $2T_b$ to the horizontal input (deflection plates). The resulting is similar to the eye of human so called as eye pattern.

→ The eye pattern provides an excellent way of accesing the quality of the received line code and the ability of the receiver to combat bit error.





The interior region of eye pattern is called eye opening.

The information provided by eye diagram.

i) The best time to sample is the instance when the eye opening is largest.

ii) Maximum distortion and ISI are indicated by the vertical width of the two branches at the sampling time.

iii) Immunity to noise (noise margin) is indicated by the width of the eye opening.

iv) Sensitivity to timing error is indicated by the rate of closing the eye.

b) Syndrome calculation in Linear block codes

For the detector to detect or correct errors by sorting bit pattern of all possible valid code words more memory is required for large block lengths. To avoid this problem syndrome decoding is used. For this purpose, parity check matrix H is used.

module-2 addition of any sequence with itself is 0.

ie $DP \oplus DP = 0$.

or, $DP \oplus CP = 0$

or; $\begin{bmatrix} D & CP \end{bmatrix} \begin{bmatrix} P \\ I_{n-k} \end{bmatrix} = 0$

or, $CH^T = 0$

$$H = \begin{bmatrix} p_{11} & p_{21} & \cdots & p_{k,1} \\ p_{12} & p_{22} & \cdots & p_{k,2} \\ \vdots & & & \\ p_{1,n-k} & p_{2,n-k} & \cdots & p_{k,n-k} \end{bmatrix}$$

$$(H) = \left[P^T / I_{n-k} \right]_{n-k,n}$$

where,

$$H^T = \begin{bmatrix} P \\ I_{n-k} \end{bmatrix}_{n \times n-k}$$

= parity check matrix.

Let the received code vector is R and E be the error vector

Then

$R = C \oplus E$

now, the term RH^T is called syndrome and is denoted by S.

ie, $S = RH^T$ (used to detect error in R)

or, $S = (C \oplus E) H^T$

or, $S = CH^T \oplus EH^T$

or, $S = EH^T$ $\left\{ \because CH^T = 0 \right.$

ie syndrome depends on error probability only and doesnot depend on particular message vector. If there is no error then syndrome S is zero.

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2070 ASADH

[NEW BACK]

1. What is source coding? Develop Huffman coding of 5 symbol source with probabilities $S_0 = 0.3$, $S_1 = 0.25$, $S_2 = 0.2$, $S_3 = 0.15$, $S_4 = 0.1$ and also calculate coding efficiency.

⇒ Conversion of output of DMS into a sequence of binary symbol (binary codeword) is source coding and the device is source encoder.

Soln,

0.3	00	0.3	00	0.45	1	0.55	0
0.25	01	0.25	01	0.3	00	0.45	1
0.2	11	0.25	10	0.25	01		
0.15	100	0.2	11				
0.1	101						

$L = 0.3 \times 2 + 0.25 \times 2 + 0.2 \times 2 + 0.15 \times 3 + 0.1 \times 3$

$= 2.25$ bits/symbol

$H = \sum P_i \log_2 \frac{1}{P_i}$

$= 0.3 \log_2 \frac{1}{0.3} + 0.25 \log_2 \frac{1}{0.25} + 0.2 \log_2 \frac{1}{0.2} +$

$0.15 \log_2 \frac{1}{0.15} + 0.1 \log_2 \frac{1}{0.2}$

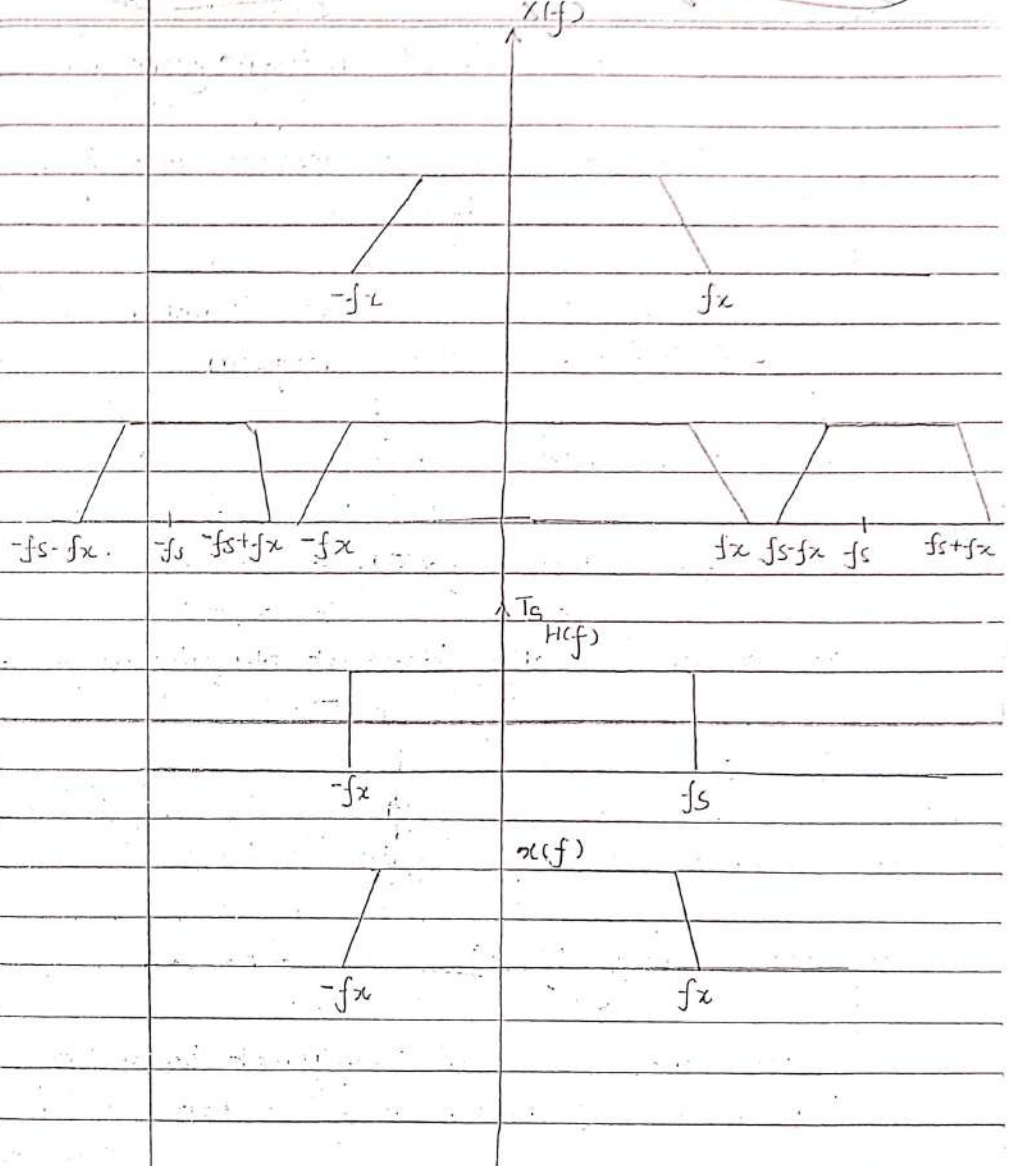
∴ H = 2.228 bits/symbol

& efficiency $= \frac{H}{L} = \frac{2.228}{2.25} \times 100$

$= 99.02\%$

2. With mathematical derivation show that original band limited signals can be reconstructed from its samples taken at Nyquist rate.

⇒ We know that the spectrum of sampled signal consists of spectrum of original signal and its periodic replica with period of Fs. If the sampling was done with the rate greater than Nyquist rate then original signal through LPF of bandwidth + fx Hz and gain Ts



Consider a ideal LPF of transfer function

$$H(f) = \begin{cases} T_s & |f| \le f_x \\ 0 & \text{otherwise} \end{cases} \quad \text{or} \quad H(f) = T_s \, \text{rect}\left(\frac{f}{2f_x}\right)$$

Corresponding impulse response is obtained by finding the fourier transform.

$$h(t) = \mathcal{F}^{-1}[H(f)]$$

$$= 2f_x T_s \, \text{sinc}(2f_x t).$$

Assuming sampling done at Nyquist rate

$$T_s = \frac{1}{2f_x};$$

or, $2T_s f_x = 1$

or, $h(t) = \text{sinc}(2f_x t).$

hence $h(t)=0$ at all Nyquist sampling interval $t = \pm nT_s$ except at $t=0$.

If $x_s(t)$ be the sampled signal of $x(t)$ then,

$$x_s(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\,\delta(t-nT_s)$$

when $x_s(t)$ is input to the ideal LPF it produces the output $x(t)$

$$\therefore \; x(t) = h(t) \otimes \sum_{n=-\infty}^{\infty} x(nT_s)\,\delta(t-nT_s)$$

or, $$x(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\, h(t-nT_s).$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s)\, \text{sinc}(2f_x(t-nT_s))$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s)\, \text{sinc}(2f_x t-n).$$

This equation is known as interpolation formula, which yields values of $x(t)$ between samples a weighted sum of all the sample values

2b) What is aliasing effect and how can it be minimized?

⇒ Refer Q.No:-2 chaitra 2070.

3a Find SQNR in uniform quantization in terms of no of bits per source sample

⇒ In PCM system, SQNR is given as

$$SQNR = \frac{\text{average signal power}}{\text{average quantization noise power}} \quad ---(i)$$

average quantization noise power

$$P_q = \frac{\Delta^2}{12} = \frac{1}{3}\left(\frac{x_{max}}{2^{n-1}}\right)^2$$

$$= \frac{x^2_{max}}{3 \times 2^{2} \times 2^{2n-2}}$$

$$= \frac{x^2_{max}}{3 \times 4^n}$$

Let $x(t)$ be the desired signal. Average power of message signal $x(t)$ is $\overline{x^2}$

so,

$$SQNR = \frac{\overline{x^2}}{x^2_{max}} \times 3 \times 4^n$$

$\frac{\overline{x^2}}{x^2_{max}}$ → normalized signal power ≤ 1 $(\tilde{x^2})$

$SQNR = 3 \times 4^n \times \tilde{x^2}$

Upper Limit

$SQNR = 3 \times 4^n$

This expression shows that SQNR increases exponentially with increasing bits per sample.

In terms of dB,

$$(SQNR)_{dB} = 10 \log_{10} (3 \times 4^n)$$

$$= 10 \log_{10} 3 + 10\, n \log_{10} 4$$

$$= (4.8 + 6n)\, dB.$$

So for each extra bit n used for representing each quantization level, SQNR is increased by 6 dB (4 times)

Further in terms of level of Quantization

$$SQNR = 3 \times 4^n$$
$$= 3 \times (2^n)^2$$
$$= 3 \times N^2$$

$$SQNR_{dB} = 10 \log 10^3 + 20 \log_{10} N$$
$$= 4.8 + 20 \log N$$

for $N >> 1$, $SQNR \approx 20 \log N$ dB

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Bandwidth requirement for PCM,

Max possible pulse duration $T_b(max) = \frac{T_s}{n}$

signalling rate(R) = no of samples/sec * no of bits/samples

$$= f_s n$$

$$R \geq 2n f_x$$

Theoretically minimum (BW) = $\frac{1}{2}$ signalling rate

$$= \frac{2n f_x}{2} = n f_x$$

Practical BW $(1+\rho) n f_x$

$$= 2n f_x \quad (\rho = 1)$$

where ρ = roll off factor

3.b) Explain functional block diagram of the PCM. Find the signalling rate of T_1 system and draw its frame diagram.

=> PCM is the technique by which the analog are converted into digitally encoded signal. The 3 essential operations in PCM system are: sampling, quantization, encoding

Analog message signal → LPF → Sampler → quantizer → encoder → PCM signal

Fig:- a

Distorted PCM signal → Regenerative repeater →--- Regenerative repeater → regenerated PCM signal

(b)

input → Regenerative ckt → Decoder → Reconstruction filter → Destination

(c).

It consists of the 3 main parts transmitter, transmission path and receiver

i) Transmitter

The essential operations in the transmitter of the PCM system are sampling, quantizing, and encoding, sampling is the operation in which analog signal is sampled through the sampling theorem, resulting in discrete time signal. The quantizing and encoding is done in same circuit known as ADC.

a) Low pass filter :-

The signal is passed through the LPF of fm Hz. This LPF blocks all the frequency above fm Hz ie signal x(t) is bandlimited to fm Hz.

b) Sample and hold circuit.

It samples the signal at the rate of fs (sampling frequency) fs is selected sufficiently above Nyquist rate to avoid aliasing.

ie $f_s \geq 2f_m$.

c) quantizer and quantization.

It converts the discrete time continuous amplitude signals to discrete amplitude signals.

- It compares input $x(nT_s)$ with fixed digital levels.
- The output of quantizer is digital level $x_q(nT_s)$

d) encoder

The quantised signal level $x_q(nT_s)$ is given to binary encoder which converts input signal to 'v' digits binary word. Thus $x_q(nT_s)$ is converted to 'v' binary bits. This encoder is also known as digitizer.

2. Transmission path.

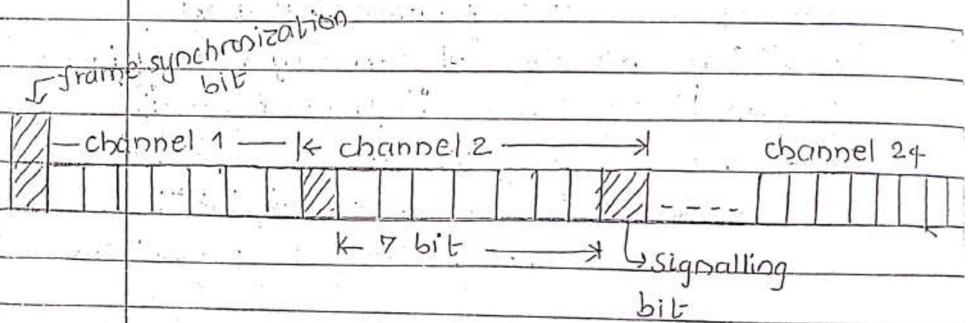
It is the path between the transmitter and receiver. The most important feature of PCM system is its ability to control the effects of noise and distortion. PCM accomplishes this capacity by means of using a chain of regenerative repeaters.

3. Receiver.

The regenerator at the start of PCM receiver reshapes the pulses and removes the noise. The signal is then converted to parallel digital words for each sample. The digital word is then converted to analog value $x_q(t)$ with the help of sample and hold circuit. This signal at the output of sample and hold circuit is allowed to pass through a low pass reconstruction filter to get appropriate original message signal $x(t)$.

Signalling rate of T_1 = 193 bits per frame $\times$ 8000 frames/sec.

= 1.544 Mbps.



T_s 125 µs

Total no of bits per frame = (24×8+1)

= 193 bits/frame.

4 a) Define information and entropy, calculate the upper limit of the channel capacity as the bandwidth of the channel B tends to infinity.

⇒ Entropy :-

Information is related to the information theory. It is rate at which source emit information.

As the bandwidth of channel 'B' tend to infinity, the channel capacity reaches to an upper limit C_{max} (instead of $C \to \infty$) because as the channel BW increased, noise power increases correspondingly. noise power is proportional to channel BW.

noise power $N = N_0 B$ ($N_0 \to$ psdf).

$$C = B \log_2 \left(1 + \frac{S}{N_0 B} \right)$$

$$= \frac{S}{N_0} \log_2 \left(1 + \frac{S}{N_0 B} \right)^{N_0 B / S}$$

$$= \frac{S}{N_0} \log_2 (1 + x)^{1/x} \quad \left\{ \frac{S}{N_0 B} = x \right.$$

as $B \to \infty$ $x \to 0$ so,

$$\lim_{B \to \infty} C = C_{max} = \frac{S}{N_0} \log_2 e \quad \left[\lim_{x \to 0} (1+x)^{1/x} = e \right]$$

$$\therefore C_{max} = 1.44 \left(\frac{S}{N_0} \right)$$

b) A discrete source emits one of 6 possible symbols per 10 µs in statistically dependent manner. The symbol probabilities are $1/4, 1/4, 1/4, 1/8, 1/16$ and $1/16$ respectively.

Calculate symbol rate, entropy and information rate.

⇒ soln,

$$\text{symbol rate} = r = \frac{1}{T} = \frac{1}{10\,\mu s} = 10^5 \text{ sym/sec}$$

$$\text{entropy} = \sum P_i \log_2 \frac{1}{P_i}$$

$$\Rightarrow \frac{1}{4} \log_2 \frac{1}{1/4} + \frac{1}{4} \log_2 \frac{1}{1/4} + \frac{1}{4} \log_2 \frac{1}{1/4} + \frac{1}{8} \log_2 \frac{1}{1/8} + \frac{1}{16} \log_2 \frac{1}{1/16} + \frac{1}{16} \log_2 \frac{1}{1/16}$$

- 27 -

$= \frac{1}{4}\log_2(4) + \frac{1}{4}\log_2(4) + \frac{1}{4}\log_2(4)$

$+ \frac{1}{8}\log_2(8) + \frac{1}{16}\log_2 16 + \frac{1}{16}\log_2 16$

$= 2.375$

& Information rate $= 2.375 \times 10^5$

$= 237500$ bps

5 a. What is DPSK and how it can be implemented?

⇒ Refer 2071 Shrawan (Back). Q.no:-6.

5 b. What is modem? Discuss the modes of operation of modes.

⇒ Modems (modulators & Demodulators) are used for carrying data over analog lines. A combined modulator and Demodulator unit is called modem.

Modes of operation

1) Simplex
- One direction data transmission.
- No path from receiver to transmitter
- Limited use, no possibility of re-transmission and error correction.

2) Half Duplex
- Reverse channel is available, but not simultaneosly
- one channel (bidirectional) required for this mode of operation.
- Transmission speed is reducing due to waiting for transmission and reception.

3) Full Duplex
- Data can be transmitted and received in both directions at the same time
- requires 2 independent channels.
- Most commonly used mode of operation.

6 a. Define noise equivalent bandwidth. Find mean and AC function at the output when the WSSP is passed through the LTI system.

⇒ Average noise power at the output of ideal LPF of BW B Hz when input is white noise with zero mean and psdf $N_0/2$

$$P_N \text{ ideal} = \frac{N_0}{4RC} e^{-|\tau|/RC} \Big|_{\tau=0}$$

$$= \frac{N_0}{4RC}$$

The filter has 3dB BW equal to $\frac{1}{2\pi RC}$

Passing signal through LTI System
Let a wide sense stationary random process (wssp) $x(t)$ is applied to the i/p of a LTI system with impulse response $h(t)$ and transfer function $H(f)$.

$$x(t) \longrightarrow \boxed{\begin{array}{c} \text{LTI} \\ h(t), H(f) \end{array}} \longrightarrow Y(t)$$

By the definition, the output of LTI system is

$Y(t) = x(t) * h(t)$

$$= \int_{-\infty}^{\infty} x(t-\alpha)\, h(\alpha)\, d\alpha \xrightarrow{\text{Stationary}} \int_{-\infty}^{\infty} x(\alpha)\, h(t-\alpha)\, d\alpha$$

ie statistics is independent on time shift.

mean value of o/p $Y(t)$ is

$$m_y = E[Y(t)] = E\left[\int_{-\infty}^{\infty} x(t-\alpha)\, h(\alpha)\, d\alpha\right]$$

as expectation operator $E[\cdot]$ is linear operation.

So,

$$m_y = \int_{-\infty}^{\infty} E[x(t-\alpha)\, h(\alpha)\, d\alpha] = m_x \int_{-\infty}^{\infty} h(\alpha)\, d\alpha$$

Because $x(t)$ input is wssp, the mean m_x is constant and independent of time shift

$E[x(t-\alpha)] = E[x(t)] = m_x$

we know that,

$$H(f) = \int_{-\infty}^{\infty} h(\alpha)\, e^{-j2\pi f\alpha}\, d\alpha$$

at $f=0$

$H(0) = \int_{-\infty}^{\infty} h(\alpha)\, d\alpha \rightarrow$ zero frequency response of system

So, $m_y = m_x H(0)$ - - - - - - - (a).

ie mean value of output is also independent of the shift in time [1st criteria for WSSP)

To find autocorrelation function of $Y(t)$

$$R_{xy}(t_1, t_2) = R_{xy}(\tau)$$

$$= E[x(t_1) \cdot Y(t_2)]$$

$$= E\left[x(t_1) \int_{-\infty}^{\infty} x(s)\, h(t_2 - s)\, ds\right]$$

$$= \int_{-\infty}^{\infty} E[x(t_1)\, x(s)\, h(t_2 - s)\, ds]$$

$$\cancel{= \int_{-\infty}^{\infty} R_{ss}}$$

$$= \int_{-\infty}^{\infty} R_{xx}(t_1 - s)\, h(t_2 - s)\, ds$$

Let $s - t_2 = u$

$$R_{xy}(\tau) = \int_{-\infty}^{\infty} R_{xx}(\tau - u)\, h(-u)\, du$$

$$= R_{xx}(\tau) * h(-\tau).$$

Now,

autocorrelation of $Y(t)$ is

$$R_{yy}(t_1, t_2) = E[Y(t_1) \cdot Y(t_2)]$$

$$= E\left[Y(t_2) \int_{-\infty}^{\infty} x(s)\, h(t_1 - s)\, ds\right]$$

$$= \int_{-\infty}^{\infty} E[x(s)\, Y(t_2)]\, h(t_1 - s)]$$

$$= \int_{-\infty}^{\infty} R_{xy}(s - t_2)\, h(t_1 - s)\, ds$$

$$= \int_{-\infty}^{\infty} R_{xy}(u)\, h(t_1 - t_2 - u)\, du$$

$$= \int_{-\infty}^{\infty} R_{xy}(u)\, h(\tau - u)\, du.$$

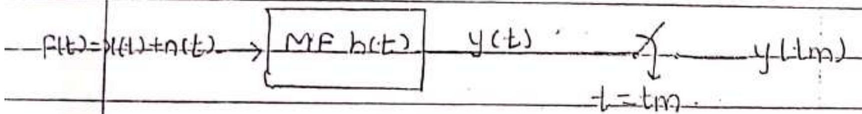
$= R_{xy}(\tau) * h(\tau).$

So, $R_{yy}(t_1, t_2) = R_{xx}(\tau)\, h(-\tau) * h(\tau)$ ---(b)

That is AC function of o/p $Y(t)$ is also independent of shift in time (only function of time lag $\tau = t_1 - t_2$)

from a and b it can be concluded that the o/p of LTI system to WSSP excitation is also WSSP.

b) Realize the matched filter with relevant mathematical support.



f(t) is noisy input signal

$f(t) = x(t) + n(t)$

$$y(t) = f(t) * h(t) = \int_{-\infty}^{\infty} f(s)\, h(t-s)\, ds$$

for matched filter $h(t) = x(t_m - t)$.

$$\therefore y(t) = \int_{-\infty}^{\infty} f(s)\, x(t_m - t + s)\, ds.$$

at $t = t_m$:

$$y(t_m) = \int_{-\infty}^{\infty} f(s)\, x(s)\, ds$$

This eqn can be represented in block diagram as;

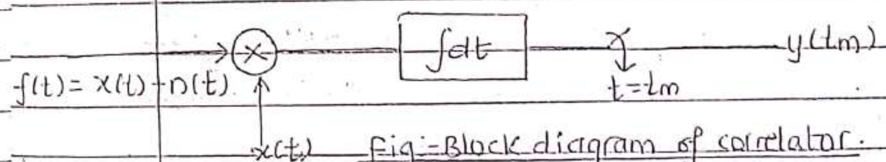


Fig:-Block diagram of correlator.

The signal $f(t)$ is multiplied by locally generated replica of i/p signal $x(t)$. Result is integrated and sampled at $t = t_m$. The above arrangement is called time correlator which is synchronous detector.

7 a. what is capture effect? calculate the gain parameter in DSB-SC with envelop detection.

⇒ If there is interference in the form of other stations with comparable carrier frequencies, the fm station will enhance one of the sections depending upon the intensities of the signal at the receiver. If the level of interference is low compared to the desired signal then FM ~~depends~~ captures the desired signal and remains locked. But if the level of interference becomes greater than desired signal then the FM captures the interference. This effect of capturing the strong station is called capture effect.

Envelope Detection.

input signal can be written as.

$$x_{AM}(t) = [A_c + x_m(t)] \cos w_c t + n_c \cos w_c t + n_s(t) \sin w_c t.$$

$$= [A_c + x_m(t) + n_c(t)] \cos w_c t + n_s(t) \sin w_c t$$

$$= v(t) \cos (w_c t + \psi(t))$$

where,

$$v(t) = \sqrt{[A_c + x_m(t) + n_c(t)]^2 + n_s^2(t)}$$

$$\psi(t) = -\tan^{-1}\left[\frac{n_s(t)}{A_c + x_m(t) + n_c(t)}\right]$$

case i:- Let us assume that the noise is small

ie $[A_c + x_m(t)] >> n_i(t)$

or, $[A_c + x_m(t) + n_c(t)]^2 >> n_s^2(t).$

$\therefore$

envelope $v(t) = A_c + x_m(t) + n_c(t).$

$\psi(t) = 0.$

-32-

at the output of envelope detector we get only envelope of signal so o/p of ideal envelope detector is;

$v(t) = A_c + x_m(t) + n_c(t)$

LPF filters out dc component ie A_c so o/p of LPF is

$x_m(t) + n_c(t)$

output signal power $P_{so} = \overline{x_m^2(t)}$

output noise power $P_{no} = \overline{n_c^2(t)}$

$\therefore SNR_o = \dfrac{\overline{x_m^2(t)}}{\overline{n_c^2(t)}}$

Detection gain $\gamma = \dfrac{SNR_o}{SNR_i}$

$= \dfrac{2\overline{x_m^2(t)}}{A_c^2 + \overline{x_m^2(t)}}$

we know that $\dfrac{\overline{x_m^2(t)}}{A_c^2 + \overline{x_m^2(t)}} = \eta$; efficiency of DSB-FC AM

$\gamma = 2\eta$

maximum efficiency is $1/3$ for 100% modulation index ($\mu = 1$)

$\therefore \gamma_{max} = 2 * 1/3 = \dfrac{2}{3} < 1.$

SNR_o is less than $SNR_i \Rightarrow$ Degrade the signal.

b) compare AM and FM in terms of power efficiency, bandwidth efficiency and system complexity, calculate the error probability of ASK

$\Rightarrow$ Comparision.

Modulation scheme	BW eff	Power eff	System complexity
DSB AM (fc)	SSB	FM	DSB-FC
DSB-SC	VSB	SSB	DSB-SC
SSB	DSB-FC / DSB-SC	VSB	FM
FM	FM	DSB-SC	VSB
		DSB-FC	SSB

a) Bandwidth efficiency
SSB is the best as channel BW is equal to the message BW so SSB-SC is used when BW is major constraint (μwave link, satellite communication). For applications requiring near to DC transmission (eg:- TV, broadcasting), VSB-SC is used.

b) Power efficiency
AM is least efficient. FM is most efficient due to high level of noise immunity (at least the cost of high channel BW). So the power critical application such as space vehicle communication, FM broadcasting is used.

For error probability.
⇒ Refer Q.no:- 11 Chaitra 2069.

8 a. Define Hamming weight and Hamming Distance.

⇒ Hamming weight:
It is defined as the no of non-zero components in the code.
ex:- c = 101100, HW = 3.

Hamming Distance.
It is the no by which a code word differ from other code word ~~by~~ of the same code vector.

b. What is binary cyclic code? Construct a (7,4) cyclic code using generator polynomial $g(x) = x^3 + x^2 + 1$ with data vector 1011

⇒ The cyclic codes are the subclass of linear codes which satisfy the following properties.

a) Linearity property :- sum of any two code words is also a code word.

b) cyclic property :- any cyclic shift of a codeword is also a codeword.

⇒ soln,

$m(x) = x^3+x^2+1$

write as the order of g(x)

$[1\times x^3+0.x^2+1.x^1+1]$

or, $x^{n-k} m(x) = x^3(x^3+x+1)$

$= x^6+x^4+x^3$

$x^3+x^2+1 \,)\, x^6+x^4+x^3 \,(\, x^3+x^2$

$x^6+x^5+x^3$

x^5+x^4

$x^5+x^4+x^2$

x^2

$r(x) = x^2$

$r = 100 \, [x^2.1+0.x+0.1]$

$\therefore C = \underbrace{1011}_{m.} \; \underbrace{100}_{r}$

2070 CHAITRA

[REGULAR]

1. Explain the importance of source encoder. write algorithm for Huffman's coding?

⇒ Refer 2069 Chaitra.

algorithm for Huffman's coding

i) List symbols in decreasing order of probability

ii) Combine the probability of two symbols having lowest probabilities and recorder the resultant probabilities. Repeat until there are two ordered probabilities remaining.

iii) Start encoding with the last reduction. Assign 0 as the first digit in code words for all source symbol associated with the first probability, assign 1 to the second probability

iv) Now go back and assign 0 and 1 to second digit for the two probabilities that were combined assignments until the first column is reached

Consider a 6 symbol source with probability

$P(x_1) = 0.3$ $P(x_4) = 0.1$

$P(x_2) = 0.4$ $P(x_5) = 0.1$

$P(x_3) = 0.06$ $P(x_6) = 0.04$

x_i	$P(x_i)$	Code							
x_2	0.4	1	0.4	1	0.4	1	0.4	1	0.6 0
x_1	0.3	00	0.3	00	0.3	00	0.3	00	0.4 1
x_5	0.1	011	0.2	011	0.2	010	0.3	01	
x_4	0.1	0100	0.1	0100	0.1	011			
x_3	0.06	01010	0.1	0101					
x_6	0.04	01011							

$H(x) = 2.1435$ bits/symbol

$L = 2.2$ bits/symbol

$$\eta = \frac{2.1435}{2.2} = 97\ \%$$

2) Explain what are the practical factors to be considered while sampling. If two band limited signals $x_1(t)$ and $x_2(t)$ have band widths of w_1 and w_2 Hertz respectively estimate the maximum sampling interval required for the signal given by $Y[t] = X_1[t] \cdot X_2[t]$

⇒ Practical issues of sampling.

(i) The sampled waveform consists of finite amplitude and duration pulses, rather than ideal pulses → aperture effect.

(ii) Reconstruction filters are not ideal (distortion)

(iii) The input waveforms are rather time limited than band limited → aliasing.

Effects of above deviations.

1) The real sampling pulse is flat topped with finite duration τ. The net result is that

$$x_s(t) = \sum_{K=\infty}^{\infty} x(KT_s)\, p(t-KT_s).$$

where $p(t)$ = sampling pulse

now,

$$x_s(t) = p(t) * \sum_{K=-\infty}^{\infty} x(KT_s)\, \delta(t-KT_s)$$

$$= p(t) * x_\delta(t).$$

Taking F.T.

$x_s(f) = P(f) * x_\delta(f)$.

$$= P(f) \cdot f_s \sum_{n=-\infty}^{\infty} x(f - nf_s).$$

Since $P(f)$ is sinc function the primary effect of flat topped sampling is the attenuation of high frequency components of the message signal. This effect is also called ~~capture effect~~ aperture effect.

Minimizing aperture effect.

(a) Passing signal through an equalizing filter with a transfer function

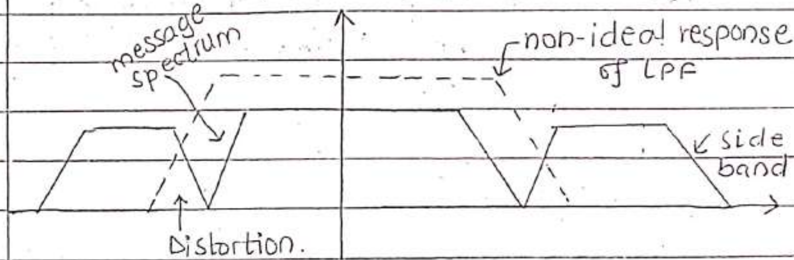
$$H_{eq}(f) = \frac{1}{P(f)}$$

$$|H_{eq}(f)| = \frac{1}{|P(f)|} = \frac{1}{A\tau \operatorname{sinc}(f\tau)}$$

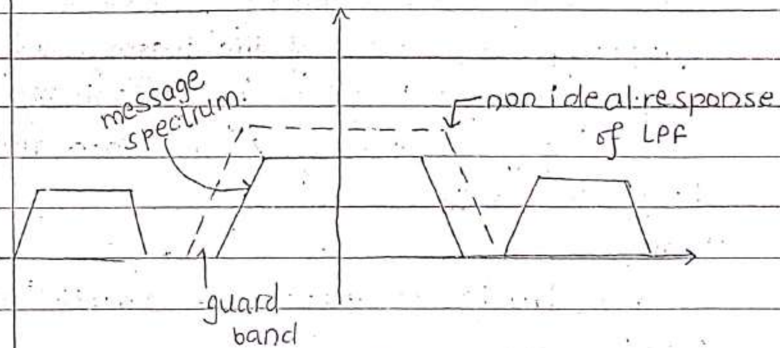
pulse amplitude

(b) If $\tau << T_s$ then $p(f)$ is more or less constant over message frequency band and aperture effect can be neglected.

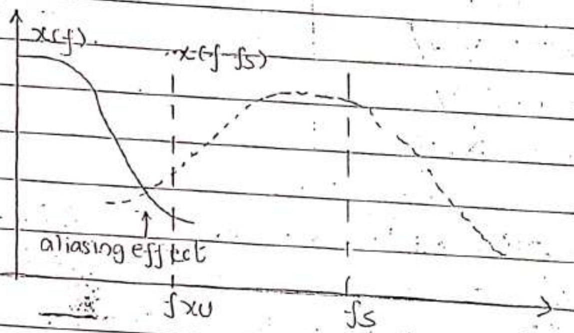
effect of non-ideal reconstruction filter is that the portion of side bands will also be filtered out along with message signal.



Good filter design may eliminate this problem. one of the other way to minimize this effect is to introduce guard band by selecting f_s slightly higher than $2f_x$



→ Real signals encountered in real life are usually time limited but not band limited. eg. a pulse of finite duration whose spectrum is theoretically unlimited. Thus there will always be aliasing effect.



To overcome this effect, a filter called pre-aliasing filter is used to attenuate frequency components of the message spectrum higher than the band of interest and by selecting fs moderately higher than the Nyquist rate.

→ soln,

modulation is equivalent to ~~mod~~ multiplication

Let $\omega_2 > \omega_1$

max in frequency = $\omega_1 + \omega_2$

sampling frequency = $2(\omega_1 + \omega_2)$

$\therefore$ Time $(T_s) = \frac{1}{f_s}$

3) What are the signalling rate and bandwidth requirement for the T_1 and E_1 digital carrier systems? Explain briefly about Differential pulse code modulation encoder.

⇒ soln,

for T_1

Total no of bits per frame = $(24 \times 8 + 1)$

= 193 bits/frame

Bit rate = 193 bits/frame * 8000 frames/sec

= 1.544 mbps

$BW_{pr} = R_b$ MHZ (practically)

= 1.544 MHZ

$BW_{the} = \frac{R_b}{2} = \frac{1.544}{2} = 0.77$ HZ (theoretically) for E1

Total no of bits per frame = 32×8 = 256 bits/frame

Bit rate = 8000 frame/sec × 256 bits/frame

= 2.048 Mbps

$BW_{pr} = R_b$ MHZ = 2.048 MHZ

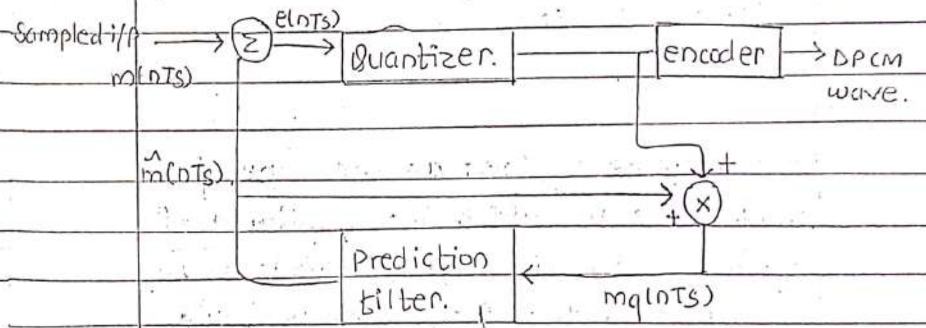
$BW_{th} = \frac{R_b}{2}$ MHZ = 1.024 MHZ.

DPCM.

In ordinary PCM, after sampling the message signal, each sample is quantized in independent manner. It means previous sample values have no effect on the quantization of new samples. But in practice band limited random signals when sampled at Nyquist rate or higher produce highly correlated sample values.

This is true in most cases except when the spectrum of message signal is flat within the bandwidth of interest. for eg:- The sample of speech signal doesn't change abruptly from sample to sample. It means there is redundancy in samples that could be removed for efficient quantization & coding.

The DPCM takes into account of these facts and the quantization is performed into the differences of i^{th} sample and its prediction derived from $(i-1)^{th}$ sample. That is instead of quantizing the whole sample value, DPCM quantizes the difference only; thus reducing the required quantization level to minimum. In other words for given no of levels per sample DPCM yield lower value of q_e than direct quantization.



$e(nT_s) = m(nT_s) - \hat{m}(nT_s)$ - - - - - (i)

$e_q(nT_s) = e(nT_s) + q_e(nT_s)$ - - - - (ii)

where $q_e(nT_s)$ is quantization error.

from fig;

$m_q(nT_s) = \hat{m}(nT_s) + e_q(nT_s)$

$\Rightarrow m_q(nT_s) = \hat{m}(nT_s) + e(nT_s) + q_e(nT_s)$ { using 2

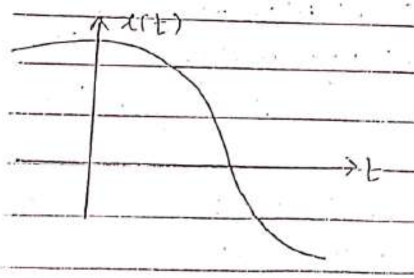
$\Rightarrow m_q(nT_s) = m(nT_s) + q_e(nT_s)$.

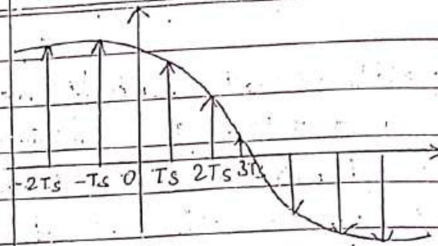
4. Define PAM, PWM, PPM with corresponding waveform. A television signal having a bandwidth of 4.8 MHz is transmitted using binary PCM system. Given that the no of quantization level is 512. Determine :-

i) code word length
ii) Transmission bandwidth
iii) final bit rate
iv) output SNR.

* PAM.

PAM may be defined as the type of modulation in which the amplitude of regularly spaced rectangular pulses vary according to the instantaneous value of modulating or message signal.





* PWM.

In PWM, the width of the modulated pulses varies in proportion with the amplitude of modulating signal.

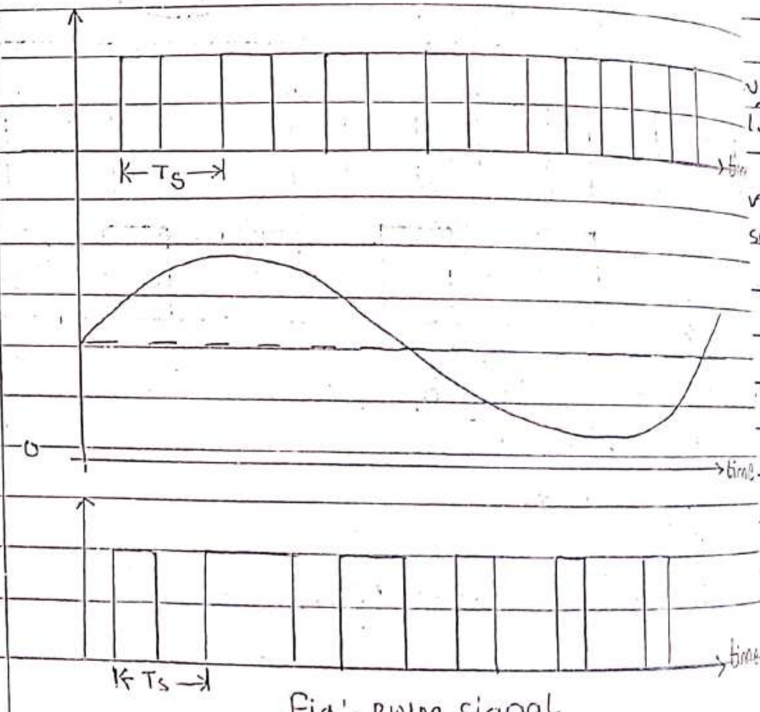
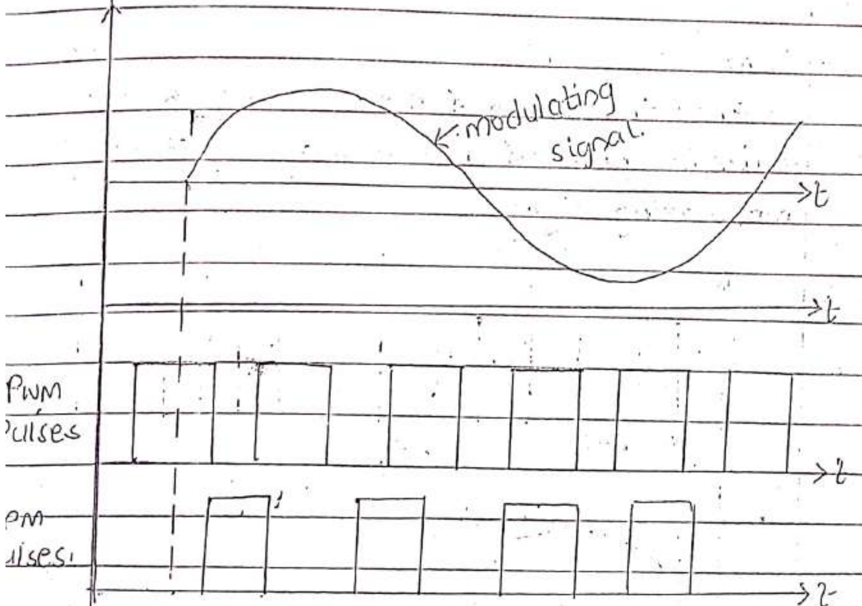


Fig:- PWM signal

* PPM.

In PPM, the amplitude and width of the pulses is kept constant but the position of each pulse is varied with the amplitude of sampled values of modulating signal.



→ code word length;

$2^n = 512$

or, $n = 9$.

→ Sampling frequency $(f_s) = 2 \times 4.8$

$= 9.6$ MHz.

→ final bit rate

$R = 9.6 \times 9$ Mbps

$= 86.4$ Mbps.

→ Transmission bandwidth $(B_{W_{PP}}) = R = 86.4$ Mbps.

→ Output SQNR $= 4.8 + 6n$

$= 4.8 + 6 \times 9$

$= 58.8$ dB.

5. Derive the expression for evaluating signal to quantization noise ratio for Delta modulation.

⇒ In DM, the output of quantizer is,

$$e_q(nT_s) = \Delta \operatorname{sgn}\{e(nT_s)\}$$

The value of $e_q(nT_s)$ may be $+\Delta$ and $-\Delta$ and total swing is 2Δ.

∴ Total noise power

$$P_q = \frac{1}{2\Delta}\int_{-\Delta}^{\Delta} e_q^2 \, de_q = \frac{\Delta^2}{3}.$$

experiments have indicated that incase of DM normalized power of $e_q(t)$ is uniformly distributed over f interval of $(0, f_s)$.

Then psdf of $e_q(t)$ is

$$G_{eq}(f) = \frac{P_q}{2f_s'} \quad \text{for } |f| \le f_s'$$

or. $$G_{eq}(f) = \frac{\Delta^2}{3 \times 2f_s'}$$

If the output receiver filter is ideal over frequency range of 0, f_x then normalized $[h_{LPF}(f) = 1]$, average power at the output of filter will be.

$$P_q(LPF) = \int_{-f_x}^{f_x} G_{eq}(f)\, df.$$

$$= \frac{\Delta^2}{3} * \frac{2f_x}{2f_s'}$$

$$= \frac{\Delta^2 f_x}{3f_s'}$$

Regarding the signal power, let us consider the case when all signal power is concentrated at the upper end of the BW of message signal is;

$x(t) = A \cos 2\pi f_x t.$

at output of ideal LPF

$x_{LPF}(t) = A \cos 2\pi f_x t$

and signal power $P_x = \frac{A^2}{2}.$

To avoid slope overload the max value of A should be equal or less than $\frac{\Delta f_s'}{2\pi f_x}$

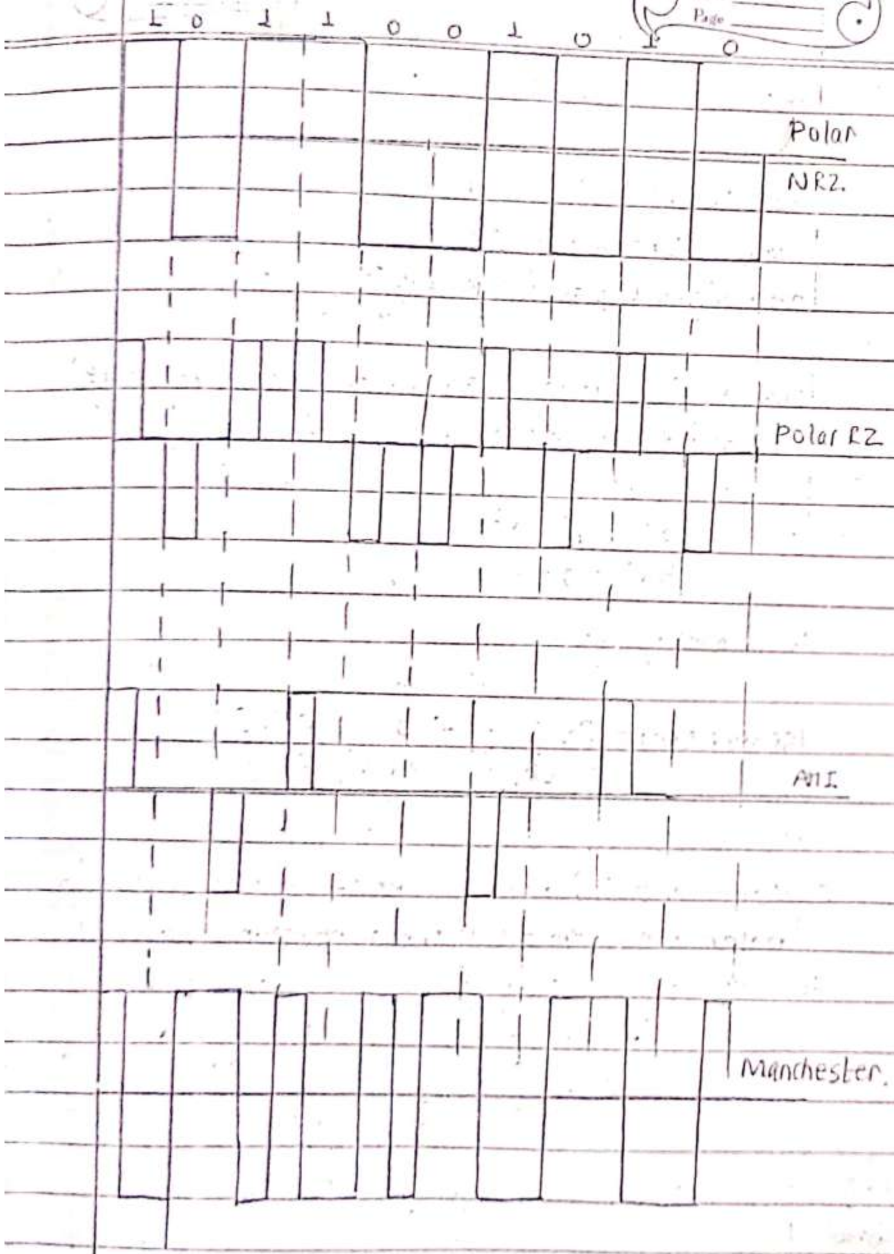
$\therefore P_x = \frac{\Delta^2}{8\pi^2}\left(\frac{f_s'}{f_x}\right)^2$

$\therefore$ SQNR of DM

$$SQNR(DM) = \frac{P_x}{P_q(LPF)} = \frac{3}{8\pi^2}\left(\frac{f_s'}{f_x}\right)^3.$$

6. Represent binary sequence 1011001010 in polar NRZ, polar RZ, manchester and AMI codes.

$\Rightarrow$ Soln,



7. Explain the modulator, demodulator and signal space diagram for QPSK modulation with relevant derivation.

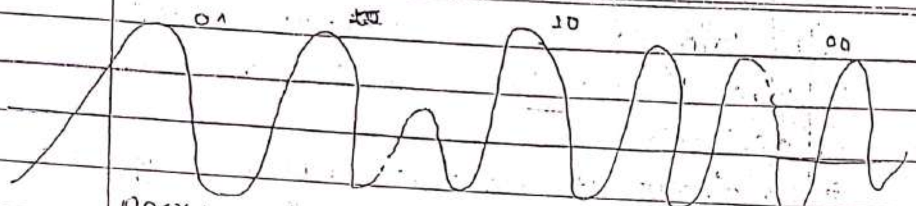
⇒ In binary data transmission only one of the 2 possible signals during each bit interval Tb is transmitted but in M-ary data transmission system any one of the M-possible signals during each signalling interval is transmitted. For almost all applications the no of possible signals $M = 2^n$ where n is an integer and signalling interval $T = nT_b$

QPSK is an example of an M-ary data transmission system more specifically M-ary PSK with $M = 2^2 = 4$. In QPSK one of the four signals is transmitted during each signalling interval with each signal uniquely related to a dibit (pair of bits). Two succesive bits in the data sequence are grouped together and possible dibits 00, 10, 11 and 01 (grey encoding) are transmitted by sinusoidal carriers written below

$$s(t) = \begin{cases} A_c \cos(2\pi f_c t - 3\pi/4) & 00 \\ A_c \cos(2\pi f_c t - \pi/4) & 10 \\ A_c \cos(2\pi f_c t + \pi/4) & 11 \\ A_c \cos(2\pi f_c t + 3\pi/4) & 01 \end{cases}$$

where, $0 \leq t \leq T$

T - T symbol duration.



QPSK waveform for binary seg 01101000

QPSK represents a special form of phase modulation in which modulated wave s(t) can be expressed as:-

$s(t) = A_c \cos(2\pi f_c t + \phi(t))$

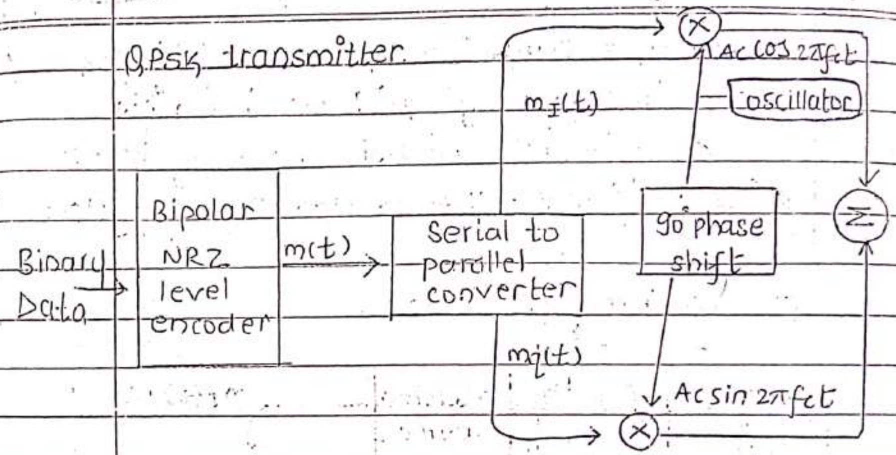
where $\phi(t)$ represent phase angle for each dibit.

The expression for s(t) can be written as;

$s(t) = A_c \cos\phi(t)\cos 2\pi f_c t - A_c \sin\phi(t) \sin 2\pi f_c t$

Thus QPSK wave s(t) has an inphase component equal to $A_c \cos\phi(t)$ and quadrature component equal to $A_c \sin\phi(t)$

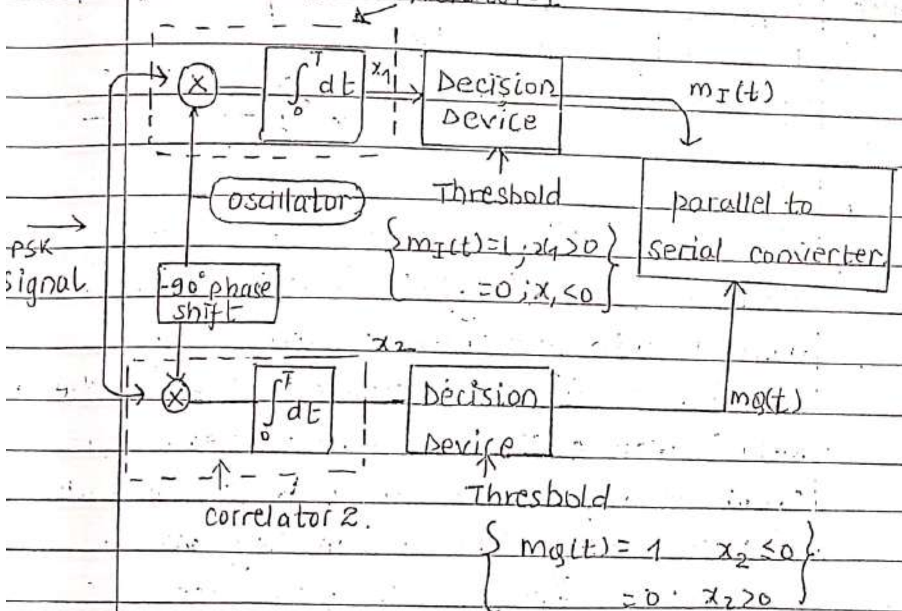
QPSK transmitter



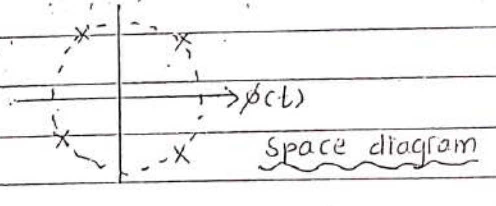
The unipolar binary message stream has bit rate R_b and is first converted into bipolar non-return to zero (NRZ) sequence using a unipolar to bipolar converter. The bit stream m(t) is then split into two bit stream $m_I(t)$ (inphase or even stream) each having a bit rate of $R_s = \frac{R_b}{2}$. These two sequence are separately modulated by two quadrature carriers which can be considered to be BPSK signal and are summed to produce QPSK signal.

The signalling interval T in a QPSK system is twice as long as the bit duration T_b of input binary data. This implies that for a given bit rate $1/T_b$, a QPSK system requires half the transmission bandwidth of corresponding PSK system. Thus for a given transmission BW a QPSK system carrier twice as many bits of information as the corresponding PSK system.

QPSK receiver :- Correlator-1



The QPSK receiver consists of two correlators connected in parallel. One correlator computes the cosine component of the carrier phase whereas the other computes the sine components. By comparing the signs of the two correlator outputs through the use of pair of decision device, a unique resolution of one of the four transmitted phase angle is made. The parallel to series converter interleaves the decision made by impulse & quadrature channels of the receiver and thereby reconstructs the binary data stream which is identical to one at the transmitter input.



Q. Differentiate between message and information? A discrete source is emitting one of the 5 possible symbols per 10 microsec. The probabilities are $1/2$, $1/4$, $1/8$, $1/16$ and $1/16$. Find a) symbol rate b) source entropy c) information rate.

⇒ A message is a sequence of symbols intended to reduce the uncertainty of the receiver. If the sequence of symbols doesn't change the uncertainty level of the receiver then the message doesn't contain any information.

Information is the branch of probability theory which deals with the study of communication system.

→ Higher the probability of occurance, the lesser the information contained of the message & vice versa.

→ Symbol rate $= \frac{1}{T} = R = \frac{1}{10\mu s}$

$= 10^5$ sym/sec

Entropy $= \sum p_i \log_2 \frac{1}{p_i}$

$\Rightarrow \frac{1}{2}\log_2\left(\frac{1}{1/2}\right) + \frac{1}{4}\log_2\left(\frac{1}{1/4}\right) + \frac{1}{8}\log_2\left(\frac{1}{1/8}\right) +$

$\frac{1}{16}\log_2\left(\frac{1}{1/16}\right) + \frac{1}{16}\log_2\left(\frac{1}{1/16}\right)$

$= 1.875.$

Information rate $= 1.875 \times 10^5$.

$= 187500$ bps.

9. Explain the approximation of the matched filter for a rectangular pulse using an ideal low pass filter with variable bandwidth.

⇒ Impulse response of matched filter for rectangular pulse is;

$$H_{MF}(t) = \begin{cases} A & \text{for } 0 \le t \le T \\ 0 & \text{otherwise.} \end{cases}$$

its transfer function is;

$H_{MF}(f) = \text{sinc}(fT)\exp(-j\pi f T)$
assuming $AT = 1$

But TF of an ideal LPF is a rectangle.

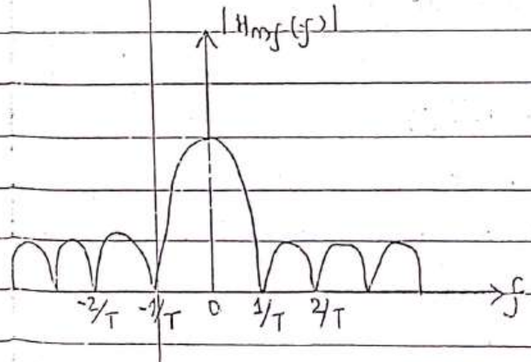


fig:- Amplitude response of filter matched to rect pulse

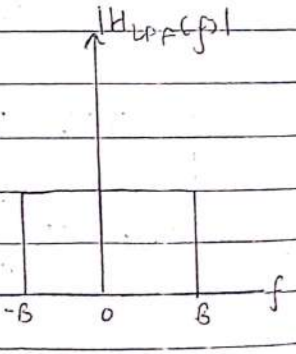


fig:- Amplitude response of ideal LPF approx the MF.

Impulse response of ideal LPF that works as distortionless transmission is

$$h(t) = \frac{\sin 2\pi B(t-t_0)}{\pi(t-t_0)}$$

$$= 2B \operatorname{sinc}[2B(t-t_0)]$$

when a rectangular pulse of amplitude A and duration T is passed through an ideal LPF its response is given by:

By convolution,

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau.$$

$$= 2BA \int_{-T/2}^{T/2} \frac{\sin 2\pi B(t-t_0-\tau)}{2\pi B(t-t_0-\tau)}\, d\tau.$$

Let, $\lambda = 2\pi B(t-t_0-\tau)$

$$y(t) = \frac{A}{\pi} \int_{2\pi B(t-t_0-\tau)}^{2\pi B(t-t_0-\tau)} \frac{\sin\lambda}{\lambda}\, d\lambda.$$

$y(t)$ is maximum at $t = T/2$ for $BT \geq 1$

$$y(t)|_{max} = \frac{2A}{\pi} \operatorname{Si}(\pi BT)$$

where, sinc signal is defined by,

$$\operatorname{Si}(u) = \int_0^u \frac{\sin\lambda}{\lambda}\, d\lambda.$$

so, LPF will satisfy first condition of realization aspect of MF (if condition is met)

Noise power at o/p of ideal LPF is $N_0/2 \times 2B = BN_0$

So,

Maximum SNR at o/p of ideal LPF is,

$$SNR_{LPF} = \frac{\left(\frac{2A}{\pi}\right)^2 \operatorname{Si}^2(\pi BT)}{BN_0}$$

we have,

Maximum SNR at output of MF is,

$$SNR_{MF} = \frac{E}{N_0/2} = \frac{2E}{N_0}$$

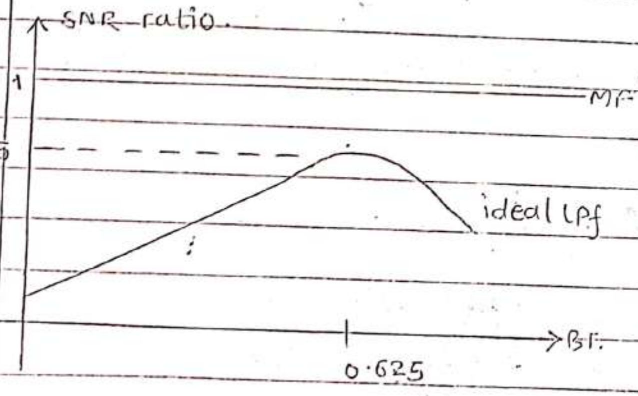
$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_0^T A^2 dt = A^2 T.$$

So,

$$SNR_{MF} = \frac{2A^2 T}{N_0}$$

ratio of SNR

$$\frac{SNR_{LPF}}{SNR_{MF}} = \frac{2}{\pi^2 BT} \sin^2(\pi BT)$$



10. Derive the expression for evaluating the error probability in binary communication system? What is threshold effect in FM? How can it be minimized.

⇒ Refer 2069 chaitra QNo 9 & 10.

11. The generator polynomial of a (7,4) cyclic code is $g(x) = 1+x+x^3$. Find the code for the message vector 1011 in a non-systematic and systematic form.

⇒ Soln,

$g(x) = 1+x+x^3$

$m(x) = 1+x^2+x^3 \; [1\times x^0 + 0\times x^1 + 1\times x^2 + 1\times x^3]$

non systemmatic code

$c(x) = m(x)\,g(x)$

$= (1+x+x^3)(1+x^2+x^3)$

$= 1+x^2+x^3+x+x^3+x^4+x^3+x^5+x^6$

$= 1+x+x^2+x^3+x^4+x^5+x^6$

$\therefore C = 1111111$

for systemmatic.

$x^{n-k}\, m(x) = x^3(1+x^2+x^3)$

$= x^3+x^5+x^6.$

$x^3+x+1 \overline{\big) x^6+x^5+x^3} \big(x^3+x^2+x+1$

$x^6+x^4+x^3$

x^5+x^4

$x^5+x^3+x^2$

$x^4+x^3+x^2$

x^4+x^2+x

x^3+x

x^3+x+1

1

$r(x) = 1$

$c(x) = r(x) + x^3 m(x)$

$= 1+x^3+x^5+x^6$

$\therefore c = 1001011$

12. Write short notes on
a) linear prediction theory
b) white noise & its pdf.

a) Linear prediction theory.
Digital speech coders can be classified into 2 catagories ; waveform coders & vocoders.
→ waveform coders use algorithm to encode & decode so that the system output is approximation of the input waveforms eg:- PCM & DPCM. Its advantage is the high quality of signal reproduced but requires relatively high bit rates.

Vocoders operate significantly at lower bit rates. vocoders encode speech signals by modelling the signal and extracting a set of parameters. These parameters are digitized and transmitted to the receiver.

At receiver end the original voice signal is predicted and synthesized using the parameters of model such technique of coding of speech signal is called Linear prediction coding or Theory.

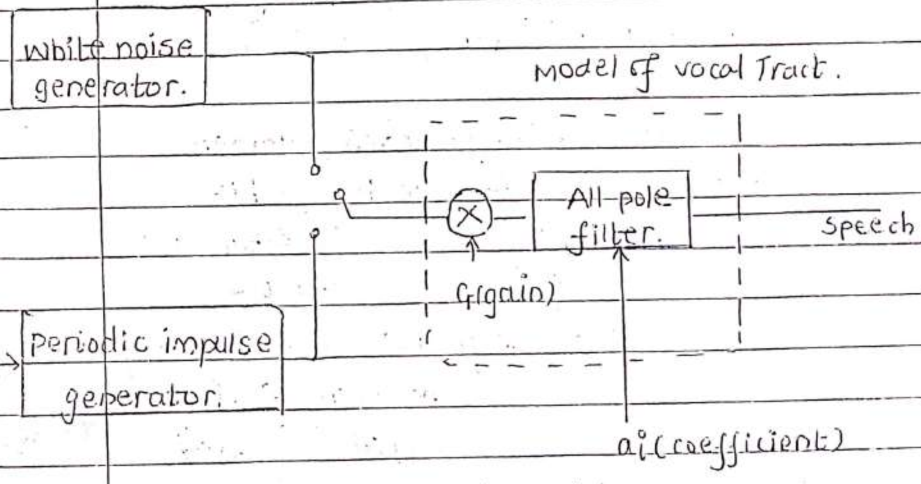


Fig:- Speech model

A speech signal can be modeled as the system consisting of short pulses of repeatation frequency. f_0 which is fundamental frequency of vibration of the vocal cords an all pole filter with coefficient a_i and gain parameter G. The unvoiced sounds are simulated by white noise source.

A property of human speech is that it is stationary for a period of 20-30ms. i.e within this period its statistical property remain constant for this period. we analyse values of f_0, a_i and G for these period, and transmit. At receiving end we use same prediction filter to reconstruct original speech signal.

Bit representation for each parameters
voiced/unvoiced information - 1 bit
pitch - 6 bit
Gain - 5 bit
filter coefficient - 8-10 bits/coff (normally first order filter is used)

example :-
voice - 4KHz f_s - 8000 samples/sec

rate in PCM - 8000 × 8 = 64 kbps

In LPC - 20ms - 1 frame 1sec - 50 frame

1 frame contain - 22 bits.

∴ Total no of bits = 22 × 50
= 1100 bits/sec → cellular telephone system

LPC at 4800 bps = 4.8 kbps for good quality of speech.

b) White noise & its pdf
⇒ Refer 2069 chaitra Qno:-8.

2065 Shrawan

[OLD COURSE]

1. State sampling theorem. What are aliasing and aperture effects in sampling? Find the expression for SQNR in PCM.

→ Sampling theorem states that "Analog signal can be reproduced from an appropriate set of its samples taken at some fixed interval of time."

→ For aliasing & aperture effect
refer 2070 chaitra Q.no:-2.

→ for SQNR
refer 2070 asadh Q.no 3a.

2. Define entropy. Derive the expression for evaluating the entropy of source emitting symbols in statistically independent manner. A discrete source emits 3 symbols with probabilities 1/3, 1/6, 1/2. Find source entropy.

→ The average information content of a sequence of symbols is called entropy.
Consider a source emits m possible symbols $S_1, S_2, ---- S_m$ whose probabilities are $P_1, P_2 ---- P_m$ respectively.

In a long sequence of N symbols, occurance of symbol S_i is $P_i N$.

$$\therefore I(S_i) = P_i N \log\left(\frac{1}{P_i}\right)$$

Then the total information content of N sequence of m symbol is;

$$I_{total} = \sum_{i=1}^{m} N P_i \log_2\left(\frac{1}{P_i}\right)$$

The average information content is

$$I_{average} = \underbrace{H}_{\text{entropy}} = \frac{I_{total}}{N} = \sum_{i=1}^{m} P_i \log_2\left(\frac{1}{P_i}\right)$$

If R_s is the symbol or message rate.

$$R_{inf} = R_s \cdot H \text{ bits/sec}$$

→ sol^n,

$$H = \sum_{i=1}^{m} P_i \log_2\left(\frac{1}{P_i}\right)$$

$$= \frac{1}{3}\log_2\left(\frac{1}{1/3}\right) + \frac{1}{6}\log_2\left(\frac{1}{1/6}\right) + \frac{1}{2}\log_2\left(\frac{1}{1/2}\right)$$

$$= 1.459.$$

3. Define ISI. State Nyquist pulse shaping criteria for zero ISI. Explain Duo binary encoding method

⇒ ISI (Refer 2069 chaitra Q.no. 6

Nyquist pulse shaping criteria for zero ISI

we have,

$$y(nT_b) = \mu A_n + \sum_{\substack{k=-\infty \\ k \neq n}}^{\infty} \mu A_k \, P(nT_b - kT_b)$$

$$= \mu \sum_{k=-\infty}^{\infty} A_k \, P(nT_b - kT_b)$$

for zero ISI

$$P(nT_b - kT_b) = \begin{cases} 1 & , k = n \\ 0 & , k \neq n \end{cases}$$

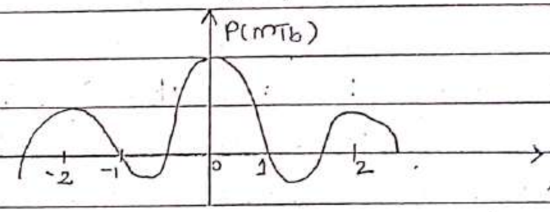
ie $$P(m T_b) = \begin{cases} 1 & , m = 0 \\ 0 & , m \neq 0. \end{cases}$$

The pulse $p(mT_b)$ with above condition is sinc function.

$p(t) = \text{sinc}(2\pi B_0 t)$

$$= \frac{\sin(2\pi B_0 t)}{2\pi B_0 t}.$$

where $B_0 = \dfrac{1}{2T_b} = \dfrac{R_b}{2}$



⇒ Duobinary encoding

Refer 2071 Q.no 5 (shrawan).

5. Prove that the output of LTI system is wide sense stationary process (WSSP) if its i/p is WSSP.

⇒ Refer 2070 asadh Q.no 6a.

6. Derive the expression for error probability for M-ary system.

⇒ Refer 2071 shrawan Q.no 11.

4. Derive the expression for the impulse response of a matched filter.

$x(t)+n(t) \longrightarrow$ [Optimum Detector / filter, $h(t), H(\omega)$] $\longrightarrow y(t)+n_0(t)$

Consider $x(t)$ and $n(t)$ be the input signal and white gaussian noise to matched filter, optimum detector and $y(t)$ and $n_0(t)$ be the corresponding outputs. Optimum detector has to maximize o/p SNR at decision making instants ($t=t_m$)

$$SNR_0 = \frac{\overline{y^2(t)}}{\overline{n_0^2(t)}} \quad \text{------(i)}$$

at $t=t_m$

$$\max^m SNR_0 = \frac{\overline{y^2(t_m)}}{\overline{n_0^2(t_m)}}$$

Then

output of optimum detector is

$$y(t) = F^{-1}\left[X(\omega)\,H(\omega)\right]$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} H(\omega)\,X(\omega)\,e^{j\omega t}\,d\omega$$

at $t=t_m$

$$y(t_m) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(\omega)\cdot X(\omega)\cdot e^{j\omega t}\,d\omega \quad \text{----(ii)}$$

The output noise power is;

$$\overline{n_0^2(t_m)} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{N_0}{2}\,|H(\omega)|^2\,d\omega$$

$$= \frac{N_0}{4\pi}\int_{-\infty}^{\infty} |H(\omega)|^2\,d\omega \quad \text{-----(a)}$$

now,

output SNR at $t=t_m$.

$$SNR_0\Big|_{t=t_m} = \frac{\overline{y^2(t_m)}}{\overline{n_0^2(t_m)}}$$

$$= \frac{\frac{1}{4\pi^2}\left|\int_{-\infty}^{\infty} H(\omega)\cdot X(\omega)\,e^{j\omega t_m}\,d\omega\right|^2}{\frac{N_0}{4\pi}\int_{-\infty}^{\infty} |H(\omega)|^2\,d\omega}$$

input $x(\omega)$ is fixed. To find $H(\omega)$ that makes SNR_o maximum we use schwarz's inequality principle

$$\frac{\left|\int_{-\infty}^{\infty} F_1(\omega)\, F_2(\omega)\, d\omega\right|^2}{\int_{-\infty}^{\infty} |F_1(\omega)|^2\, d\omega} \leq \int_{-\infty}^{\infty} |F_2(\omega)|^2\, d\omega.$$

This equality hold true only if

$$F_1(\omega) = k\, F_2^*(\omega) \text{ ------- (b)}$$

where

$k \rightarrow$ arbitary constant.

$F_2^*(\omega) \rightarrow$ complex conjugate of $F_2(\omega)$

let, $F_1(\omega)$ is $H(\omega)$ and $F_2(\omega)$ is $x(\omega) e^{j\omega t_m}$.

so,

$$\frac{\left|\int_{-\infty}^{\infty} H(\omega) \cdot x(\omega)\, e^{j\omega t_m}\, d\omega\right|^2}{\pi N_0 \int_{-\infty}^{\infty} |H(\omega)|^2\, d\omega} \leq \frac{1}{\pi N_0} \int_{-\infty}^{\infty} |x(\omega)|^2\, d\omega.$$

$$\left\{ \because \left|e^{j\omega t_m}\right| = 1. \right.$$

$$\therefore SNR_o \leq \frac{1}{\pi N_0} \int_{-\infty}^{\infty} |x(\omega)|^2\, d\omega.$$

The max^m value of SNR would be.

$$SNR_o\big|_{max} = \frac{1}{\pi N_0} \int_{-\infty}^{\infty} |x(\omega)|^2\, d\omega \text{ ---- (iii)}$$

This expression remain valid only if the pre-requisite condition of the schwarz's inequality remain valid ie.

$$H(\omega) = k\left\{x(\omega)\, e^{j\omega t_m}\right\}^*$$

$$= k\, x^*(\omega)\, e^{-j\omega t_m}$$

$$= k\, x(-\omega)\, e^{-j\omega t_m}. \text{ ------ (IV)}$$

The impulse response is,

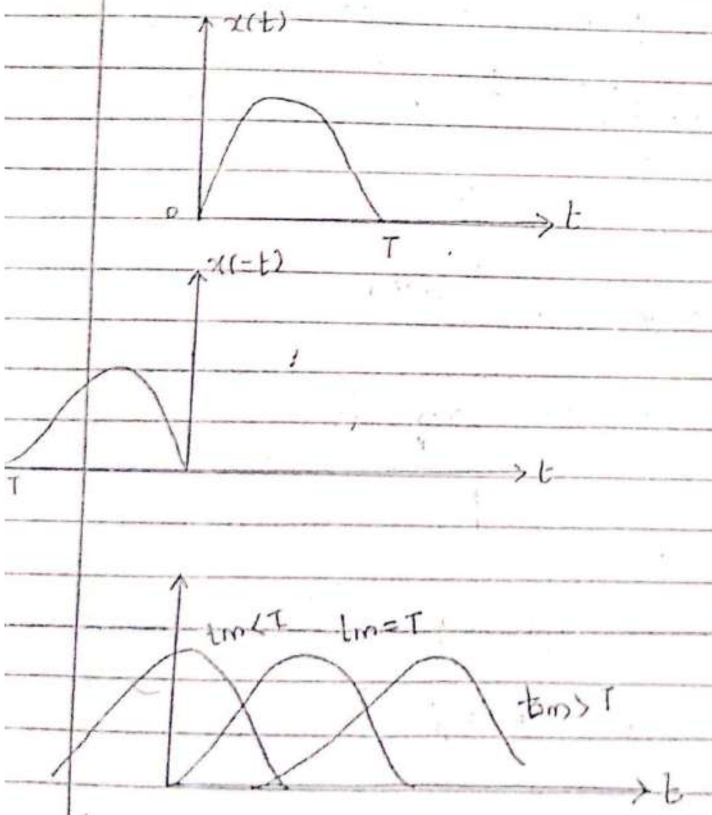
$$h(t) = F^{-1}\left[H(\omega)\right]$$

$$h(t) = F^{-1}\left[k\, x(-\omega)\, e^{-j\omega t_m}\right]$$

$$= kx\left[-t + t_m\right]$$

$$= kx\left[t_m - t\right]$$

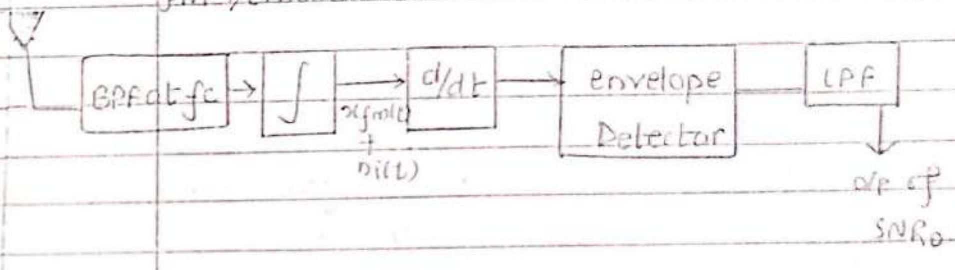
This is the impulse response of the required optimum detector.
for K=1; the impulse response of the optimum detector is the replica of incoming signal shifted by t_m.
This filter whose response is time shifted replica of input signal is called matched filter.



for $t_m < t$ impulse response $h(t) = x(t_m - t)$ is non causal and so practically non realizable so for practical purpose $t_m \geq T$.

7. Derive the expression for evaluating the gain parameter of FM Detector.

⇒ consider the standard limiter Descriminator FM Demodulator.



The input signal of Demodulator is;

$$x_i(t) = x_{FM}(t) + n_i(t).$$
$$= A_c \cos\left[\omega_c t + 2\pi K_f \int_0^t x_m(t)\,dt\right] + n_i(t).$$

So,
Input signal to noise power is,

$P_{si} = \frac{A_c^2}{2}$

$P_{ni} = \overline{n_i^2(t)} = 2B \times N_0$ ----(i)

where,

B = BW of message signal

N_0 = psd of noise.

$N_c(f) = N_s(f) = N_0$

$\therefore SNR_i = \frac{A_c^2}{4BN_0}$ -----(ii)

To find output SNR, let us first consider the case when i/p noise is absent.

i.e,

$n_c(t) = n_s(t) = 0$; $n_i(t) = 0$

then,

$x_i(t) = A_c \cos(w_c t + \phi(t))$.

where $\phi(t) = 2\pi k_f \int_{-\infty}^{t} m(t)\, dt$.

output of Descriminator is

$\frac{dx_i(t)}{dt} = A_c\left(w_c + \frac{d\phi(t)}{dt}\right) \sin(w_c t + \phi(t))$

Envelope Detector gives envelope of above signal at output and LPF removes dc components and components centred at w_c

so,

output of LPF is $A_c \frac{d\phi(t)}{dt} = A_c 2\pi k_f x_m(t)$

$\therefore P_{so} = 4\pi^2 A_c^2 k_f^2 \overline{x_m^2(t)}$ -----(iii)

now,

assuming message signal is absent in i/p then $x_m(t) = 0$ so input is only unmodulated carrier and noise then,

$x_i(t) = A_c \cos w_c t + n_i(t)$

$= [A_c + n_c(t)] \cos w_c t + n_s(t) \sin w_c t$

let,

$R_n(t) = \sqrt{[A_c + n_c(t)]^2 + n_s^2(t)}$

$\theta(t) = -\tan^{-1}\left\{\frac{n_s(t)}{A_c + n_c(t)}\right\}$

$\therefore x_i(t) = R_n(t) \cos(w_c t + \theta(t))$.

We can assume that $R_n(t)$ will be removed by limiter descriminator.

Assume $A_c >> n_c(t)$

then,

$$\theta(t) = -\tan^{-1}\left[\frac{n_s(t)}{A_c}\right] \approx \frac{n_s(t)}{A_c}$$

so, the input to the descriminator is,

$$x_{dis}(t) = \cos\left(w_c t + \frac{n_s(t)}{A_c}\right) \quad \text{-----(a)}$$

& output of Descriminator is;

$$\frac{d x_{dis}(t)}{dt} = \left\{w_c + \frac{1}{A_c}\frac{d n_s(t)}{dt}\right\}\sin\left\{w_c t + \frac{n_s(t)}{A_c}\right\}$$

The signal is passed through envelope detector and LPF so o/p of LPF is,

$$n_o(t) = \frac{1}{A_c}\frac{d n_s(t)}{dt}.$$

But there is no mathematical representation of $n_s(t)$ so we use spectrum method that is the psdf approach.

$$\underset{S_{ns}(f)}{n_s(t)} \longrightarrow \boxed{\begin{matrix}\text{Differentiator}\\ H(f)\end{matrix}} \longrightarrow \underset{S_{no}(f)}{n_o(t)}$$

The psdf of the signal at the output of the differentiator is

$$S_{no}(f) = S_{ns}(f)\;|H(f)|^2$$

Transfer function of differentiator.

$$H(f) = j\,2\pi f$$

so,

$$S_{no}(f) = N_o|j2\pi f|^2$$

$$= 4\pi^2 N_o f^2.$$

This shows that the noise psd at the output of the descriminator is proportional to f^2

$\therefore$ The noise power within message BW is,

$$P_{no} = \frac{1}{A_c^2}\int_{-B}^{B} S_{no}(f)\,df = \frac{1}{A_c^2}\,4\pi^2 N_o\int_{-B}^{B} f^2\,df$$

$= \dfrac{8\pi^2 N_0 B^3}{3A_c^2}.$

So

$SNR_o = 4\pi^2 A_c^2 K_f^2 \overline{x_m^2(t)} * \dfrac{3A_c^2}{8\pi^2 N_0 B^3}$

$= \dfrac{3A_c^4 K_f^2 \overline{x_m^2(t)}}{2N_0 B^3}$

$\therefore$ Detection gain $(\gamma) = \dfrac{SNR_o}{SNR_i}$

$= \dfrac{3A_c^2 K_f^2 \overline{x_m^2(t)}}{2N_0 B^3} * \dfrac{4BN_0}{A_c^2}$

$\therefore \gamma = \dfrac{6A_c^2 K_f^2 \overline{x_m^2(t)}}{B^2}$ ---- (*)

8. Compare PCM and DPCM

A linear Delta modulator is designed to operate on speech signals limited to 3.4 KHZ. The specified actions of the modulator are as follows.

- Sampling rate = 10 f_N where f_N = Nyquist rate of speech signal.
- Step size Δ = 100 mV.

The modulator is tested with a 1 KHZ sinusoidal signal. Determine the maximum amplitude of this test signal required to avoid slope overload.

⇒ For DPCM refer 2070 chaitra Q.no:-3.

→ Soln,

$\Delta = 100mV$

$A 2\pi f_x \leq \dfrac{\Delta}{T_s'}$

or, $A \leq \dfrac{\Delta f_s'}{2\pi f_x}$ $\left\{ f_s' = \dfrac{1}{T_s'} \right.$

$\therefore A_{max} = \dfrac{\Delta f_s'}{2\pi f_x} = \dfrac{100 \times 10^{-3} \times 10 \times 2 \times 3.4 \times 10^3}{2\pi \times 10^3 \times 1}$

$= 1.0822$ V.

9. The generator polynomial of a (7,4) cyclic code is $g(x) = 1 + x + x^3$. Find the code vector for the message vector 1011 in non-symmetric & symmetric form

⇒ Refer 2070 chaitra Q.no:-11.

10) Write short notes on
i) Linear prediction theory
ii) Eye diagram

i) Linear prediction theory
→ Refer ~~2069~~ 2070 chaitra Q.no 12(b)

ii) Eye diagram.
→ Refer 2069 chaitra Q.no: 12(c).

2066 BHADRA

[REGULAR]

1. State and explain Nyquist sampling theorem. A bandpass signal centered at 40 MHz and having total bandwidth of 60 kHz is to be sampled. Calculate the minimum sampling frequency.

⇒ Sampling theorem states that "Analog signal can be reproduced from an appropriate set of its samples taken at some fixed intervals of time."

The sampling theorem can be stated in two parts:

a) For transmitting end

A strictly band limited signal ($f \geq B$) is completely described by the sample of the signal taken at the instance of time separated by $\frac{1}{2B}$ seconds where B is the signal bandwidth.

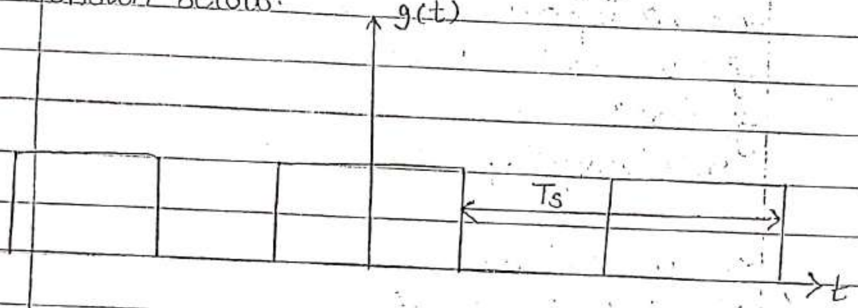
b) For receiving end.

The original signal may be recovered if we know its values taken at the rate of 2B / second.

If the signal $x(t)$ to be sampled is band limited then the sampled signal can be represented as;

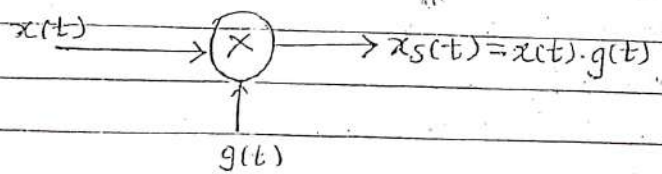
$x_s(t) = x(t) \cdot g(t)$

where $g(t)$ is the sampling function shown below.



where T_s = sampling period

τ = duration of sampling pulse.



The gate function $g(t)$ can be expressed in terms of fourier series;

$$g(t) = C_0 + \sum_{n=1}^{\infty} 2C_n \cos n\omega_s t \text{ ------(i)}$$

where $C_0 = \frac{\tau}{T_s}$

$\omega_s = 2\pi f_s$.

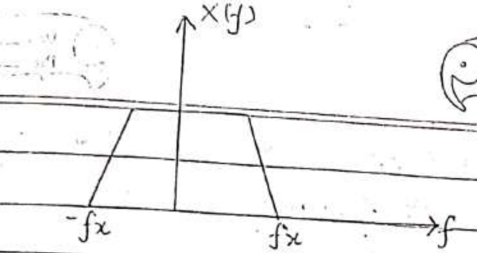
$C_n = f_s \tau \,\text{sinc}(n f_s \tau)$.

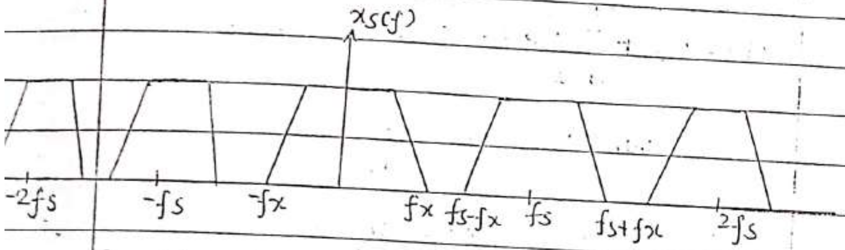
$\therefore x_s(t) = C_0 x(t) + 2C_1 \cos \omega_s t \; x(t) + 2C_2 \cos \omega_s t \; x(t) + \cdots$

Taking fourier transform

$X_s(f) = C_0 X(f) + C_1 X(f - f_s) + C_2 X(f - 2f_s) + \cdots$

The above series can be graphically represented as;





For Distortionless recovery of ~~signal~~ original message signal from spectrum of sampled signal

$f_s - f_x \geq f_x$

or, $f_s \geq 2f_x$

sampling frequency ≥ twice the message frequency

→ soln,

$f_c = 40\ MHz = 40\,000\ KHz$

$BW = 60\ KHz.$

$f_{xu} = f_c + \frac{BW}{2}$

$= 40{,}000 + \frac{60}{2} = 40030\ KHz.$

$m = \left[\frac{40030}{2}\right] \left[\frac{f_{xu}}{BW}\right]$

$= 67$

∴ sampling frequency $= \frac{2 f_{xu}}{m}$

$= 2 * \frac{40030}{667}$

$= 120\ KHz.$

2. Find the expression for evaluating SQNR for the case of uniform quantization. Discuss any one of the companding techniques used in non-uniform quantization.

⇒ For SQNR 2070 Asadh Q no:-3a.
for companding Refer 2069 chaitra Qno:-4.

3. State and explain shannon's channel capacity theorem for binary channel. Derive the expression for theoretical limits of this theorem.

⇒ According to this theorem, the capacity of a channel c to transmit information without error is limited by its bandwidth & noise in the channel.

Mathematically,

$C = B \log_2 (1+SNR)$ bits/sec. - - - - (i)

where

C = channel capacity

B = channel BW

SNR = signal to noise ratio.

Implications.

→ Indicates the upper limit of data transmission for reliable communication.

→ Trade off between B and SNR for given C.

The channel capacity is limited by various extrageneous factors that the designer can not play with for example maximum frequency that can be transmitted over a part of cable is limited by its construction But B and SNR is in the hand of designer

Example :-

Case I :-

R = 10,000 bits/sec

B = 5000 Hz to design.

B = 3000 Hz - available

$\therefore C_{min} = R$

$SNR = 2^{C/B} - 1$

$= 2^{10,000/3,000} - 1$

$= 9$

It means that signal power must be 9 times higher than noise power.

Case II :-

B = 10,000 Hz

SNR ≃ 1

It means if B is higher same quality of signal transmission can be achieved with less signal power (9 times).

In other words, bandwidth compression from 10,000 to 3.000 Hz is possible but at cost of increasing signal power by 9 times.

Theoretical limits of shannon's channel capacity theorem.

a) As noise in channel tends to zero value of SNR will tend to infinity. Subsequently the channel capacity c will tend to infinity. It means that the noise less channel has an infinite capacity. This type of channel is reffered to as ideal channel.

b) As the BW of channel B tends to infinity the channel capacity reaches an upper limit Cmax. This is because noise power is proportional to bandwidth.

ie

i) If $N \to 0$ $SNR \to \infty$

& $C \to \infty$

ii) If $B \to \infty$ $C \to C_{max}$.

As BW is increased noise power also increases correspondingly.

$\therefore C = B \log_2 (1 + SNR)$

$= B \log_2 (1 + S/N)$

$= B \log_2 \left(1 + \frac{S}{\eta B}\right)$

The above expression can be written as,

$$C = \left(\frac{S}{\eta}\right) \log_2 \left(1 + \frac{S}{\eta B}\right)^{B \cdot \eta / S} \text{ - - - - - (ii)}$$

let $r = \frac{S}{\eta B}$

as $B \to \infty$ $r \to 0$.

$$\therefore \lim_{B \to \infty} \left(1 + \frac{S}{\eta B}\right)^{\eta \cdot B/S} = \lim_{r \to 0} (1 + r)^{1/r} = e.$$

$$\therefore \lim_{B \to \infty} C = C_{max} \left(\frac{S}{\eta}\right) \log_2 e$$

$$= 1.44 \left(\frac{S}{\eta}\right).$$

4. Define white noise. Find its autocorrelation function. Explain Rc filtering of white noise with necessary derivations.

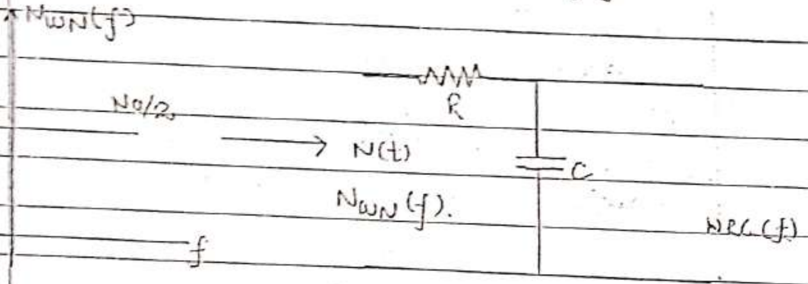
$\Rightarrow$ For white noise

→ Refer 2069 chaitra Q.no:- 8.

RC filtering of white noise.

Let the white noise with power spectrum density function (psdf)

$N_{WN}(f) = \frac{N_0}{2}$ is passed through the RC LPF as shown in the fig.



Frequency response of RC filter is;

$$H(f) = \frac{1/j\omega C}{R + 1/j\omega C}$$

$$= \frac{1}{1 + j2\pi f RC}$$

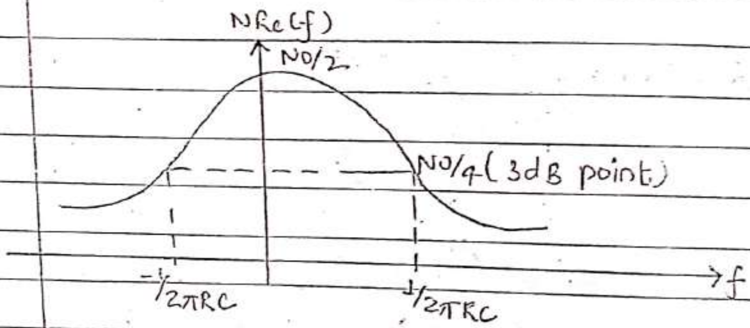
$$|H(f)|^2 = \frac{1}{1 + (2\pi f RC)^2}$$

The psdf of noise at o/p of RC filter is

$$N_{RC}(f) = N_{WN}(f)\,|H(f)|^2$$

$$= \frac{N_0}{2} \cdot \frac{1}{1 + (2\pi f RC)^2}$$

$$= \frac{N_0/2}{1 + (2\pi f RC)^2}$$



As the function of the o/p noise is inverse fourier transform of psdf

$$R_{RC}(\tau) = \int_{-\infty}^{\infty} N_{RC}(f)\, e^{j2\pi f\tau}\, df$$

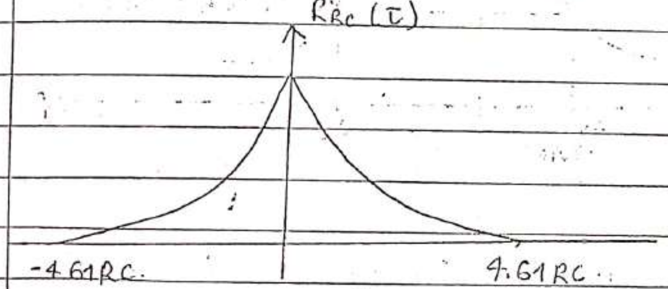
$$= \int_{-\infty}^{\infty} \frac{N_0/2}{1 + (2\pi f RC)^2}\, e^{j2\pi f\tau}\, df.$$

$$= \frac{1}{2\pi} \frac{N_0}{2} \int_{-\infty}^{\infty} \frac{1}{|1+(\omega RC)|^2} e^{j\omega\tau} d\omega.$$

$$= \frac{w_0}{4\pi(RC)^2} 2\int_{-\infty}^{\infty} \frac{\cos\omega\tau}{(1/RC)^2+\omega^2} d\omega$$

$$= \frac{w_0}{4\pi(RC)^2} \frac{2.\ \pi}{2.\ 1/RC.} e^{-|\tau|/RC}$$

$$= R_{RC}(\tau) = \frac{w_0}{4RC} e^{-|\tau|/RC.}$$



6. Derive the expression for impulse response of matched filter

⇒ Refer 2065 shrawan q no:- 4.

5. Derive the expression for evaluating error probability for the case of binary systemmatic channel with additive noise.

⇒ Refer 2069 chaitra q no:- 9.

7. A (7,4) non systemmatic cyclic code generator polynomial has the form $g(x) = 1+x^2$. Find the code words for message blocks (1101) and (0010).

⇒ soln,

for 1101

$m(x) = 1 + x + 0.x^2 + x^3$

$= 1+x+x^3.$

$c(x) = m(x).g(x)$

$= (1+x+x^3)(1+x^2)$

$= 1+x^2+x+x^3+x^3+x^5 \quad \left\{ \because x^3+x^3 = 0 \right.$

$= 1+x+x^2+x^5$

$\therefore c = 1110010.$

for 0010.

$m(x) = 0 + 0.x + 1.x^2 + 0.x^3$

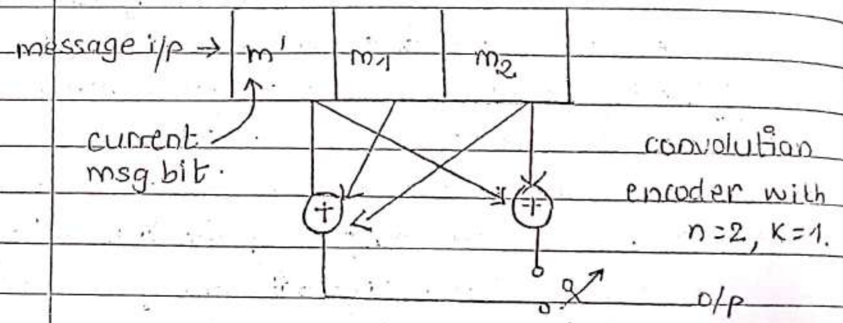
$= x^2.$

$\therefore C(x) = m(x) \cdot g(x)$
$= x^2 (1+x^2)$
$= x^2 + x^4.$

8. Write short notes on
a) convolution coding
b) eye diagram.

a) convolution coding.
In most application and in data communication message bit arrive continuously. convolution coding is a scheme that codes the data 'on the fly' continuous fashion (no storage is required except N shift register)
In convolution codes the message bits and check bits are continuously interleaved and the code word for the i^{th} message bit depend not only upon i^{th} input but also upon previous $(N-1)^{th}$ bit.
where N = no of shift registers used
constraint length $l = k(m-1)$
where, m = no of memory registers
k = no of i/p bits.
n = no of code bits per k input message bit.

These codes can be devised for correcting random error burst error and both. convolution encoding can be accomplished by using simple shift registers as shown.



whenever the message bit is shifted to position 'm' output are generated (v_1, v_2) m_1 and m_2 store the previous message bit and current bit in 'm' register.

b) eye diagram
⇒ Refer 2069 chaitra Qno 12 (a).

ROLL SHRAWAN

[NEW BACK].

Date ______
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1. Explain the importance of source coding in digital communication system. A discrete memory memoryless source emits one of the eight symbols with probabilities 0.25, 0.20, 0.2, 0.15, 0.08, 0.07, 0.03, 0.02. If the output symbols are encoded using Huffman code, find the coding efficiency and o/p bit if the symbol rate of the source is 1000 symbols per second.

⇒ Source coding is importance as it is the conversion of the output of the DMS into a sequence of binary symbol (binary code word)...

According to the source coding theorem average code word length L per ~~second~~ symbol is bounded as

$L \geq H(x)$.

Huffman coding.

0.25	01	0.25	01	0.25	01	0.25	01	0.35	00	0.4	1	0.6	0
0.20	10	0.20	10	0.20	10	0.20	10	0.25	01	0.35	00	0.4	1
0.2	11	0.2	11	0.2	11	0.2	11	0.2	10	0.25	01		
0.15	001	0.15	001	0.15	001	0.2	000	0.2	11				
0.08	0001	0.08	0001	0.12	0000	0.15	001						
0.07	00000	0.07	00000	0.08	0001								
0.03	000010	0.05	00001										
0.02	000011												

Date ______
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$\Rightarrow L = \{0.25\times2 + 0.20\times2 + 0.2\times2 + 0.15\times3 + 0.08\times4 + 0.07\times5 + 0.03\times6 + 0.02\times6\}$

$= 2.72$

$H = 0.25\log_2\frac{1}{0.25} + 0.20\log_2\frac{1}{0.2} + 0.2\log_2\frac{1}{0.2} +$

$0.15\log_2\frac{1}{0.15} + 0.08\log_2\frac{1}{0.08} + 0.07\log_2\frac{1}{0.07} +$

$0.03\log_2\frac{1}{0.03} + 0.02\log_2\frac{1}{0.02}$

$\therefore$ Code efficiency $= \frac{H}{L}\times100\%$

$= \frac{2.66}{2.72}\times100\%$

$= 97.79\%$

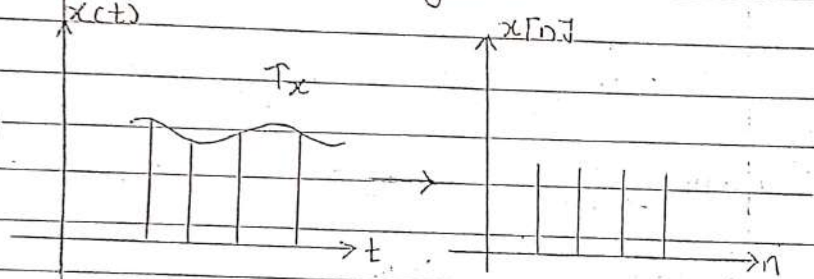
output bit rate $= H\times1000$

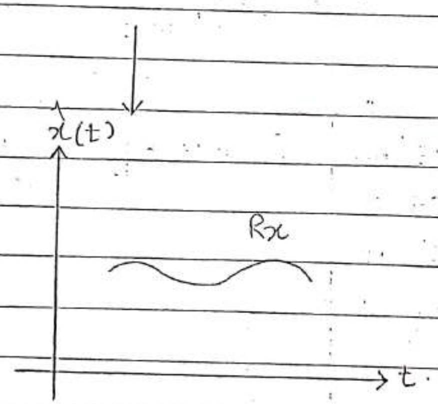
$= 2.66\times1000 = 2660$ bps

2. State and explain Nyquist - Kotelnikov sampling theorem with time domain & frequency domain analysis. Define aliasing & aperture effect.

⇒ Sampling theorem provides the mechanism for representing a continuous time signal by discrete time signal.

- By sampling we can reduce redundancy in continuous time signal.





at Tx end.

A band limited signal ie $x(f)=0$ for $|f|>B$ is completely described by its sample values at uniform interval of $1/2B$ seconds.

at Rx end.

A bandlimited signal can be recovered if samples taken at the rate of 2B per second is known OR if minimum values of sampling frequency is twice the maximum frequency component present in the signal to be sampled

Proof:-

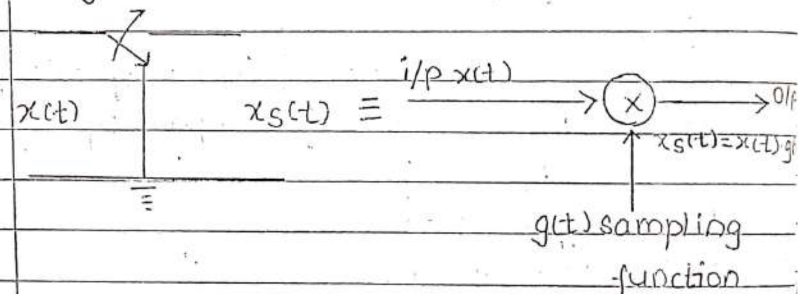
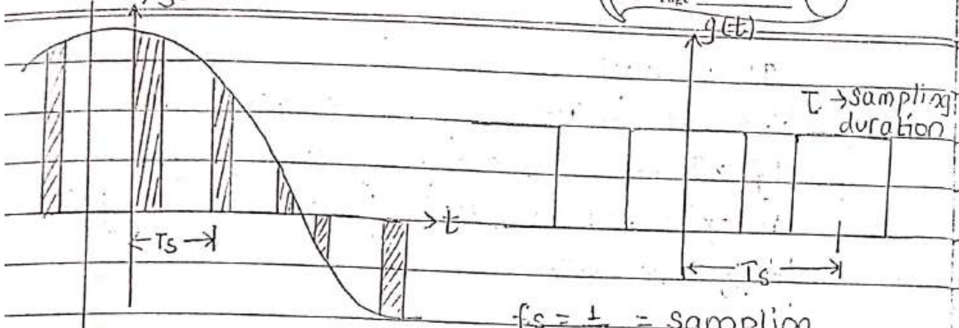


Fig:- schematic diagram of sampler.



$f_s = \frac{1}{T_s}$ = sampling frequency.

consider a band limited signal $x(t)$ and sampling function $g(t)$

ie $x(f) = 0$ for $|f| > f_x$

$$g(t) = \begin{cases} 1 ; & \text{for sampling time} \\ 0 ; & \text{otherwise.} \end{cases}$$

Sampling may be accomplished by multiplying $x(t)$ by gate pulse $g(t)$ repeating periodically T_s seconds.

so, sampled signal.

$$x_s(t) = x(t) \cdot g(t) \quad \text{----- (i)}.$$

$g(t)$ is a periodic signal so it can be expressed in terms of fourier series as;

$$g(t) = c_0 + \sum_{n=1}^{\infty} 2 c_n \cos n \omega_s t$$ for real periodic signal

where $c_0 = \frac{\tau}{T_s}$.

$$c_n = \frac{\tau}{T_s} \operatorname{sinc}\left(\frac{n\tau}{T_s}\right) = f_s \tau \operatorname{sinc}(n f_s \tau) \cdots$$

so,

$$g(t) = c_0 + 2c_1 \cos \omega_s t + 2 c_2 \cos 2 \omega_s t + \cdots \quad (2)$$

$$c_0 = \frac{1}{T_s} \int_{T_s} x(t)\, dt$$

$$= \frac{1}{\tau} \int_{-\tau/2}^{\tau/2} 1\, dt$$

$$= \frac{\tau}{T_s}.$$

&

$$c_n = \frac{1}{T_s} \int_{T_s} x(t)\, e^{-jn\omega_s t}\, dt.$$

$$= \frac{1}{T_s} \int_{-\tau/2}^{\tau/2} e^{-j\omega_s n}\, dt.$$

$$= \frac{-1}{j\omega_s n T_s}\, e^{-jn\omega_s t}\Big|_{-\tau/2}^{\tau/2}$$

$$= \frac{1}{j\omega_s n T_s}\left[e^{j\omega_s n \tau/2} - e^{-j\omega_s n \tau/2}\right]$$

$$= \frac{2}{j\omega_s T_s}\left[\frac{e^{j\omega_s n \tau/2} - e^{-jn\omega_s \tau/2}}{2j}\right]$$

$$= \frac{2}{n\omega_s T_s}\sin\left(n\omega_s \frac{\tau}{2}\right)$$

$$= \frac{2\sin(n\omega_s \tau/2)}{n\omega_s T_s}$$

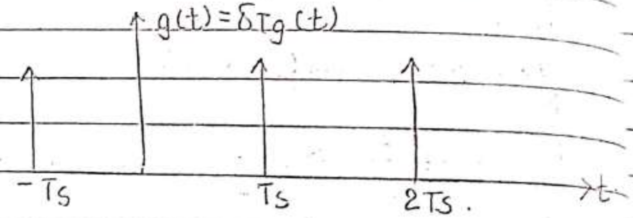
$$= \frac{2\sin(n 2\pi f_s \tau/2)}{n 2\pi f_s T_s}$$

$$= \frac{\tau}{T_s}\,\frac{\sin(\pi n f_s \tau)}{n f_s \tau}$$

$$= \frac{\tau}{T_s}\,\mathrm{sinc}\, n f_s \tau$$

$$= f_s\tau\ \mathrm{sinc}(n f_s \tau).$$

for the case of periodic impulse train $\delta_{T_g}(t)$.



$b_n = 0$

$$C_0 = \frac{1}{T_g}\int_{-T_s/2}^{T_s/2} \delta(t)\, dt = \frac{1}{T_s}$$

$$C_n = \frac{1}{T_s}\int_{-T/2}^{T_s/2} \delta(t)\cdot e^{-jn\omega_s t}\, dt = \frac{1}{T_s}.$$

For the case of gate pulse input signal is weighted by sinc function (so causes alias effect) ie different values. But for the case of impulse train, only amplitude of desired signal is scaled by $1/T_s$ factor.

from eqⁿ ① and ②

$$x_s(t) = C_0\, x(t) + 2 c_1 x(t) \cos \omega_s t + 2 c_2 x(t) \cos 2\omega_s t + \dots$$

fourier transform.

$$x(t) \xleftrightarrow{FT} X(f)$$

$$\cos \omega_0 t \xleftrightarrow{FT.} \frac{1}{2}\left[\delta(f-f_0) + \delta(f+f_0)\right]$$

$$x(t) \cos \omega_0 t \xleftrightarrow{FT.} \frac{1}{2}\left[x(f-f_0) + x(f+f_0)\right]$$

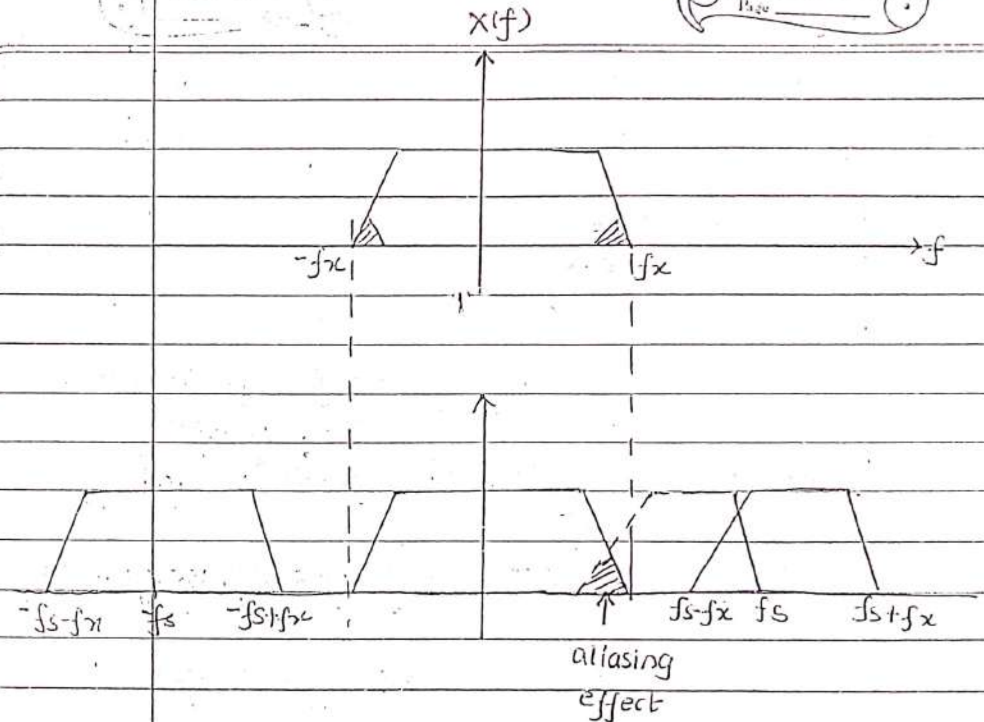
modulation property of F-T

so,

The fourier transform of $x_s(t)$ is

$$x_s(f) = C_0\, x(f) + c_1 x(f-f_s) + c_1 x(f+f_s) + c_2 x(f-2f_s) + \dots$$

This means spectrum of sampled signal $x_s(t)$ consists of original signal $x(f)$ repeating periodically with period $f_s = \frac{1}{T_s}$



From the above figure it is clear that the spectrum of original signal can be recovered from sampled original by passing it through LPF of cut off frequency f_x. For distortionless recovery of original signal from spectrum of sampled signal following condition should be met.

$$f_s - f_x \geq f_x$$

or, $f_s \geq 2 f_x$.

also,

Sampling interval $T_s = \frac{1}{f_s}$

$$T_s \leq \frac{1}{2f_x}$$

The minimum sampling rate $f_s = 2f_x$ is called Nyquist rate and corresponding sampling interval $T_s = \frac{1}{2f_x}$ called Nyquist interval

where f_x is maximum frequency component

* for aliasing and aperture effect refer earlier.

3. A message signal $x(t) = 6\cos(5000\pi t)$ is quantized in 128 levels using Nyquist sampling rate.

a) Find SQNR of the PCM signal.

b) Find the sampling frequency required when same signal uses Delta modulation for same SQNR.

c) If the system uses DM using Nyquist sampling rate, find SQNR degradation in DM as compared to PCM.

⇒ soln,

$2^n = 128$

or, $2^n = 2^7$

or, $n = 7$

a) $SQNR = 4.8 + 6n$

$= 4.8 + 6 \times 7$

$= 46.8$ dB.

b) $3 \times 4^n = 3 \times 4^7 = 49152$

We know,

$$x = \frac{3}{8\pi^2}\left(\frac{f_s'}{f_x}\right)^3$$

$\pi t 5000 = 2\pi f t$

or, $f_x = 2500$.

$$49152 = \frac{3}{8\pi^2}\left(\frac{f_s'}{2500}\right)^3$$

or, $f_s' = 272401.75$

$\therefore f_s' = 272.401$ KHZ.

c) $SQNR = \frac{3}{8\pi^2}\left(\frac{2fx}{fx}\right)$

$= \frac{3}{8\pi^2} * 2^3$

$= 0.30$

In dB, $10\log_{10} SQNR = -5.22$ dB.

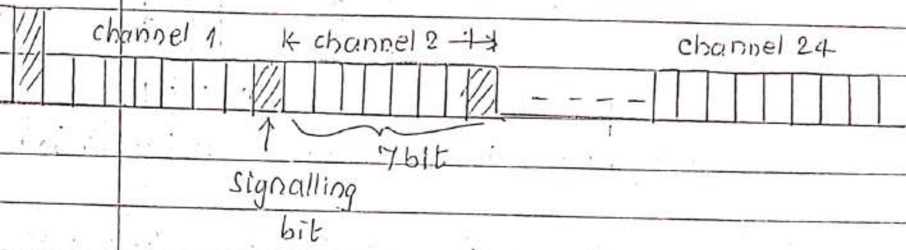
4. What do you mean by companding? Explain T_1 hierarchy of TDM-PCM telephony.

→ Companding

→ Refer 2069 chaitra Q.no:-4.

T_1 hierarchy of TDM-PCM Telephone system.

- It accomodates 24 voice channels. Each of TDM signal is converted into PCM with companding ie 24 voice channels are sampled at $fs = 8$ KHz ($Ts = 125\ \mu s$). Each sample is quantized and converted into 7 bit PCM codeword.
- The 8^{th} addition bit is added for synchronization purpose
- To pavide synchronization an extra bit called framing bit is transmitted at begining of each frame.



Total no of bits/frame $= (24 \times 8) + 1$

$= 193$ bits.

Bit rate $= 193$ bits/frame $* 8000$ frames/sec

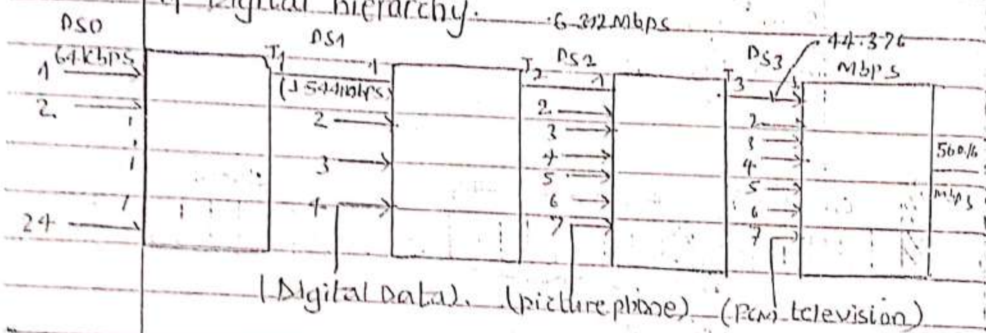
$= 1.544$ Mbps.

Duration of each bit

$T_{b\,max} = \frac{1}{193 \times 8000}$ sec

$= 0.6476\ \mu s$.

T1 Digital hierarchy.



Group	No of voice channels	bits rate (Mbps)
T-1	24	1.544
T-2 ($4T_1$)	96	6.312
T-3 ($7T_2$)	672	44.736
T-4 ($6T_3$)	4032	560.160

5. What is ISI? state Nyquist pulse shaping criteria for zero ISI. Explain Duo-binary coding with example.

⇒ Refer 2069 chaitra Qno:-6.

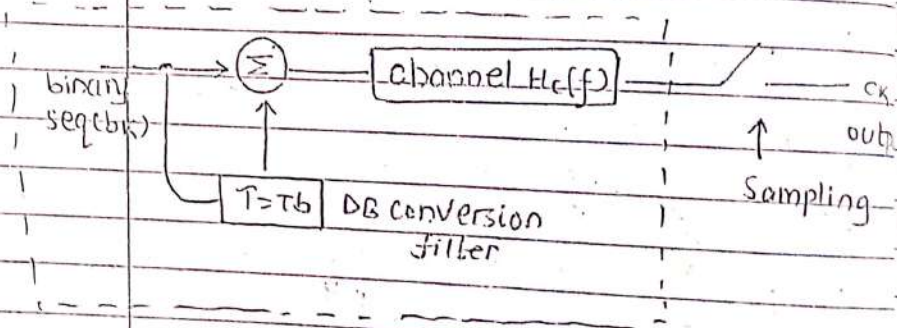
Duo-binary encoding.

In duo-binary encoding input bit stream b_k represented as +A for digit 1 and -A for digit 0 is converted into 3 level bit sequence c_k having 3 levels +2A, 0 & -2A by employing following technique

$$c_k = b_k \oplus b_{k-1} \quad \text{------} \quad (1)$$

Such that;

$$c_k = \begin{cases} +2A; & \text{if } b_k \text{ and } b_{k-1} \text{ are both } 1 \\ 0; & \text{if } b_k \text{ and } b_{k-1} \text{ are different} \\ -2A; & \text{if } b_k \text{ and } b_{k-1} \text{ are both } 0 \end{cases}$$



Now, the frequency response of filter consisting of DB conversion of channel

$$H(f) = H_{DB}(f) \cdot H_c(f).$$

where H_{DB} = frequency response containing of adder & Delay line network

$$H_{DB}(f) = 1 + \exp(-j2\pi f T_b)$$

$$\therefore H(f) = H_c(f)\left[1 + \exp(-j2\pi f T_b)\right]$$

$$= H_c(f)\left[\exp(j2\pi f T_b) + \exp(-j2\pi f T_b)\right] \exp\left[(-j2\pi f T_b)\right]$$

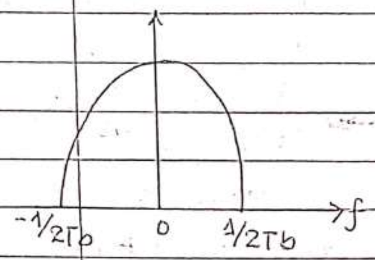
$$= 2H_c(f)\cos(\pi f T_b)\exp(-j\pi f T_b)$$

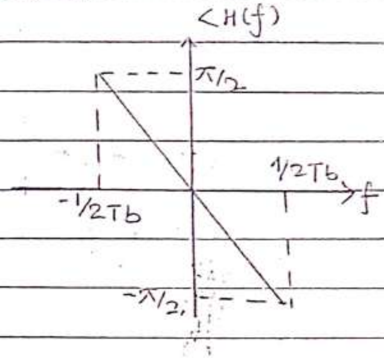
Assuming $H_c(f)$ be ideal

$$H_c(f) = \begin{cases} 1; & \text{for } |f| \le \frac{1}{2T_b} = B_0 \\ 0; & \text{for } |f| > \frac{1}{2T_b}. \end{cases}$$

$$H(f) = \begin{cases} 2\cos(\pi f T_b)\exp(-j\pi f T_b) & \text{for } |f| \le \frac{1}{2T_b} \\ 0, & \text{elsewhere.} \end{cases}$$

The advantage of $H(f)$ is that for transmission of R_b signals that required BW is only B_0 and that $H(f)$ is easily approximated & implemented

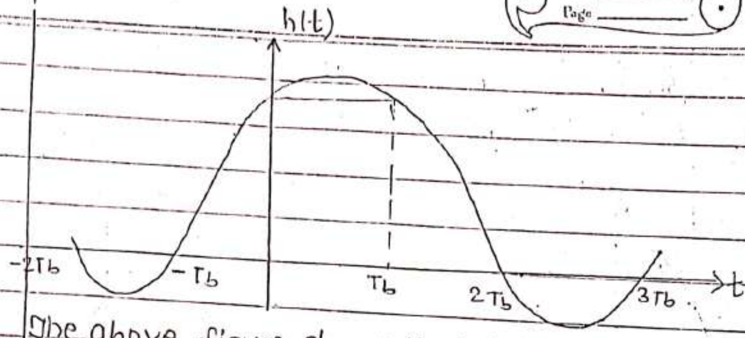




The impulse response of Duo binary conversion filter with transfer function $H(f)$ is

$$h(t) = \frac{\sin(\pi t/T_b)}{\pi t/T_b} + \frac{\sin(\pi(t-T_b)/T_b)}{\pi(t-T_b)/T_b}$$

$$= \frac{T_b^2 \sin(\pi t/T_b)}{\pi t(T_b - t)}$$



The above figure shows that h(t) has 2 distinguished values at sampling instance -2Tb and -Tb.

6. What do you understand by differential coding? Explain Differential phase shift keying modulation & Detection with example & diagram.

⇒ Differential coding is means for avoiding the error propagation. It is usually used before the duo binary encoding to reduce the error.

DPSK.

→ Differential PSK is the non-coherent form of phase shift keying which avoids the need of a coherent carrier at receiver. In DPSK system the input binary sequence is first differentially encoded and then modulated using a BPSK modulator.

The digital information content of the binary data b_k is encoded in terms of signal transistions i.e. Differentially encoded sequence d_k is generated from the input binary sequence b_k by complementing the modulo-2 sum of b_k and d_{k-1}. The effect is to leave the symbol d_k unchanged from the previous symbol if the incoming binary symbol b_k is 1 and to toggle d_k if b_k is 0.

$$d_k = \overline{b_k \oplus b_{k-1}}$$

$$d_k = \overline{0 \oplus d_{k-1}} = \overline{d_{k-1}}$$

$$d_k = \overline{1 \oplus d_{k-1}} = d_{k-1}$$

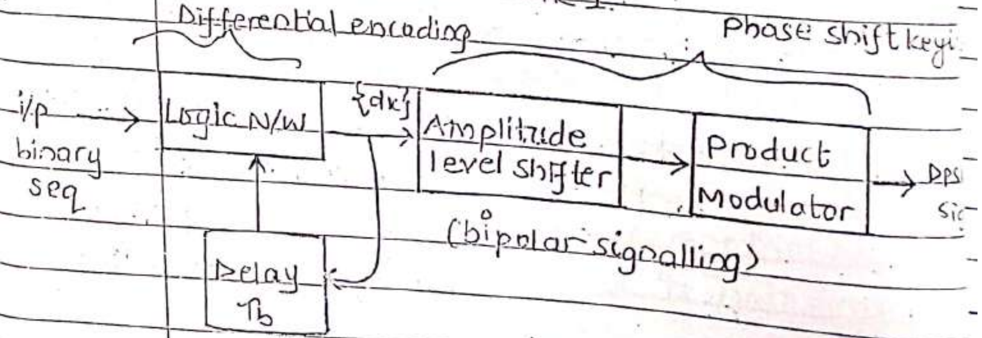


Fig:- DPSK transmitter.

It consists of a one bit delay element and a logic circuit and a logic circuit interconnected so as to generate the differentially encoded sequence from the input binary sequence. The output is passed the product modulator after level shifting to obtain the DPSK signal.

Example :- Illustration of DPSK for 10010011

$\{b_k\}$		1	0	0	1	0	0	1	1
$\{d_k\}$	1	1	0	1	1	0	1	1	1 = 1 d_{k-1} (assume)
phase ϕ		0	1	0	0	1	0	0	0
or									
$\{d_k\}$	π	π	0	π	π	0	π	π	π

From b_k and phase (ϕ) row it is clear that for transmitting '1' the carrier phase is unchanged. Similarly for transmitting 0 the carrier phase is changed by π radian.

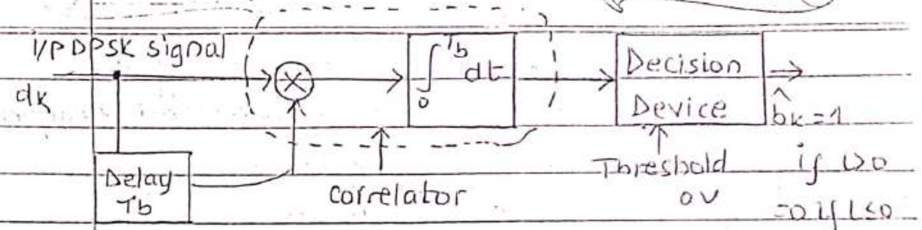


Fig:- Receiver for det^n of DPSK signal.

- At the receiver input, the received DPSK signal and its delayed version (delayed by T_b or 1 bit duration) are applied to the correlator.
- The correlator output is proportional to $\cos\phi$, where ϕ is the difference between the carrier phase angle, in the received DPSK signal and its delayed version.
- When $\phi = 0$ (corresponding to symbol 1) the integrator output is +ve.
- When $\phi = \pi$ (corresponding to symbol 0) the integrator output is -ve.
- Comparing correlator output with the decision level of 0 volts the receiver can reconstruct the binary sequence b_k.

7. Prove that the impulse response of the matched filter is the time reversed delayed version of the input signal.

⇒ Refer 2066 Bhadra Q no:-6.

8. With necessary derivations compare the noise performance of DSB-SC and SSB-SC modulation in analog communication system.

⇒ DSB-SC

Detection method for DSB-SC is always synchronous.

Input to the Demodulator is;

$x(t) = x_m(t)\cos w_c t + n_i(t)$ ---- (i)

input signal and noise power are;

$$P_{si}(t) = \frac{\overline{x_m^2(t)}}{2}$$

$$P_{ni}(t) = \overline{n_i^2(t)}$$

So

$$SNR_i = \frac{\overline{x_m^2(t)}}{2\overline{n_i^2(t)}} \quad \text{----(a)}$$

Output of the synchronous Detector is

$[x_m(t)\cos w_c t + n_i(t)]\cos w_c t$

$= \{(x_m(t) + n_c(t))\cos w_c t + n_s(t)\sin w_c t\}\cos w_c t$

$= \frac{x_m(t) + n_c(t)}{2} + \frac{x_m(t) + n_c(t)}{2}\cos 2w_c t + \frac{n_s(t)}{2}\sin 2w_c t$

Now,

Output of LPF is,

$\frac{x_m(t)}{2} + \frac{n_c(t)}{2}$

Output signal and noise power are;

$P_{so} = \frac{\overline{x_m^2(t)}}{4}$ —— (b)

&

$P_{no} = \frac{\overline{n_c^2(t)}}{4}$ —— (c)

$\therefore SNR_o = \dfrac{\overline{x_m^2(t)}}{\overline{n_c^2(t)}}$

$\therefore \gamma = \dfrac{SNR_o}{SNR_i} = 2 \quad \left(\because \overline{n_c^2(t)} = \overline{n_i^2(t)} \right)$

$\therefore$ DSB-SC is better than DSB-FC.

SSB.

We know that the SSB wave is;

$x_{SSB}(t) = x_m(t)\cos\omega_c(t) \pm \hat{x}_m(t)\sin\omega_c t$

$+ \rightarrow$ USB

$- \rightarrow$ LSB.

after passing through channel, the input to Demodulator is;

$x(t) = x_m(t)\cos\omega_c t \pm \hat{x}_m(t)\sin\omega_c t + n_i(t)$

$\hat{x}_m(t)$ = Hilbert transform of $x_m(t)$.

input signal and noise power are;

$P_{si} = \dfrac{\overline{x_m^2(t)}}{2} + \dfrac{\overline{\hat{x}_m^2(t)}}{2}$

$= \overline{x_m^2(t)} \quad \left\{ \because \overline{x_m^2(t)} = \overline{\hat{x}_m^2(t)} \right.$

$P_{ni} = \overline{n_i^2(t)}$

$\therefore SNR_i = \dfrac{\overline{x_m^2(t)}}{\overline{n_i^2(t)}}$

output of the synchronous Demodulator (for USB)

$= \{x_m(t)\cos\omega_c t + \hat{x}_m(t)\sin\omega_c t + n_i(t)\}\cos\omega_c t$

$= (x_m(t) + n_c(t))\cos^2\omega_c t + (\hat{x}_m(t) + n_s(t))\sin\omega_c t \cdot \cos\omega_c t.$

&

O/P of the LPF will be.

$\dfrac{1}{2}x_m(t) + \dfrac{1}{2}n_c(t).$

$\therefore P_{so} = \dfrac{\overline{x_m^2(t)}}{4}$

& $P_{no} = \dfrac{n_c^2(t)}{4}.$

So, $SNR_o = \frac{2m^2(t)}{n_c^2(t)}$

$\therefore \gamma = \frac{SNR_o}{SNR_i} = 1$

SSB doesnot provide the noise improvement in noise performance (but reduce BW required).

In case of DSB-SC the i/p noise power is twice than that in case of SSB so performance of DSB-SC are identical form from noise improvement point of view.

9. Define the random process? show that the output of LTI is WSSP if the input is also a WSSP

⇒ Refer 2069 chaitra Q no:- 8

&

2070 asadh Q no 6(a).

10. What is the importance of hamming Distance and haming weight in coding theory. explain the example of syndrome decoding method in linear block coding.

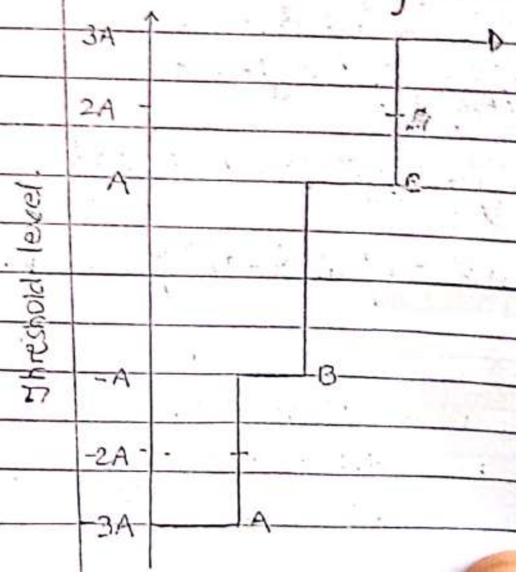
⇒ Refer 2070 Asadh Q no:- 8 a

&

2069 chaitra Q no:- 12(b).

11. Derive the expression of error probability in case of M-ary system. compare binary and M-ary schemes in terms of BW, efficiency and system complexity

⇒ For the case of M = 4.



Decoding algorithm is;

$y(t_m) \leq -2A \rightarrow$ symbol A

$-2A < y(t_m) \leq 0 \rightarrow$ symbol B

$0 < y(t_m) \leq 2A \rightarrow$ symbol C

$y(t_m) > 2A \rightarrow$ symbol D.

here symbol A, B, C and D are equiprobable

so; $P(A\,sent) = P(B\,sent) = P(C\,sent) = P(D\,sent) = 1/4$.

Now,

Probability of error

$P_e = P(error/D\,sent) * P(D\,sent) + P(error/C\,sent) * P(C\,sent) + P(error/B\,sent) \cdot P(B\,sent) + P(error/A\,sent) * P(A\,sent)$

Probability of error when B sent.

$= \frac{1}{4}\Big\{ P(y(t_m) \leq 2A / A_m = 3A) + P(y(t_m) > 2A$ or $\leq 0 / A_m = A) + P(y(t_m) > 0$ or $\leq -2A / A_m = -A) + P(y(t_m) > -2A / A_m = -3A) \Big\}$

since we know that;

$y(t_m) = A_m + n_o(t_m)$ so the expression can be written in terms of noise as;

$P_e = 1/4 \Big\{ P(n_o(t_m) < -A) + P(n_o(t_m) > A$ or $n_o(t_m) < -A) + P(n_o(t_m) > A$ or $n_o(t_m) < -A) + P(n_o(t_m) > A) \Big\}$

$= \frac{1}{4}\Big\{ (|n_o(t_m)| > A) + P(|n_o(t_m)| > A) + P_e(|n_o(t_m)| > A) \Big\}$

from 1st & 4th term,

$= 3/4 \Big\{ P(|n_o(t_m)| > A) \Big\}$ ------(i)

Assuming $n_o(t)$ follow gaussian Distribution

$$P_e = \frac{3}{4} \times 2 \int_{x=A}^{\infty} \frac{1}{\sqrt{2\pi N_0}} \left(\frac{-x^2}{2N_0} \right) dx$$

$$= \frac{3}{4} \times 2 \int_{A/\sqrt{2N_0}}^{\infty} \frac{1}{\sqrt{\pi}} \exp(-\tau^2)\, d\tau$$

$$= \frac{3}{4}\, erfc\left(\frac{A}{\sqrt{2N_0}} \right)$$

In general;

$$P_e = \frac{(M-1)}{M}\, erfc\left(\frac{A}{\sqrt{2N_0}} \right) \text{ ------(ii)}$$

Transmission bandwidth depends only on the pulse rate and not on pulse amplitudes. The signalling interval duration $T_s = 1/R_s$ is same for both binary and M-ary systems so, minimum absolute BW required for both is $R_s/2$ Hz. Price paid by M-ary signalling is that high power ie $\frac{M^3}{3\log_2 M}$ times greater than binary system required to transmit the signal. System is more complex as it requires multiple comparators at the receiving end.

-82-

Some important numerical examples

(i) A memoryless Discrete information source emits one of 3 possible symbols per miliseconds in statistically independent manner. The probabilities of symbols are 1/4, 1/4 and 1/2. Calculate the symbol rate, entropy and information rate.

⇒ soln

$$H = \sum P_i \log_2 \frac{1}{P_i}$$

$$= \frac{1}{4}\log_2\left(\frac{1}{1/4}\right) + \frac{1}{4}\log_2\left(\frac{1}{1/4}\right) + \frac{1}{2}\log_2\left(\frac{1}{1/2}\right)$$

$$= 1.5$$

also

$$1\text{ ms} = 1 \text{ symbol}$$

$$1\text{ sec} = \frac{1}{10^{-3}} \text{ sym}$$

∴ symbol rate (R) = 10^3 sym/sec.

& Information rate = R × H

$$= 10^3 * 1.5$$

$$= 1500 \text{ bps.}$$

Date ____ Page ____

(ii) A video signal having the BW of 6 MHz is to be transmitted using BPCM. Assuming no of uniform quantization levels to be equal to 256. estimate (a) code word length (b) absolute minimum and practical BW of signal (c) final data rate in kbps d) output of SQNR.

⇒ soln,

a) $256 = 2^n$

or, $n = 8$.

b) $BW_{The} = \frac{R}{2} = 48000$

$BW_{pra} = R = 96000$

c) $R = 8 \times 2 \times 6 \times 10^6 = 96000000$

$= 96000$ Kbps

d) $SQNR = 4.8 + 6n$

$= 4.8 + 6 * 8$

$= 51.8$ dB.

(iii) A signal having the dynamic range of ±5V is to be uniformly quantized to 128 representation level. Estimate the required step size, power of quantization noise produced and maximum SQNR that can be achieved.

⇒ soln,

$\Delta = \frac{+5-(-5)}{128} = 0.0781$ V.

$n = 7$

$P_q = \frac{\Delta^2}{12} = \frac{(0.0781)^2}{12} = 0.508$ mwatt

$SQNR = 4.8 + 6n$

$= 4.8 + 6 * \log_2 N$

$= 46.8$ dB.

(iv) An eathquake data recorder traces the signals that changes its polarity a max^m of thirty times each 10 sec estimate the Nyquist sampling frequency & the data rate if this signal is to be converted into a 10 bit PCM signals.

⇒ soln,

⇒ soln,

$f_{max} = 1\,Hz$

$f_s = 2 f_{max}$

$\therefore R = n f_s$

Total cycle in 10 sec = 15 cycle

$1\,sec = \frac{15}{10}$ cycle

= 1.5 cycle/sec.

$\therefore f_{max} = 1.5\,Hz$

$f_s = 2 \times 1.5 = 3\,Hz$

$R = 10 \times 3 = 30.$

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(v) A signal $x(t) = 10\cos(2\pi\,2000t) + 4\cos(2\pi\,3000t)$ is to be sampled and quantized using 256 levels, calculate the minimum sampling frequency and sampling period.

⇒ soln,

$f_x = 3000$ (max freq)

minimum sampling freq $(f_s) = 2 f_x = 2 \times 3000$

$= 6000$

$= 6\,KHz.$

sampling period $= 1/f_s$

$= 1/6000$

$= 0.1667$ msec.

(vii) An analog signal bandlimited to 10 KHz is sampled at Nyquist rate and quantized in 8 levels with probabilities of $1/4, 1/5, 1/5, 1/10, 1/16, 1/20, 1/20, 1/20$ respectively. calculate the entropy & information rate.

⇒ soln,

$H = \sum P_i \log \frac{1}{P_i}$

$= \frac{1}{4}\log_2\left(\frac{1}{1/4}\right) + \frac{1}{5}\log_2\left(\frac{1}{1/5}\right) + \cdots$

$= 2.741.$

Information rate $= H \times 2 \times 10$

$= 2.741 \times 2 \times 10$

$= 54.82$ kbps

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1. Explain the importance of source coding in digital communication system. A discrete memoryless source emits four symbols with probabilities p = {0.125, 0.125, 0.25, 0.5}. If the output symbols are encoded using Shanon Fano code, find the coding efficiency and compare the coding efficiency with that of BCD code.

Ans:→ Source coding is the process of conversion of the output of the DMS into a sequence of binary symbol (BCD). It helps in data compression of naturally redundant message for efficient storage or transmission.

Shannon fano coding:

Symbol x_i	Probabilities $P(x_i)$	step I	Step II	Step III	Code
x_1	0.5	0			0
x_2	0.25	1	0		10
x_3	0.125	1	1	0	110
x_4	0.125	1	1	1	111

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Here,

$$H(X) = -\sum_{i=1}^{4} P(x_i)\log_2(P(x_i))$$

$$= -(-0.5 - 0.5 - 0.375 - 0.375)$$

$$= 1.75$$

and,

$$L = \sum_{i=1}^{4} P(x_i) n_i =$$

$$= 1*0.5 + 2*0.25 + 3*(0.125)*2$$

$$= 1.75$$

then, $\eta_{sf} = \frac{H(X)}{L} = \frac{1.75}{1.75} * 100\%$

$\therefore \boxed{\eta_{sf} = 100\%}$ — (1)

~~Now, for~~

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Now, for BCD Coding :-

In BCD coding length of Codeword is 4 digits.

So, $L = 4*(0.5 + 0.25 + 0.125 + 0.125)$
$= 4$

$$\eta_{BCD} = \frac{H}{L} \times 100\%$$

$$= \frac{1.75}{4} \times 100\%$$

$$\boxed{\eta_{BCD} = 43.75}$$ — (2)

From (1) and (2), It is clear that efficiency of shannon fano coding is greater than that of BCD coding.

2. State Nyquist Sampling theorem. Determine the Nyquist rate and Nyquist interval for a continuous time signal
$x(t) = 6\cos 50\pi t + 20\sin 300\pi t - 10\cos 100\pi t$
is to be sampled and quantized using 512 levels.

Ans:- The statement of Nyquist sampling theorem:

(i) A band limited signal of finite energy, which has no frequency-component higher than Fm Hz, is completely described by its sample values at uniform intervals less than or equal to $\frac{1}{2F_m}$ seconds apart. Which known as Nyquist Rate.

(ii) A band limited signal of finite energy, which has no frequency components higher than Fm Hz, may be completely recovered from the knowledge of its samples taken at the rate of $2F_m$ samples per second. This is called as Nyquist Rate.

Numerical Part:

given, $x(t) = 6\cos 50\pi t + 20\sin 300\pi t - 10\cos 100\pi t$ — ①

Let the three frequency present in eqn ① be $\omega_1, \omega_2, \omega_3$.

So, that the new equation for signal — ① is:

$x(t) = 6\cos\omega_1 t + 10\sin \omega_2 t - \cos\omega_3 t$ — ⑪

Comparing ① & ⑪ we get,

$\omega_1 t = 50\pi t \rightarrow \omega_1 = 50\pi$

$2\pi f_1 = 50\pi$, $\therefore f_1 = 25$ Hz

Similarly,

$f_2 = 150$ Hz

and, $f_3 = 50$ Hz

Also, Signal ① is sampled and quantized using 512 levels.

So, $q = 2^v$

or, $512 = 2^v$

on solving we get

$\therefore v = 9$ bits.

$\because F_2 > F_3 > F_1$

$\therefore F_m = F_2 = 150$ Hz

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Nyquist rate $(f_s) = 2f_m$
$= 2 \times 150$
$= 300\ Hz$

Nyquist interval $(T_s) = \frac{1}{f_s} = \frac{1}{2f_m}$
$= \frac{1}{300}$
$= 0.0037$
$= 3.7 \times 10^{-3}$ sec

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3. Explain E1 digital telephony hierarchy as related to telephony system. Evaluate the expression of SQNR in uniformly quantized PCM.

Ans → Refer:- 2072 chaitra, Ques No:→ 3.

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4. Why DPCM is superior over PCM? Explain its working principle with necessary figure and equations.

Ans:- DPCM is superior over PCM because in PCM standard there are redundancy of information due to which bit rate is high and no. of bits required to transmit one sample is very high. But in case of ~~DPCM~~ DPCM the redundancy is reduced, so, bit rate is ~~high~~ decreased and no of bits required per sample is also reduced.

Working Principle:-
The differential pulse code modulation works on the principle of prediction. The value of the present sample is predicted from the past samples. The prediction may not be exact but it is very close to the actual samples value. Figure (a) below shows the transmitter of differential pulse code modulation (DPCM) system. The sampled signal is denoted by $x(nT_s)$ and the predicted signal is denoted by $\hat{x}(nT_s)$. The comparator

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Finds out the difference between the actual sample value $x(nT_s)$ and predicted sample value $\hat{x}(nT_s)$. This is known as prediction error and it is denoted by $e(nT_s)$. It can be defined as,

$$e(nT_s) = x(nT_s) - \hat{x}(nT_s) \quad \text{—— (a)}$$

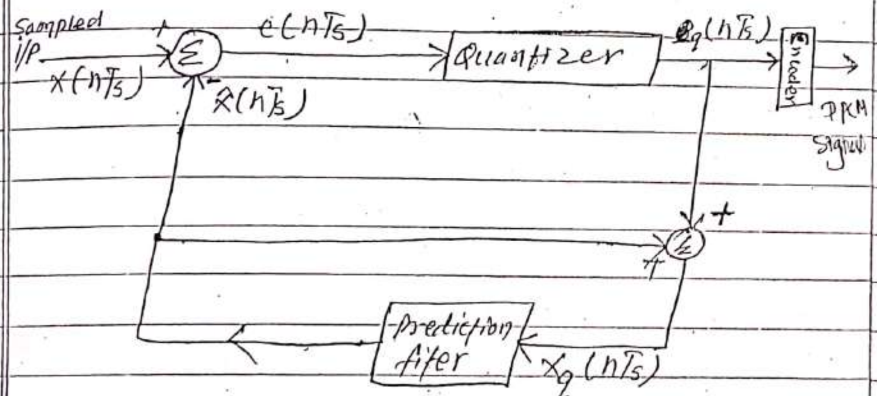

Fig:- (a) A differential PCM.

Thus, error is the difference between unquantized i/p sample $x(nT_s)$ and prediction of it $\hat{x}(nT_s)$. The predicted value is produced by using a prediction filter. The quantizer o/p signal gap. $e_q(nT_s)$ and previous prediction is added and given as i/p to the prediction filter. This signal is called $x_q(nT_s)$. This makes the prediction

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more close to the actual sampled signal. we can observe that the quantized error signal $e_q(nT_s)$ is very small and can be encoded by using small number of bits. Thus number of bits per sample are reduced in DPCM.

The quantizer o/p can be written as,

$$e_q(nT_s) = e(nT_s) + q(nT_s) \quad \text{—— (i)}$$

Here, $q(nT_s)$ is the quantization error.

From figure, the prediction filter i/p $x_q(nT_s)$ is obtained by sum $\hat{x}(nT_s)$ and quantizer o/p.

$$x_q(nT_s) = \hat{x}(nT_s) + e_q(nT_s) \quad \text{—— (ii)}$$

from (i), put $e_q(nT_s)$ in (ii).

So, $$x_q(nT_s) = \hat{x}(nT_s) + e(nT_s) + q(nT_s) \quad \text{—— (iii)}$$

So, eqn (a) becomes as,

$$e(nT_s) = x(nT_s) - \hat{x}(nT_s)$$

$$x(nT_s) = e(nT_s) + \hat{x}(nT_s) \quad \text{—— (iv)}$$

Hence, eqn (iii) becomes;

$$x_q(nT_s) = x(nT_s) + q(nT_s)$$

Hence, the quantized version of the signal $x_q(nT_s)$ is the sum of original sample value quantization error $q(nT_s)$.

5. What is ~~Sh~~ Shannon's channel capacity theorem? Write down theoretical omitations of this theorem.

Ans:- Refer :- Ch 2 - Chaitra Ques No :- 4.

Q. 6. State Nyquist Criteria for zero ISI in both time and frequency domain. What are two major difficulties with duo binary encoding method and Explain how can they be solved?

Ans:- In Time domain

For zero ISI,

$y(t_i) = \mu a_i$ — — (i)

This expression shows that under these conditions, the i^{th} transmitted bit can be decoded correctly. In order to minimize the effects of ISI, we have to design the transmitting and receiving filters properly.

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In Frequency domain :-

If we do Fourier transform of eqn then we get,

$$P_\delta(f) = F[P(nT_b)] = R_b \sum_{n=-\infty}^{\infty} P(f-nR_b)$$

where, $R_b = \frac{1}{T_b}$ (bitrate)

for zero ISI it can represented as

$$\sum_{n=-\infty}^{\infty} P(f-nR_b) = 1/R_b = T_b$$

This is the frequency domain Representation for zero ISI in the absence of noise.

Two major difficulties with duo binary encoding method are given below:-

(i) If a carrier is modulated by a duo binary encoded waveform, then the bandwidth of modulated signal will be smaller than if the unencoded data were used.

(ii) If there is an error in the received signal then it will propagate in the system

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To solve the above difficulties:

To eliminate the first difficulty we must have to modify the duo binary encoder as below.

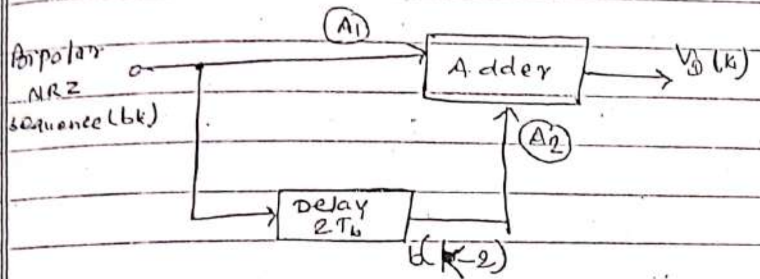

Fig. a. modified duo binary encoder

In this modified duo binary encoder, ~~that~~ ~~to~~ the Correlation between binary digits takes place over duration of $2T_b$ instead of T_b.

But still there is a ~~...~~ limitation of error presence.

In modified duo binary encoding the present in received signal may ~~propagat~~ propagated through the whole system.

To eliminate the error it must be precoded. Hence, a modified duobinary encoder with precoder Can solved the above two major difficulties.

The basic arrangement is shown below.

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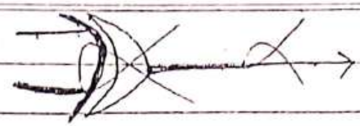

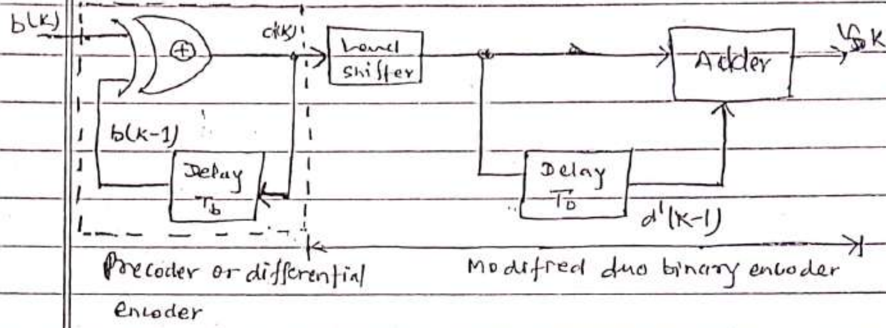

Fig. Block diagram of duo binary encoder with precoder.

7. Represent binary sequence 1001001101 in polar, NRZ, polar RZ, Manchester and AMI codes.

Ans: $\Longrightarrow$ see Question 5.(b) of 2072 chaitra.

8. Define moment and central moment of continuous random variable. Show that first central moment is always zero. Determine the noise equivalent bandwidth of RC-LPF and that of ideal LPF of zero frequency response on e. Also, find output noise power of this RC-LPF when input is white noise.

Moment

Ans:- The n^{th} moment of a random variable is defined as the mean value of X^n. Thus, the expression for the n^{th} moment of x given by,

$$n^{th} \text{ moment of } x = E[x^n] = \overline{x^n} = \int_{-\infty}^{\infty} x^n f_x(x)\,dx \quad \text{--- (I)}$$

Central Moment :-

The central moment is the expected value of the difference between the random variable X and it's mean value m_x. thus, n^{th} central moment is defined as under:

$$n^{th} \text{ central moment of } X = E[(X - m_x)^n]$$

$$= \int_{-\infty}^{\infty} (x - m_x)^n f_x(x)\,dx \quad \text{--- (II)}$$

~~from eqn (II) we have,~~

~~n^{th} moment of X = $E[x^n]$~~

$$~~= \int_{-\infty}^{\infty} x^n f_x(x)\,dx~~$$

~~For 1^{st} moment, $n = 1$,~~

$$~~\therefore E[x] = \int_{-\infty}^{\infty}~~$$

from eqn (I),

m_x is the average value of x.

and ~~this value~~ 1

Now, For 1^{st} central moment, $n = 1$.

$$1^{st} \text{ central moment of } x = E[(x - m_x)^1]$$

$$= E[(X)] - E(m_x)$$

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Since the 1^{st} moment of x is given by

$$E(x) = \int_{-\infty}^{\infty} x^1 f(x)\,dx = m_x$$

1^{st} central moment of $x = m_x - E(m_x)$

$$= m_x - m_x$$

$$= 0$$

Hence, 1^{st} central moment of continuous Random variable is zero.

Next para:-

The noise power at the o/p of a RC-LPF filter is theoretically infinite and is defined only by transfer function of RC filter.

$$P_{NRC} = \frac{N_0}{2}\int_{-\infty}^{\infty} |H_{RC}(f)|^2\,df$$

$$= N_0 \int_0^{\infty} |H_R(f)|^2\,df \quad --- (i)$$

Whereas, the noise power at the o/p of and LPF (ideal) is finite and proportional to the bandwidth of filter.

$$P_{N\,ideal} = \frac{N_0}{2} * 2B = N_0 B \quad --- (ii)$$

To generalize the noise power, we need to define some standard parameter called noise equivalent bandwidth (BN) that can be used to calculate average noise power. In case of RC filter with an ideal LPF having transfer function $H(0)$ and bandwidth equal to BN. So that the noise power at an output of this idealized filter and RC filter are equal.

$$P_{N\,ideal} = N_0 B_N H^2(0) \quad --- (iii)$$

$$\& \quad P_{N\,real} = N_0 \int_0^{\infty} |H(f)|^2\,df \quad --- (iv)$$

Thus we get,

$$B_N = \frac{\int_0^{\infty} |H(f)|^2}{|H^2(0)|} \quad --- (v)$$

Similarly for Bandpass filter Noise equivalent BW equals

$$B_N = \frac{\int_0^{\infty} |H(f)|^2\,df}{|H^2(f_c)|} \quad --- (vi)$$

In general Equivalent noise bandwidth expressed as;

$$B_N = \frac{1}{g_a}\int_0^{\infty} |H(f)|^2\,df \quad --- (vii)$$

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where, g_a is maximum available power gain of filter.

∴ In terms of equivalent noise BW the noise power at the o/p of any filter is

$$P_N = N_0 B_N g_a \quad \text{--- eqn (viii)}$$

9. What do you mean by optimum detector? Find the impulse response of optimum detector in the presence of additive white noise.

Ans:- See. 2072 - ~~shrawan~~ chaitra Quest. No:- 6
or See last pages

10. Derive the expression for evaluating the gain parameter ~~of optimum~~ (SNRo / SNRi) of non-coherent FM detector.

Refer:- 2073 Shrawan Quest No:- 8

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11. With necessary assumption, derive the expression for bit error probability for Binary ASK System.

Ans:- In Binary ASK, the i/p signals can be represented as;

$$s_1(t) = 0 \quad \text{--- eqn (i)}$$
$$s_2(t) = A\cos w_c t \quad \text{--- eqn (ii)}$$

Subtracting eqn (i) and (ii)

$$s_2(t) - s_1(t) = A\cos W_c t \quad \text{--- eqn (iii)}$$

The o/p signal for each case is,

$$S_{o1}(kT_b) = \int_0^{T_b} s_1(t)[s_2(t) - s_1(t)]\,dt =$$

$$S_{o2}(kT_b) = \int_0^{T_b} s_2(t)[s_2(t) - s_1(t)]\,dt$$
$$= \int_0^{T_b} s_2(t)[s_2(t) - s_1(t)]\,dt$$
$$= \int_0^{T_b} A^2\cos^2 w_c t\,dt$$
$$= \frac{A^2}{2} \cdot T_b \quad \text{--- (1)}$$

||ly

The maximum value of SNR, γ will be

$$\gamma^2_{max} = \frac{\int_{-\infty}^{\infty} [s_2(t) - s_1(t)]^2 dt}{N_0/2}$$

$$= \frac{2}{N_0} \int_0^{T_b} A^2 \cos^2 \omega_c t \, dt.$$

$$= \frac{2}{N_0} \frac{A^2 T_b}{2} = \frac{A^2 T_b}{N_0} \quad \text{---- eqn (ii)}$$

$\therefore$ Error probability will be

$$P_{emin} = erfc\left(\frac{\gamma_{max}}{2}\right)$$

or, $P_e = erfc\left(\sqrt{\frac{A^2 T_b}{4 N_0}}\right)$ --- (iv)

Since S_1 and S_2 are equiprobable the average signal power will be,

$$S_{av} = \frac{A^2}{4} \quad \text{--- (v)}$$

In terms of average signal power,

$P_e = erfc\left(\sqrt{\frac{S_{av} T_b}{N_0}}\right) = erfc\sqrt{\frac{E_{av}}{N_0}}$ — (vi)

And, the average signal energy power per bit information is.

$E_{av} = S_{av} \cdot T_b$ —— (vii)

12. Define Hamming weight and Hamming distance for a code vector $x = (0111000)$ and the parity check matrix H given below. Prove that, the given code is valid.

$$H = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}_{3\times 7}$$

Same as: 2072 - Chaitra Ques No:- 10

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1. If a source emits symbols x_i = {A, B, C, D, E, F} in the BCD format with

a) Probabilities $P(X_i)$ = {0.3, 0.1, 0.02, 0.15, 0.4, 0.03} at a Rate R_s = 14.4 Kbaud, Find the following:

i) Information rate

ii) Coding efficiency both with BCD and Huffman Coded Signal

b) Explain Huffman codes with Examples.

1(a) Ans:- using ~~Huffman~~ BCD coding:

Symbol x_i	probabilities $P(x_i)$	BCD code	n_i
A	0.4	1010	4
B	0.3	1011	4
C	0.15	1100	4
D	0.1	1101	4
E	0.03	1110	4
F	0.02	1111	4

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For Huffman Coding :-

x_i	$P(x_i)$	step-I	Step-II	step-III	step-IV
x_5	0.4	→ 0.4	→ 0.4	→ 0.4	→ 0.6 0
x_1	0.3	→ 0.3	→ 0.3	→ 0.3 0	→ 0.4 1
x_4	0.15	→ 0.15	→ 0.15 0	→ 0.3 1	
x_2	0.1	→ 0.1 0	→ 0.15 1		
x_6	0.03 0	→ 0.05 1			
x_3	0.02 1				

Now,

x_i	$P(x_i)$	Huffman code	n_i
x_5	0.4	1	1
x_1	0.3	01	2
x_4	0.15	001	3
x_2	0.1	0000	4
x_6	0.03	00010	5
x_3	0.02	00011	5

Calculation :-

$$L_{Huff} = 0.4*1 + 0.3*2 + 0.15*3 + 0.1\times4 + 0.03*5 + 0.02*5$$

$$= 2.1 \text{ bits/symbol}$$

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$$H = \sum_{i=1}^{6} P_i \log_2 \frac{1}{P_i}$$

$$= 0.4 \log_2 \frac{1}{0.4} + 0.3 \log_2 \frac{1}{0.3} + 0.15 \log_2 \frac{1}{0.15} + 0.1 \log_2 \frac{1}{0.1} + 0.03 \log_2 \frac{1}{0.03} + 0.02 \log_2 \frac{1}{0.02}$$

$$= 2.057 \text{ bits/symbol}$$

(i)

$\Rightarrow R_s = 14.4$ k baud.

$= 14400$ baud.

O/P bit rate / information rate $= H \times R_s$

$= 2.057 \times 14400$

$= 29620$ bps

(ii)

$$\Rightarrow \eta_{Huff} = \frac{H}{L_{Huff}} * 100\%$$

$$= \frac{2.057}{2.1} \times 100\%$$

$$= 97.95\%$$

$L_{BCD} = 4\times0.4 + 4\times0.3 + 4\times0.15 + 4*0.1 + 4*0.03 + 4*0.02$

$= 4$

$\eta_{BCD} = \frac{2.057}{4} * 100\%$

$= 0.514 * 100\%$

$= 51.4\%$

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1 (b) :-

Ans:- Huffman coding Algorithm :-

1. For a given list of symbols, develop a corresponding list of frequency count so that each symbol's relative frequency of occurence is known.
2. Arrange the probability in descending order.
3. Combine the probability of two symbols ~~combi~~ having lowest probable and re-order the resultant probability in the next step. This step is repeated until there are two ordered probabilities remaining.
4. Start encoding with the last reduction. Assign 0 as the first in codewords for all source symbols associated with the first probability: assign 1 to the second probability
5. Now go back and assign 0 and 1 to the second digit for the two probabilities that were combined in the previous reduction step, retaining all assignments made in step 3.
6. Keep regressing this way until the first column is reached.

Example :-

Symbol (x_i)	probabilities $P(x_i)$	step-I	step-II	step-III	code
x_1	0.4	0.4	0.4	0.6 0	
x_2	0.19	0.25	0.35 0	0.4 1	
x_3	0.16	0.19 0	0.25 1		
x_4	0.15 0	0.16 1			
x_5	0.1 1				

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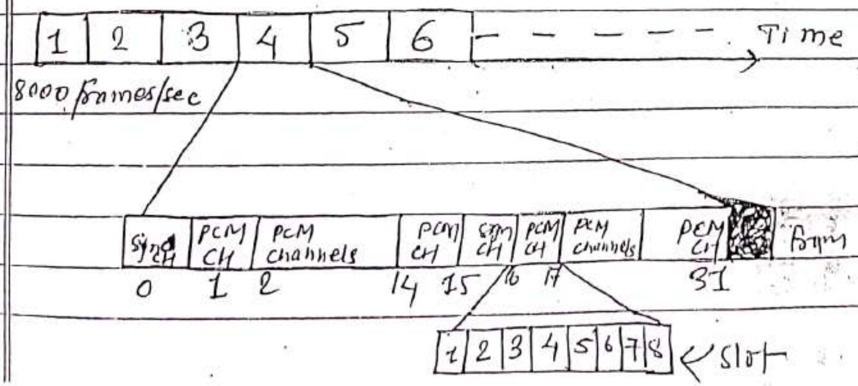
2. State Nyquist sampling theory. Determine the Nyquist rate and Nyquist interval for a continuous time signal $x(t) = 6\cos 50\pi t + 20\sin 300\pi t - 10\cos 100\pi t$ is to be sampled and quantized using 512 Levels.

Ans:- ~~Refer:- 2071 Chaitra Question No. 1~~
~~Refer:- 2071 Chaitra Ques No. 1~~

Refer:- 2071 - chaitra ques No. - 2.

3. Explain E1 digital hierarchy as related to telephony system. Evaluate the expression of SQNR in uniformly quantized PCM system.

Ans:- E1 digital hierarchy as related to telephony system:



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VF band frequency:- 4 kHz (300Hz - 3400Hz)
Sampling frequency:- 8 kHz

Sampling rate = 8000 cycles per second.
i.e. 8000 frames per second.

In each frame;

↳ Time slot 0 or channel 0:- Synchronous channel or frame allignment channel

↳ Time slot 16 or channel 16:- Signalling channel for all 30 Vf channels.

↳ Time slot or VF channel $\left\{\begin{matrix}1-15\\17-31\end{matrix}\right\}$ 30 Vf or Telephony channels.

↳ each time slot or channel consists of 8 bit digital signal.

↳ 0 channel → framing code

↳ 16 channel → Signaling code
channel $\left.\begin{matrix}1-15\\17-31\end{matrix}\right\}$ Information.

↳ Total no. of bits in a frame
= 32 × 8 = 256 bits/frame

i.e. 125 μ => 256 bits T_x

Thus, bit rate = 2.048 Mbps.

↳ Channel Speed = 8 bits/sample × 8000 samples/sec

= 64 Kb/sec.

SQNR in Uniformly quantized PCM Systems:-

We know that in a PCM system for linear quantization the signal to quantization noise ratio is given as,

$$\frac{S}{N} = \frac{\text{Normalized Signal power}}{\text{Normalized Noise power}}$$

But, Normalized noise power has been calculated as $\frac{\Delta^2}{12}$.

Therefore, $\frac{S}{N} = \frac{\text{Normalised Signal power}}{(\Delta^2/12)}$ — (i)

~~Let us assume~~ We know that the number of bits 'v' and quantization levels are related as,

$$q = 2^v \quad \text{— (ii)}$$

Let us assume that i/p $x(nT_s)$ to a linear quantizer has continuous amplitude in the range $-x_{max}$ to $+x_{max}$. Therefore, total amplitude range.

$$= x_{max} - (-x_{max}) = 2x_{max}$$

Now, the step size will be,

$$\Delta = \frac{2x_{max}}{q} \quad \text{— (iii)}$$

Here, substituting the value of q from equation (ii) in eq^n (iii)

$$\Delta = \frac{2x_{max}}{2^v}$$

Now, substituting this value in eq^n (i), we get

$$\frac{S}{N} = \frac{\text{Normalized Signal power}}{\left(\frac{2x_{max}}{2^v}\right)^2 \cdot \frac{1}{12}}$$

Let the Normalized signal power be denoted as 'P'.

Then, $$\frac{S}{N} = \frac{P}{\frac{4x_{max}^2}{2^{2v}} \times \frac{1}{12}} = \frac{3P}{x_{max}^2} \cdot 2^{2v} \quad \text{— (iv)}$$

Hence eqn (iv) is the required relation for signal to quantization noise ratio for linear quantization in a PCM system.

~~Hence Signal to Quantization Noise Ration.~~

This expression shows that signal to noise power ratio of quantizer increases exponentially with increasing bits per second.

4. Explain shannon chanel capacity theorem. Write down theoretical limitations of this theorem.

Ans → The bandwidth and the noise power limits the rate of Information that can be transmitted by a channel. It says that in a channel which is disturbed by a white Gaussian noise, can transmit information ~~only~~ at a rate of C bits per second.

C = Channel Capacity

Mathematically,

$$C = B \log_2 \left(1 + \frac{S}{N} \right)$$

In this expression,

B = Bandwidth of channel in Hz

S = Signal power

N = Noise power.

Theoretical limitations :-

1. For a noiseless channel N = 0 then the ~~[illegible]~~ value of SNR is infinite and the channel capacity is infinite. But practically N is finite and Hence, C is finite.

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2\. This is true even if the bandwidth B is infinite. The noise signal is a white noise with a uniform power density spectrum over the entire frequency range. Therefore, as the bandwidth B is increased, N also increases and Hence the channel capacity remains finite even if $B = \infty$.

5\. Define Inter Symbol Interference (ISI) in baseband digital communication system. Explain the ideal and practicle solution for Zero ISI.

Ans:- ISI :—

The ISI is the process of spreading of pulse to its neighbour pulse.

It is arises due to the imperfections in the overall frequency response of the system.

Ideal solution for Zero ISI:-

We have,

$$\sum_{n=-\infty}^{\infty} P(f-nR_b) = 1/R_b = T_b \quad \text{—— (i)}$$

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above eqn represents Nyquist criterian for zero and distortionless baseband Transmission.

Also, In LHS of eqn (i) represents a series of shifted spectrums.

For $n=0$, the LHS corresponds to $P(f)$ and it represents a frequency function with the narrowest bandwidth which satisfies eqn (i).

The Range of frequency for $P(f)$ will extend from $-B_0$ to B_0 where B_0 corresponds to half the bit rate.

Hence, $B_0 = R_b/2$ —— (ii)

Therefore, $P(f)$ Can be specified in the following form.

$$P(f) = \frac{1}{2B_0} \operatorname{rect}\left(\frac{f}{2B_0}\right) \quad \text{—— (iii)}$$

and it has shown graphically in figure (a). This is the Spectrum of a signal which produces zero ISI. Hence, the signal that produces zero ISI can be obtained by taking the IFT of $P(f)$.

This means that we have,

$$p(t) = F^{-1}[P(f)]$$

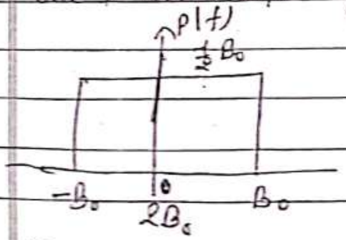
$$p(t) = F^{-1}\left[\frac{1}{2B} \text{rect}\left(\frac{f}{2B_0}\right)\right]$$

$$p(t) = \text{sinc}\,(2B_0 t) \quad \text{---} (iv)$$

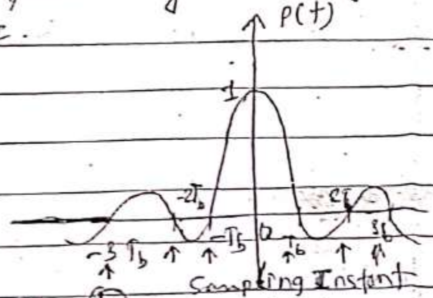
Figure ⓑ shows the plot of eqn (iv).

This function $p(t)$ can be regarded as IFT impulse response of an ideal LPF with bandwidth B_0.

Thus, the shape of a pulse should be a sinc pulse rather than being a rectangular one to eliminate the ISI.



ⓐ Graphical representation of P(f)



ⓑ Time-domain represent

Practical solution for zero ISI:-

We know that the fourier transform of a sinc pulse is a rectangular function. Hence, to preserve all the frequency components, the frequency response of the filter must be exactly flat in the pass band and zero in the attenuation band as shown in figure ⓐ below. which has roll of factor $\alpha = 0$.

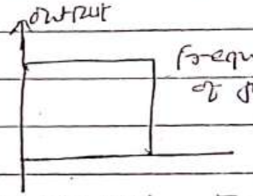


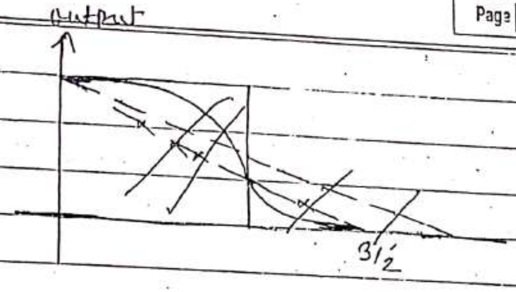
Fig (a) Frequency response of filter.

However, this type of filter is not possible. Hence, in practice the response modified using the roll of factor,

$$\alpha = 1 - \frac{f_1}{f_m}$$

Which is given in the figure ⓑ

↓



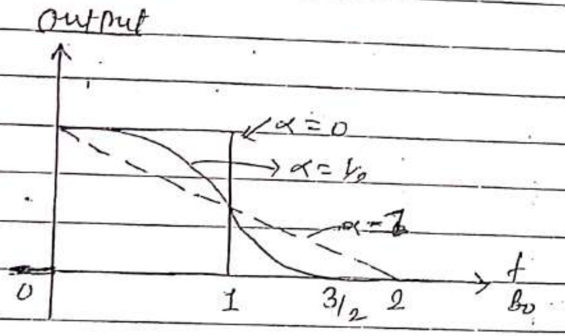


Fig: (b) illustration of practicle filter Characteristics.

5(b). Represent binary sequence 1001001101 in polar, NRZ, polar RZ, Manchester and AMI codes.

→ Refer :- 2073 Shrawan. Question 7

6. What do you understand by Optimum detection? Show that the impulse response of the optimum detector network is the time shifted replica of the incoming signal.

Ans: The word optimum detection Refers the output signal received at receiver filter having very less probability of errors.

Which is actually done by optimum filters and matched filters.

Next part :- Refer 2073 Sharawan Question No:- 6

7. Find the error probability of coherent ASK and PSK detections and show that ASK requires double the average power than PSK for same ~~pro~~ error probability.

Ans:- coherent detection of ASK:

In binary ASK, the i/p signals can be represented as:

$$s_1(t) = 0 \quad \text{--- eq}^n\text{(i)}$$
$$s_2(t) = A\cos w_c t \quad \text{--- eq}^n\text{(ii)}$$

Subtracting (i) from (ii).

$$s_2(t) - s_1(t) = A\cos w_c t \quad \text{--- eq}^n\text{(iii)}$$

The O/P signal for each case is:

$$s_{o_1}(KT_b) = \int_0^{T_b} s_1(t)[s_2(t)-s_1(t)]\,dt = 0$$

$$s_{o_2}(KT_b) = \int_0^{T_b} s_2(t)\,[s_2(t)-s_1(t)]\,dt$$

$$= \int_0^{T_b} A^2\cos^2 w_c t\,dt$$

$$= \frac{A^2}{2}\cdot T_b \quad \text{--- eq}^n\text{ (a)}$$

The max^m value of SNR, γ will be

$$\gamma^2_{max} = \int_{-\infty}^{\infty} \frac{[s_2(t) - s_1(t)]^2}{N_0/2}\,dt$$

$$= \frac{2}{N_0}\int_0^{T_b} A^2\cos^2 w_c t\,dt$$

$$= \frac{2}{N_0}\,\frac{A^2 T_b}{2} = \frac{A^2 T_b}{N_0} \quad \text{--- eq}^n\text{ (b)}$$

$\therefore$ the error probability will be

$$P_{e\,min} = \operatorname{erfc}\left(\frac{\gamma_{max}}{2}\right)$$

$$= \operatorname{erfc}\left(\sqrt{\frac{A^2 T_b}{4N_0}}\right) \quad \text{--- eq}^n\text{(iv)}$$

Since, s_1 and s_2 are equiprobable the average signal power will be,

$$S_{av} = \frac{A^2}{4} \quad \text{--- (V)}$$

$$\text{So, } P_e = \operatorname{erfc}\left(\sqrt{\frac{S_{av}\,T_b}{N_0}}\right) = \operatorname{erfc}\left(\frac{E_{av}}{N_0}\right)$$

— (V)

Now, Coherent detection of PSK :-

Let i/p signals are :

$s_1(t) = A\cos\omega_c t$ - - (1)

$s_2(t) = A\cos\omega_c t$ - (2)

So, their respective o/p will be;

$s_{o1}(kT_b) = -A^2T_b$

$s_{o2}(kT_b) = A^2T_b$

& the max^m SNR,

$$\gamma^2_{max} = \int_{-\infty}^{\infty} \frac{[s_2(t) - s_1(t)]^2 dt}{N_0/2}$$

$$= \frac{4}{N_0}A^2T_b \quad \text{--- eq}^n \text{ (A)}$$

$$\text{or } \gamma = 2A\sqrt{\frac{T_b}{N_0}} \quad \text{— eq}^n \text{ (B)}$$

and, $S_{av} = A^2/2$ (average signal power)

Now, error probability in terms of average energy per bit it's,

$$P_{emin} = \text{erfc}\left(\frac{\gamma_{max}}{2}\right) = \text{erfc}\sqrt{\frac{A^2T_b}{N_0}} \quad \text{-(3)}$$

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and using Save we get eq^n (3) as,

$$P_e = \text{erfc}\sqrt{\frac{2S_{av}T_b}{N_0}}$$

$$= \text{erfc}\sqrt{\frac{2E_{av}}{N_0}} \quad \text{— eq}^n \text{ (4)}$$

on comparing eq^n (1) & (4)

we get that ASK requires double the average signal power than PSK for same error probability.

8. Explain the modulator, demodulator and Signal Space diagram for FSK Modulation.

Ans:- Modulator :-

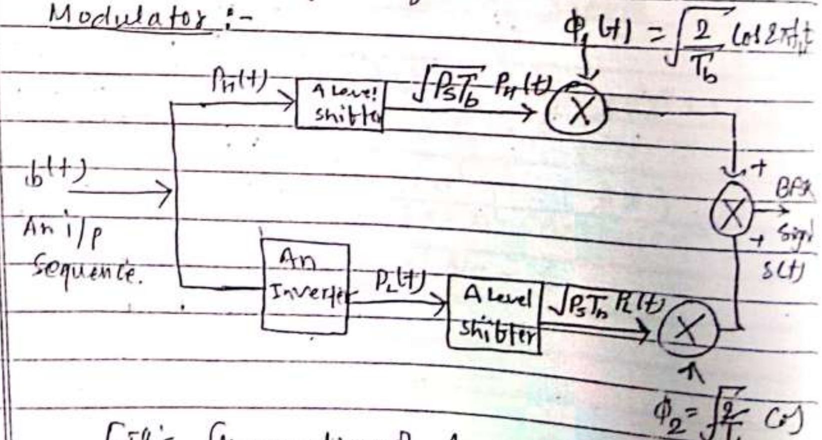

Fig:- Generation of Binary FSK. (erfc)

we know that i/p sequence $b(t)$ is same as $P_H(t)$. An inverter is added after $b(t)$ to get $P_L(t)$. The level $P_H(t)$ and $P_L(t)$ are unipolar signals. The level shifter converts the '+1' level to $\sqrt{P_s T_b}$. Zero level is ~~unaff~~ uneffected.

When a binary '0' is to be transmitted $P_L(t) = 1$ and $P_H(t) = 0$, and for a binary '1' to be transmitted, $P_H(t) = 1$ and $P_L(t) = 0$. So, Transmitted signal will have a frequency of either f_H or f_L.

And ~~Also~~ The product modulators after level shifter. The two carrier signals $\phi_1(t)$, $\phi_2(t)$ are used. These are orthogonal to each other. In one bit period of i/p signal (i.e. T_b), $\phi_1(t)$ or $\phi_2(t)$ have integral number of cycles. The modulated signal have continuous phase as shown below.

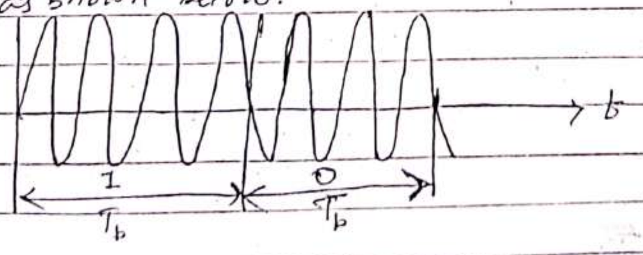

Fig: The BFSK signal.

Demodulator of Binary FSK :-

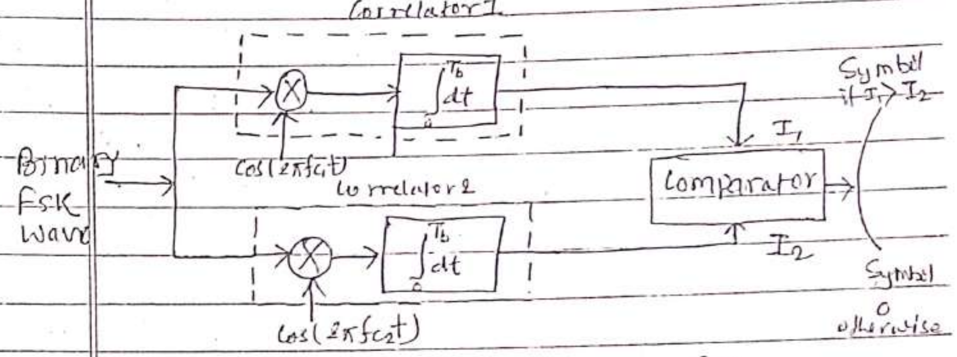

Fig: Demodulation of Binary FSK

The above demodulator consists of two correlators that are individually tuned to two different carrier frequency to represent symbols '1' and '0'. A correlator consists of a multiplier followed by an integrator. Then, the received binary FSK signal is applied to the multiplerers ~~followed by an integrator~~ of both the correlators. To the other i/p of multipliers, carriers with frequency f_{c1} & f_{c2} are applied as shown in above Figure. The multiplied o/p of each multiplier is subsequently passed through integrators, generating output I_1 and I_2 in the two paths. →

The o/p of the integrators are then fed to the decision making device. It is a comparator which compares I_1 and I_2. If $I_1 > I_2$ then detector makes a decision in favour of symbol '1' otherwise in favour of symbol '0'.

Signal space diagram:-

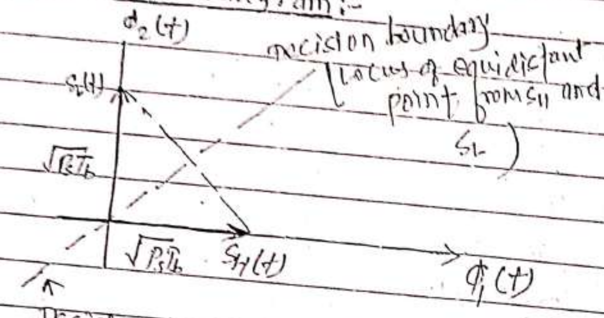

Fig:- Signal Space diagram for Binary FSK

The different signal points are represented geometrically in ϕ_1 ϕ_2-plane. For Geometrical representation of FSK, these carriers are required. $\phi_1(t)$ and $\phi_2(t)$ have different frequencies F_H and F_L. To make $\phi_1(t)$ and $\phi_2(t)$ orthogonal, the frequencies F_H and F_L must be same integer multiple of band frequency F_b.

i.e. $\begin{bmatrix} F_H = mF_b \\ F_L = nF_b \end{bmatrix}$

9. With necessary derivation, compare noise performance of DSB-AM, DSB-SC, SSB-SC.

Ans:- Noise performance of DSB-AM, DSB-SC and SSB-SC can be compared according to their gain parameter individually. The detection method having high gain γ has better noise improvement. So, let's finding out the γ parameter for each.

DSB-AM

Let i/p to the demodulator is the sum of signal noise.

$$x_i(t) = \{A_c + x_m(t)\}\cos w_c t + n_i(t) \quad \text{---eq(i)}$$

Then, signal power at the i/p is;

$$P_{Si} = \frac{A_c^2}{2} + \frac{\overline{x_m^2(t)}}{2} \quad \text{--- (a)}$$

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Similarly, Noise power $(P_{Ni}) = \overline{n_i^2(t)}$ — eq (b)

So, $SNR_i = \dfrac{A_c^2 + \overline{x_m^2(t)}}{2\,\overline{n_i^2(t)}}$ ---- (11)

Now, using synchronous detection, the received AM signal is multiplied by $\cos w_c t$ and then passed to the LPF.

$$x_i(t)\Big|_{LPF} = x_i(t).\cos w_c t$$
$$= \frac{1}{2}\cdot x_m(t) + \frac{1}{2} n_c(t)$$

Now, the o/p signal power is,

$$P_{So} = \frac{\overline{x_m^2(t)}}{4} \text{ ---- (x)}$$

& o/p noise power is

$$P_{No} = \frac{\overline{n_c^2(t)}}{4} \text{ --- (y)}$$

So, SNR at the output is,

$$SNR_o = \frac{\overline{x_m^2(t)}}{\overline{n_c^2(t)}} \text{ --- (1)}$$

$$\gamma = \frac{SNR_o}{SNR_i} = \frac{2\,\overline{x_m^2(t)}}{A_c^2 + \overline{x_m^2(t)}} \text{ --- (2)}$$

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from (2) $\gamma \uparrow$ as $A_c \downarrow$.

So, for distortionless transmiss.
$A_c \geq |x_m(t)|_{max}$.

& $\eta = \dfrac{\overline{x_m^2(t)}}{A_c^2 + \overline{x_m^2(t)}}$

So, $\gamma = 2\eta$

when modulation index is 100%. and modulating signal is ~~100%~~. sinusoidal.

$$\eta = \frac{1}{3}$$

$$\gamma = 2 \times 1/3 < 1.$$

[i.e. gain parameter $(\gamma) = < 1$ for even 100%. modulation.] — (3)

Similarly for DSB-SC:
The i/p signal is:
$$x_i(t) = x_m(t)\cos w_c(t) + n_i(t)$$

So & $P_{Si} = \dfrac{\overline{x_m^2(t)}}{2}$, $P_{Ni} = \overline{n_i^2(t)}$ → $SNR_i = \dfrac{\overline{x_m^2(t)}}{2\,\overline{n_i^2(t)}}$ — (4)

Similarly at the o/p of demodulator the usefull signal component is $\frac{x_m(t)}{2}$.

$$\therefore\ P_{so} = \frac{\overline{x_m^2(t)}}{4} = \frac{P_{si}}{2}$$

&

$$P_{N_o} = \overline{n_c^2(t)}$$

So, $SNR_o = \frac{P_{so}}{P_{N_o}} = \frac{\overline{x_m^2(t)}}{\overline{n_c^2(t)}}$ — (5)

from (4) & (5), we get,

$$\gamma = \frac{\overline{x_m^2(t)}}{\overline{n_c^2(t)}} \times \frac{2\overline{n_i^2(t)}}{\overline{x_m^2(t)}} = 2 \text{ — (6)}$$

Similarly for SSB-SC :

at i/p:

$$P_{si} = \overline{x_{mi}^2(t)} \ \& \ P_{Ni} = \overline{n_i^2(t)}$$

$$SNR_i = \frac{P_{si}}{P_{Ni}} = \frac{\overline{x_m^2(t)}}{\overline{n_i^2(t)}}$$

at o/p :

$$P_{so} = \overline{x_m^2(t)}$$

$$P_{N_o} = \overline{x_c^2(t)} = \overline{n_i^2(t)}$$

$$SNR_o = \frac{\overline{x_m^2(t)}}{\overline{n_{ni}^2(t)}}$$

The detection gain is,

$$\gamma = \frac{SNR_o}{SNR_i} = \frac{\overline{x_m^2(t)}}{\overline{x_{ni}^2(t)}} \Big/ \frac{\overline{x_m^2(t)}}{\overline{x_{ni}^2(t)}}$$

$$\therefore \boxed{\gamma = 1} \text{ — (7)}$$

from eqn (4), (6) & (7) we have the greatest value of γ in case DSB-SC so, DSB-SC has better Noise performance than the other two DSB-AM and SSB-SC. Again γ for SSB-SC > γ for DSB-AM so, SSB-SC have better Noise Improvement over DSB-AM.

10. Define Hamming weight and Hamming distance for a code vector X = 0111000 and the parity check matrix H given below. Prove that, the given code is valid.

$$H = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}_{3\times 7}$$

Ans:- For Definition Refer 073 Shrawan. Ques.No. → 10.

given, Code vector (X) = 0111000

$$\text{parity Check matrix } (H) = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}_{3\times7}$$

The transpose Response of the given parity matrix H, Can be obtained by interchanging the rows and columns as under:

$$H^T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}_{7\times}$$

for the given to be valid, it must satisfy the condition below.

$$XH^T = (0,0,\dots,0)$$

So, $XH^T = [0111000]_{1\times7} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}_{3\times7}$

$XH^T = [(0\times1)\oplus(1\times1)\oplus(1\times1)\oplus(1\times0)\oplus(0\times1)\oplus(0\times0)\oplus(0\times0),\ (0\times1)\oplus(1\times1)\oplus(1\times0)\oplus(0\times0)\oplus(0\times1)\oplus(0\times1),\ (0\times1)\oplus(1\times0)\oplus(1\times1)\oplus(1\times1)\oplus(0\times0)\oplus(0\times0)\oplus(0\times1)]$.

$XH^T = 0\oplus1\oplus1\oplus0\oplus0\oplus0\oplus0,\ 0\oplus1\oplus0\oplus1\oplus0\oplus0\oplus0,\ 0\oplus0\oplus1\oplus1\oplus0\oplus0\oplus0$

$XH^T = [0,0,0]_{1\times3} = (0,0,0)$

Hence, This is valid code word.

11. Write short Notes:

i) M-ary Base band data communication system.

ii) Eye diagram.

↳ Refer 2069 chaitra 12 (a).

11) M-ary Base band data communication

Modern Modulation techniques exploit the fact that digital baseband may be sent by varying both the envelop and phase of an RF carrier. Because the ~~phase~~ envelop and phase afferture degree of freedom., such modulation techniques map basebands data into four or more possible RF carrier signal.

Such modulation Techniques are called M-ary modulation since they can represent more signals than if just the amplitude or phase were varied alone.

In this signaling scheme, two or more bits are grouped together to form symbols and one of M possible signals $s_1(t), s_2(t), \ldots$ $s_m(t)$ is transmitted during each symbol period of duration T_s.

depending on whether the amplitude, phase or frequency of the carrier is varied, the modulation scheme is called M-ary ASK, M-ary PSK or M-ary FSK.

It achieves better bandwidth efficiency at the expense of power efficiency.

BR Binod.

2073 Shrawan *

1\. Explain the functional block diagram and the basic elements of a digital communication system. Explain shannon-Fano coding.

Ans:→ Digital Communication System is the type of communication system in which the message signal to be transmitted is digital in nature.

The basic Block diagram of digital communication which ~~the message signal to be transmitted is digital in nature~~ integrates its basic structure and functionality is given below.

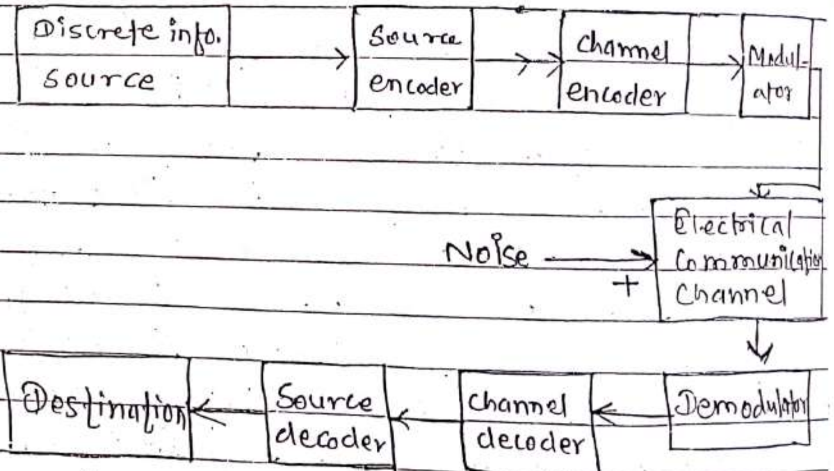

fig:- Block diagram of a digital communication system.

Explanation:-

i) Discrete Information Source:-

The information Source produces a message signal which is not continuously varying with time. Rater the message is intermittent with respect to time. The output of discrete information sources such as a teletype or the numerical o/p of a computer consists of a sequence of discrete symbols or letters.

ii) Source Encoder and Decoder:-

The symbols produced by the information source are given to the source encoder. These symbols can not be transmitted directly. They are first converted into digital form (i.e. binary sequence of 1's and 0's) by the source encoder. Each binary '1' and '0' is known as a bit. The group of bits is called a codeword. ~~The source~~ The source encode assigns codewords to the symbols. similarly source decoder are used to perform the inverse process than a source encoder.

iii) Channel Decoder and Encoder:-

After converting the message or information signal in the form of binary sequence by the source encoder, the signal is transmitted through the channel. The communication channel adds noise and interference to the signal being transmitted. Hence, error are introduced in the binary sequence received at the receiver. ~~Therfe~~ Therefore, errors are also introduced in the symbol generated by these binary codewords. Thus, the channel coding is done to avoide these types of error. The channel encoder adds some redundant bits (binary) to the i/p sequence. These redundant bits are always added with some properly defined logic. The channel decoder receives the sequence (with redundant bits) and retrives the actual bits sequence.

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IV) Digital Modulator and Demodulator :-

Digital demodulator are used for modulation. Modulation is the process, ~~of~~ In which some parameters of the carrier (high frequency) is varied in accordance with the modulating signal (message signal). If the modulating signal is digital, this technique is simply refered as Digital ~~communication~~ Modulation. The digital Demodulator demodulates modulated signal to obtain the message signal.

V) Communication channel :-

Communication channel provides a connection betn transmitter and receiver. They are actual path to carry the modulated signal which may be wirelines, wireless or fiber optic channel. However, communication channel is always prone to noise and interference.

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Explanation of Shanon fano coding :

1. For a given list of symbols, develop a corresponding list of frequency count so that each symbol's relative frequency ~~count~~ ~~output~~ ~~each symbol's~~ ~~rep~~ of occurence is known.

2. Arrange the probability in descending order.

3. Divide the list into two parts, with the total frequency counts of left part being as close to the ~~...~~ right as possible.

4. The left part of the list is assigned the binary 0, and the right part is assigned 1.

5. Recursively apply 3 and 4 to each two halves, subdividing the groups and adding bits to the codes until each symbol has become a corresponding code ~~least~~ leaf ~~at~~ on the tree (Independent).

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Q. State and prove Sampling theorem. Define aliasing effect and aperture effect.

Ans:- Sampling theorem:

Statement:-

"Analog signal can be reproduced from an appropriate set of its samples taken at some fixed intervals of time".

↳ In broad sense the sampling theorem can be stated in two parts:-

1) For transmission End:-

→ A strictly band limited signal (i.e. For $f > B$, there is no energy) is completely described by the samples (values) of the signal at instants of time seperated by $\frac{1}{2B}$ seconds.

2) For Receiving end:-

The original signal may be recovered if we know its values (samples) taken at the rate of $2B$ per second.

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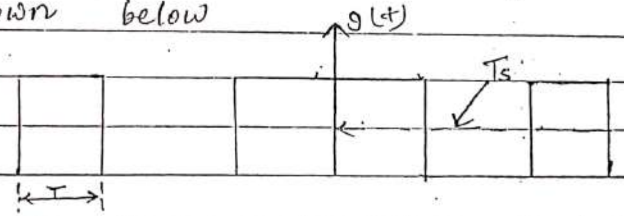
proof :

If the signal $x(t)$ to be sampled is band limited then the sampled signal can be represented as :

$$X_s(t) = x(t) * g(t)$$

Where $g(t)$ is the sampling function, shown below



Where, T_s is called sampling period and T- duration of sampling pulse.

The sampler can be implemented using the following arrangement.

$$x(t) \longrightarrow (X) \longrightarrow X_s(t) = x(t) \cdot g(t) \quad \text{(a)}$$
$$\uparrow g(t)$$

The gate function $g(t)$ can be expressed in terms of fourier series as :—

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$$g(t) = C_0 \sum_{n=1}^{\infty} 2C_n \cos n\omega_s t$$

where, $C_0 = T/T_s$, $C_n = f_s T \operatorname{sinc}[n f_s T]$,

$$\omega_s = 2\pi f_s$$

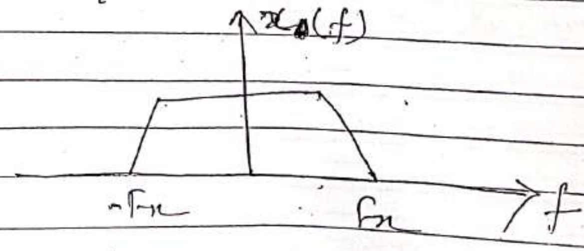
Then the $X_s(t)$ can be expressed as:-

$$X_s(t) = C_0 x(t) + 2C_1 x(t) \cos \omega_s t + 2C_2 x(t) \cos 2\omega_s t + \cdots + 2C_n x(t) * \cos n\omega_s t \cdots$$

The fourier transform of the above series is :-

$$X_s(f) = C_0 X(f) + 2C_1 X(f - f_s) + 2C_2 X(f - 2f_s) + \cdots$$

The above series can be graphically represented as:-



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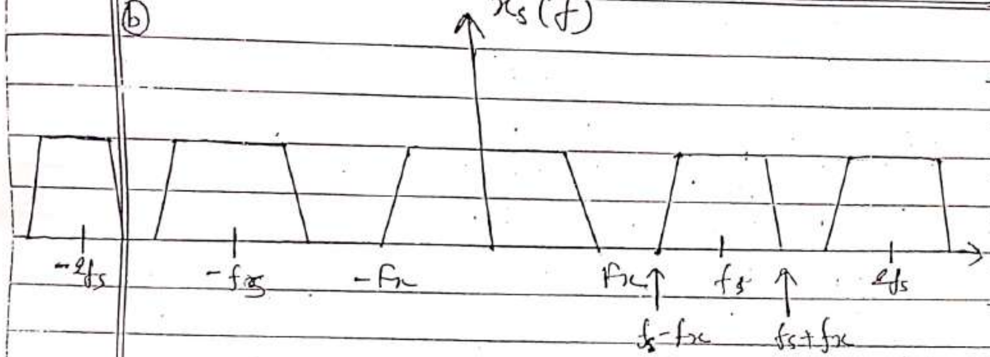
(b)



Fig(a) represent the fourier transform of the original signal $x(t)$ and figure(b) is the spectrum of the signal at the output of the sampler. It is clear from the figure (b) that the spectrum of the sampled signal contains the spectrum of the original message signal.

Now,

For distortion less recovery of original message signal from the spectrum of the sampled signal, the following condition should be met:

$$f_s - f_x \geq f_x$$
$$\text{or, } f_s \geq 2f_x$$

i.e. sampling frequency $\geq$ twice of message frequency.

Proved.

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Next part:-

Aliasing effect:-

When a continuous-time band limited signal is sampled at a rate lower than Nyquist rate $f_s < 2f_m$, then the successive cycles of the spectrum $G(\omega)$ of the sampled signal gets overlap with each other as ~~shown in fig.~~ is known as aliasing effect.

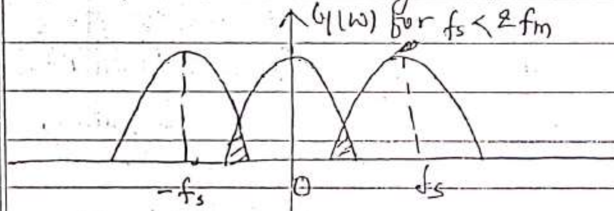

Fig:- showing Aliasing effect.

Aperture Effect:-

In a Flat top sampled signal the distortion caused by the use of pulse-amplitude modulation to transmit an analog information bearing signal is referred to as the apperture effect.

3(i) Explain working principle of PCM with necessary figures and equations.

Ans:- Pulse - Code modulation is known as a digital pulse modulation Technique. Infact, the pulse code modulation (PCM) is ~~in sense~~ quite complex compared to PAM., PWM and. PPM. in the sense that the message Signal is subjected to a great number of operations.

Elements of a PCM System:-

The basic elements of a PCM system is shown below. It consists of three main parts i.e. transmitter, transmission path and receiver. The essential operations in the transmitter of a PCM System are sampling, quantizing and encoding as shown in figure. ~~As discuss~~ below. Sampling is the operation in which an analog (ie. Continuous-time) signal is sampled according to the sampling theorem resulting in a discrete-time signal. The quantizing and encoding operations are usually performed in the same ckt which is known as an analog-to-digital converter (ADC)

Also, the essential operations in the receiver are regeneration of impaired signals, decoding and demodulation of the train of quantized samples. These operations are usually performed in the same circuit which is known as a digital to Analog converter (DAC).

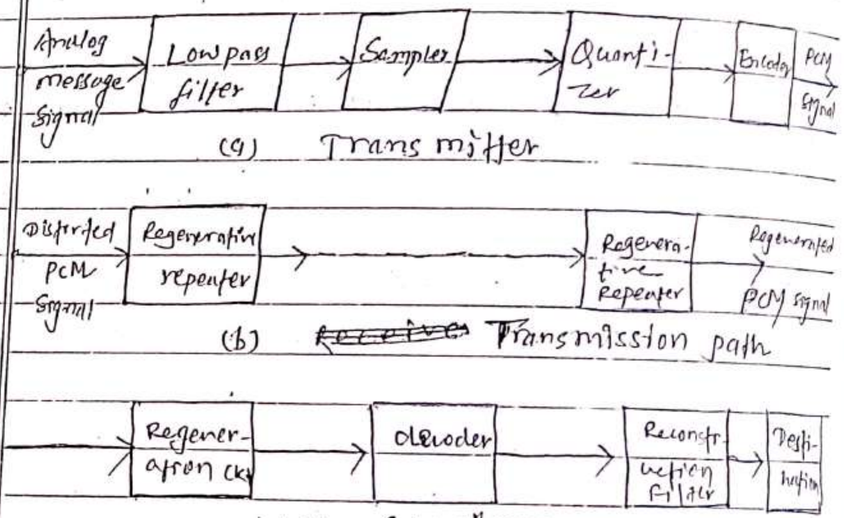

Fig:- The basic elements of a PCM System.

For the Transmission Route from the transmitter to the receiver, regenerative repeaters are used to reconstruct the transmitted sequence of coded pulse. In order to combat the accumulated effects of Signal distortion and noise.

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3 (b) A PCM System uses a uniform quantizer followed by a 7 bit binary encoder. The bit rate of the system is equal to 50×10^6 bits/sec.

(i) What is the maximum message signal bandwidth for which the system operates satisfactory?

Ans:- Let us assume that the message bandwidth is f_m Hz.

So, sampling frequency f_s is given as

$$f_s \geq 2f_m$$

no. of bits, $v = 7$ bits.

&

Sampling Rate R_s, is given as

$$R_s \geq v \cdot f_s$$

$$R_s \geq 7 * 2 f_m$$

given that $R_s = 50 \times 10^6$ bits/sec

So, $50 \times 10^6 \geq 14 f_m$

$$f_m \leq 3.57 \text{ MHz}$$

Hence, the maximum message bandwidth is 3.57 MHz.

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4. Explain the necessity of non-Uniform quantization for speech signal. Derive the expression for signal to quantization noise ration in delta modulation.

Ans:- The Speech Signals are Generally non-uniform because they occur at non-uniformly spaced time intervals and also some speech signals are slow while ~~some~~ other are fast. Hence, Uniform sampling of speech parameters is thus not efficient.

So, that we use non-uniform quantization procedure for efficient coding of speech signal. And Also enhance speech signal by reducing redundancy.

For Next part : Refer 2070 Chaitra Ques → 5.

5. (i) Given the binary sequence 1011001010 represent it in polar NRZ, Polar RZ, Manchester, AMI Codes.

Ans:-

↓

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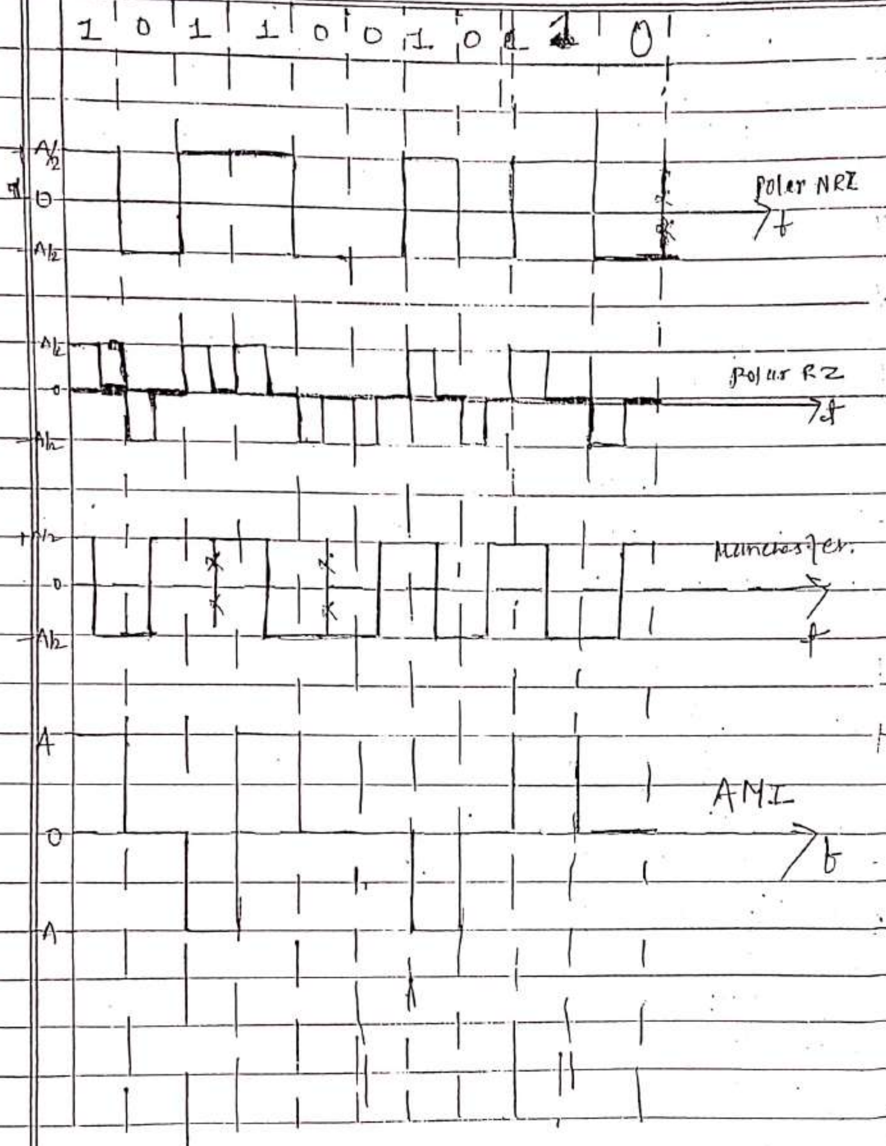
5(b)

Ans!-



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What do you mean by Intersymbol Interference? Explain Duobinary coding technique with the procedure and illustrate it using binary i/p sequence 0010110.

Inter Symbol Interference is the process of spreading of pulse to its neighbour pulse.

It is arises due to the Imperfections in the overall frequency response of the system.

Duobinary - Encoder with precoder (Differential - Encoder)

In Modified duobinary system. If there is error at received signal then it will propagate in the other values of b(k). The duobinary system with precoder eliminates this problem.

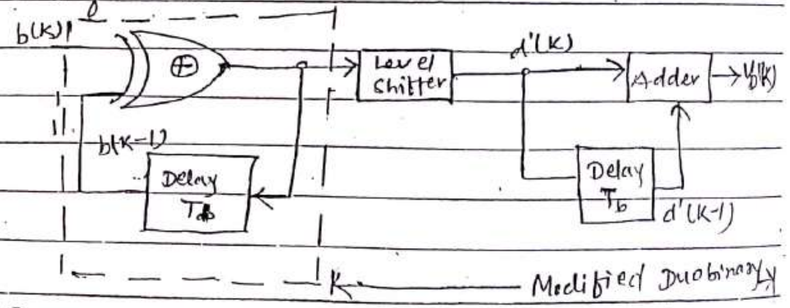

Fig:- Block diagram of duobinary Encoder with precoder. Encoder

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The precoder added to the modified duobinary encoder to obtain the duobinary encoder with procedure as given in Table. The precoder consists of an Ex-OR gate along with a delay unit that introduces a delay of one bit duration T_b.

The precoder output $d(k)$ is given by;

$$d(k) = b(k) \oplus d(k-1) \quad \text{—} (9)$$

where, $b(k)$ is the unipolar stream of binary digits.

above eqn (9) shows that precoder o/p is equal to zero, If both i/ps are alike (both '0' or '1') and it is '1' if any one of the i/p is '1'.

Illustration using Example 0010110

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6. Prove that the impulse response of the matched filter is reverse delayed version of the i/p signal.

Ans → For optimum Filter, we have Generalised Gaussian noise: when this noise is white Gaussian noise is white Gaussian noise, then the optimum filter is known as matched filter.

For the white Gaussian Noise, the power spectral density ~~function~~ is given as

$$S_{ni}(f) = \frac{N_0}{2} \quad \text{—} (i)$$

Now, Calculation for Impulse Response for the matched filter:-

The transfer function of the optimum filter is expressed as,

$$H(f) = K \cdot \frac{X^*(f)}{S_{ni}(f)} e^{-j2\pi fT} \quad \text{—} (ii)$$

Substituting (i) in (ii) we get

$$H(f) = K \cdot \frac{X^*(f)}{N_0/2} * e^{-j2\pi fT}$$

or $H(f) = \dfrac{2K}{N_0} X^*(f)\, e^{-j2\pi f T}$ — (III)

From the property of Fourier Transform, we know that,

$$X^*(f) = X(-f)$$

using this property, we can write eqn (III) as under.

$$H(f) = \frac{2K}{N_0} X(-f)\, e^{-j2\pi f T}$$

Now, The Impulse Response of matched filter can be calculated by taking Inverse Fourier Transform of above eqn.

$$h(t) = IFT[H(f)]$$

$$= IFT\left[\frac{2K}{N_0} X(-f)\, e^{-j2\pi f T}\right] \text{ — (IV)}$$

Also, we have,

$$FT[x(-t)] = X(-f)$$
$$FT[x(T-t)] = X(-f)\, e^{-j2\pi f T}$$

Note: $IFT[X(-f)] = x(-t)$
& ~~IDFT~~ $[e^{-j2\pi f T}]$ represents Time shift

using the above properties of Fourier Transform, the equation — (IV) can be written as,

$$h(t) = \frac{2K}{N_0}\, x(T-t) \text{ — (V)}$$

If $x(t) = x_1(t) - x_2(t)$ then eqn (V) can be written as.

$$h(t) = \frac{2K}{N_0}\left[x_1(T-t) - x_2(T-t)\right]$$

This is the required expression of matched filter.

which proves that the Impulse response of the matched filter is reverse delayed version of the Input signal.

Proved.

7\. Find the detection gain for ~~evaluation~~ SSB-SC demodulation and compare it with DSB-SC.

⇒ Refer 2071 shrawan Ques. No:- 8.

8. Derive the expression for evaluating the gain parameter (SNR_o/SNR_i) of non-Coherent FM detector.

Ans:- Refer:- 2065 Shrawan Ques. No.:-7

9. With necessary assumption, derive the Expression for bit error probability for binary ASK system and Extend it to M-ary.

Ans:- Refer:- 2071 Chaitra Question No:- 11.

10. Define Hamming weight and Hamming distance with Examples. validate the code if received code vector is [100011] given that

$$H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Ans:-

Hamming distance:

→ If we consider two code words (or vectors) having the same number of elements then the haming distance is defined as number of locations in which their respective elements differ.

Example:

Code word 1 :- 1 1 0 1 0 1 0 0

Code word 2 :- 0 1 0 1 1 1 1 0

So, Haming distance = 3.

Hamming weight:

The hamming weight of a code word x is defined as the number of non-zero elements in the code word. And also Hamming weight of a code vector (code word) is the distance between that code word and an all-zero code vector.

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Example:- 1 1 0 1 0 1 0 0

0 0 0 0 0 0 0 0

Hamming distance = 4

So, Hamming weight = 4.

OR/

Simply counts the non zero elements in the code word. which is equal to 4.

Next part:

we have,

$$\text{Parity check Matrix}(H) = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}_{3\times 6}$$

$$\text{Received code}(Y) = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}_{1\times 6}$$

Now, The received code word is valid if it is equal to transmitted code word X.

i.e. $X = Y$.

$$\text{Now, The transpose of } H^T = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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Also, the product XH^T is given by,

$$\text{Syndrome vector} = XH^T = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}_{1\times 6} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times 6}$$

or $XH^T = [(1\times 1)\oplus(0\times 0)\oplus(0\times 1)\oplus(0\times 1)\oplus(0\times 0)\oplus(0\times 0),\ (1\times 0)\oplus(0\times 1)\oplus(0\times 1)\oplus(0\times 0)\oplus(1\times 1)\oplus(1\times 0),\ (1\times 1)\oplus(0\times 1)\oplus(0\times 0)\oplus(0\times 0)\oplus(1\times 0)\oplus(1\times 1)]$

$S = 1\oplus 0\oplus 0\oplus 0\oplus 0\oplus 0,\ 0\oplus 0\oplus 0\oplus 0\oplus 1\oplus 0,\ 1\oplus 0\oplus 0\oplus 0\oplus 0\oplus 1$

$S = [1, 1, 0]_{1\times 3}$

Since, Syndrome vector has not all zero elements So, it has some error bits at received signal.

i.e. $X \neq Y$

So, it is not valid code word.

BK Binod

2072 Kartik

1. Write down the significant of variable length coding with relevant example. A fixed length source encoder generates six symbols with probabilities {0.2, 0.15, 0.25, 0.05, 0.3, 0.05} and encoded by unique code word. Find entropy and maximum code effiency.

Ans:- In coding theory, a variable length code is a code which maps source symbols to a variable no. of bits.

Variable-Length codes can allow source to be compressed and decompressed with zero error (Lossless data compression) and still be read back symbol by symbol.

Example:

Huffman coding

Lempel-ziv coding.

Given, source encoder generates six symbol having probabilities

0.2, 0.15, 0.25, 0.05, 0.3, 0.05

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Using Shannon-fano coding

Symbol x_i	probabilities $P(x_i)$	step I	step II	step III	code
x_1	0.3	0	0		00
x_2	0.25	0	1		01
x_3	0.2	1	0	0	100
x_4	0.15	1	0	1	101
x_5	0.05	1	1	0	110
x_6	0.05	1	1	1	111

Now,

$$L = \sum_{i=1}^{6} P(x_i)\, n_i$$

$$= 2\times 0.3 + 2\times 0.25 + 3\times 0.2 + 3\times 0.15 + 3\times 0.05 + 3\times 0.05$$

$$= 2.45$$

Entropy $H(x) = -\sum_{i=1}^{6} P(x_i) \log_2 (P(x_i))$

$$= 0.3 \log_2 \frac{1}{0.3} + 0.25 \log_2 \frac{1}{0.25} + 0.2 \log_2 \frac{1}{0.2}$$

$$+ 0.15 \log_2 \frac{1}{0.15} + 2 * 0.05 \log_2 \frac{1}{0.05}$$

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$= \cancel{2.3}$

$= 2.328$ bits/symbol

$$\eta = \frac{H(x)}{L} * 100\%$$

$$= \frac{2.328}{2.45} * 100\%$$

$$= 95.02\%$$

Hence, Entropy = 2.328 bits/symbol

Code efficiency = 95%

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(2) Explain Sub-sampling theorem. A signal $x(t) = \text{Sinc}(5\pi t)$ is sampled (using uniformly spaced impulses) at a rate of 10 Hz.
(i) Sketch the sampled of signal (not to scale)
(ii) Sketch the spectrum of the sampled signal for the range $|F| < 30$ Hz.
(iii) Explain wether you can recover the signal $x(t)$ from the sampled signal.

Ans: Statement:

It states that If a band pass signal ~~Sampling theorem which states~~ is sampled

$$f_{ss} = \frac{2F_{x_4}}{m} \quad \text{where,} \quad m = \frac{F_{x_4}}{B} \text{ is}$$

the largest integer below $\frac{F_{x_4}}{B}$.

then the original signal can be recovered without distortion. This theorem is also called Sub-sampling theorem. The sub-sampling ~~theorem~~ frequency F_{ss} is much less than the Nyquist rate $F_s > 2F_{x_4}$.

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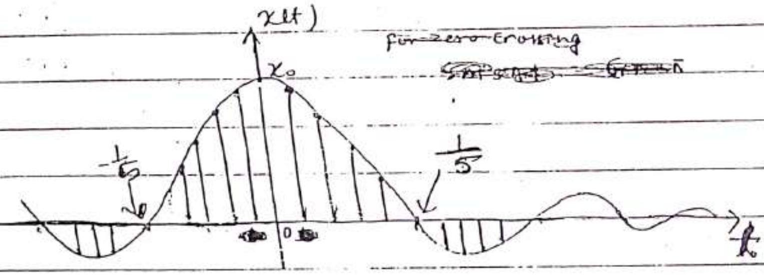
Next part !

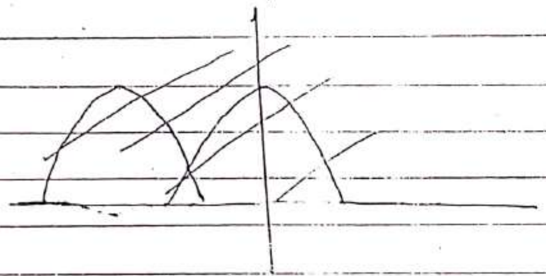
given signal $x(t) = \text{Sinc}(5\pi t) = \text{Sinc}(2 \times \pi \times 2.5 t)$

Impulse rate = 10 Hz

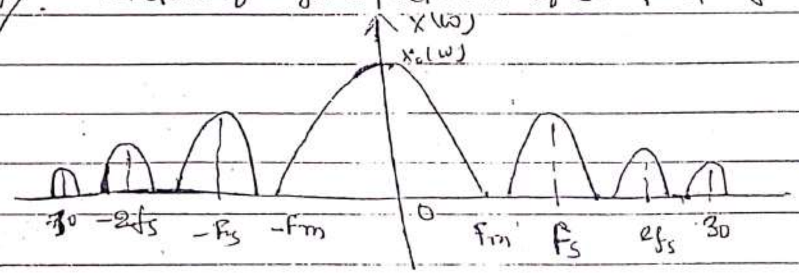
(i) Sketch of sampled signal:



(ii)



(ii) Sketch of the spectrum of sampled signal:-



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(iii) given signal $x(t) = \text{sinc}(5\pi t)$

on comparing with $\text{sinc}(2\pi f_m t)$

$$f_m = 2.5 \text{ Hz}$$

$$f_s = 10 \text{ Hz}$$

If $f_s > 2f_m$ then the original signal can be recovered.

$$10 > 2 \times 2.5$$

$$10 > 5$$

So, $x(t)$ can be recovered.

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3. Explain the basic process of Non-Uniform quantization including companding technique of its realization. An audio signal of frequency 4 KHz and maximum dynamic range of ±2.4V is digitized by PCM system with its bit rate of 64 KHz. Calculate numbers of bits per sample, quantization noise power and $SQNR_{dB}$. Estimate the minimum bandwidth required for TDM of 10 such audio signals (assume no extra framing and synchronization bits).

Ans:- If the quantizer characteristics is nonlinear and the step size is not constant instead if it variable, dependent on the amplitude of i/p signal then the quantization is known as non uniform quantization. In non-uniform quantization, the step size is ~~small, therefore the quantization is thus varied according to the signal level to~~ reduced with the reduction in signal level.

The non-uniform quantization is practicly achieved by the process of companding. companding is non-uniform quantization. It is required to be implemented to improve the signal to quantization ratio of weak signals.

we have, Quantization Noise, $\left[N_q = \frac{\Delta^2}{12}\right]$ — (1)

eqn (1) shows that in the uniform quantization, once the step size is fixed, the quantization noise power remains constant. However the signal power is not constant. Therefor the quantization noise for week signal is very poor. This will effect the quality of signal. The remedy is to use companding. companding is a term derived from two words i.e, Compressing + Expanding.

In this method weak signals are amplified and strong signal are attenuated before applying them to a quantizer. This process is called compression.

At the receiver exactly opposite is followed which is called expansion.

~~At a whole~~, The compression of signal at transmitted and Expansion at receiver is called companding.

i/p → Compressor → Uniform quantizer → Expander → o/p

Fig : Companding Model.

Next - part :-

given,

max^m i/p Audio Signal frequency f_m = 4 KHz

minimum dynamic range = ± 2.4V

bit rate = 64 KHz

bit per sample = ?

quantization Noise power = ?

$SQNR_{dB}$ = ?

minimum bandwidth = ?

Now,

No. of bits per sample $v = \frac{\log_{10} 64000}{\log_{10} 2}$

$\therefore v = 16$ bits

$x_{max} = 2.4v$

quantization Noise power. $P = \frac{A_m^2}{2}$

$= \frac{x_m^2}{2}$

$= \frac{(2.4)^2}{2}$

$= 2.88$

$\frac{S}{N} = \frac{3P}{}$

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Signal to quantization Ratio:

$$S/N = \frac{3P2^{2v}}{x_{max}^2}$$

$$= \frac{3 \times 2.88 \times 2^{2\times16}}{(2.4)^2}$$

$$= 6442450944$$

$$\left(\frac{S}{N}\right)_{dB} = 10\log_{10}\frac{S}{N}$$

$$= 10\log_{10} 6442450944$$ Ans:-

$$\approx 98\,dB.$$

For min bandwidth:

maximum frequency;

$f_m = \cancel{\ } 4KHz$

transmission bandwidth; $BW \geq v f_m$

Since, there are 10 ~~such~~ such audio signals are multiplexed so, $BW \geq 10\,v f_m$.

$BW \geq 10 \times 16 \times 4\ KHz$

$BW \geq 640KHz$ Ans:

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4. A Delta modulator is used to encode Speech signal band-limited to 3KHz with sampling frequency 10KHz. For maximum signal Amplitude of $A_{max} = 1$, Find:

(i) Minimum step size to avoid slope overloading.

(ii) Assuming the speech signal to be sinusoidal, Find signal to quantization noise ratio.

(iii) Determine the minimum transmission bandwidth.

given,

$f_m = 3KHz$

$f_s = 10KHz$

So, Nyquist rate $= 2f_m = 2\times3KHz$

$= 6KHz$

Sampling interval $(T_s) = \frac{1}{f_s}$

$= \frac{1}{10000}$ Sec

$= \cancel{0.167\times10^{-3}}$ sec.

$A_{max} = 1$

we have

Min. step-size $(\Delta) = A_m 2\pi f_m T_s$

$= 1\times2\pi\times3K\times\frac{1}{10000}$

$= 6\pi/10$

$= 1.88\,V$

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ii) $\left(\frac{S}{N_q}\right) = \frac{3}{3\pi^2 f_m^2 f_M T_s^3}$

let $f_M = f_m = 3KHz$

$= \frac{3}{3\times\pi^2 \times 3^2 \times 3 \times 10^9 \times 10^{-12}}$

$= 3.75$

(iii) transmission bandwidth:

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5. What is ISI? State Nyquist pulse shaping criteria for zero ISI. Explain duo-binary encoding with example.

Ans:- for definition Refer 2073 Shrawan 5(b)

Nyquist pulse shaping Criteria For zero ISI:

It ~~has~~ has been proved that the function which produces a zero intersymbol interference is a sinc function. Hence, instead of a rectangular pulse if we transmit a sinc pulse then the ISI can be reduced to zero. This is known as Nyquist pulse shapping. The sinc pulse transmitted to have a zero ISI has been in fig ①.

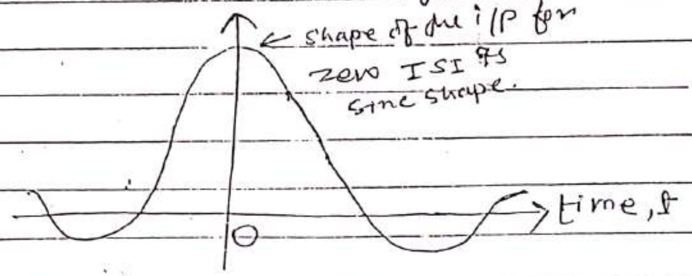

Fig:- ideal pulse shape for Zero ISI

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Next Part:-

Refer → 071 - Shrawan

Question:- 5

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6. Why DPSK is preferred than PSK? Explain the modulator, Demodulator and Signal Space diagram for DPSK System.

Ans:-

DPSK is preffered than PSK because DPSK does not need carrier at the receiver end i.e. eliminate the need for phase synchronisation of coherent receiver with PSK such that it reduces the complicated circuitary.

Modulator :-

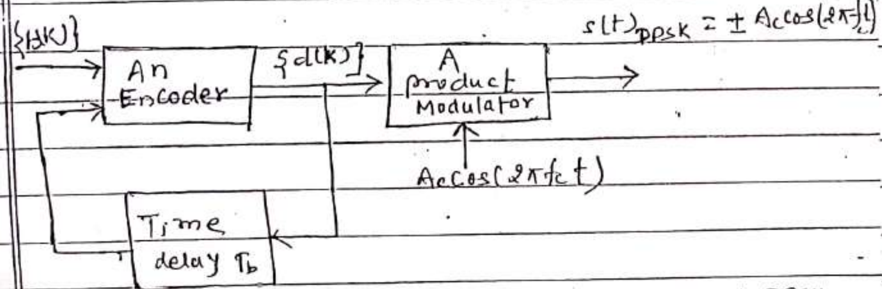

Fig:- illustration of the scheme to generat DPSK.

The above shows DPSK modulator. In order to elliminate the need for phase synchronization of coherent receiver with PSK, a differential encoding system can be used with PSK. The digital information content of the binary data is encoded in terms of signal transitions. As an example 0 may be used to represent transition in a given binary sequence (with respect to previous encoded bit) and symbol '1' to indicate no transition. This new signaling technique which combines differential encoding with phase-shift keying (PSK) is known as differential phase-shift keying (DPSK).

Demodulation of DPSK :-

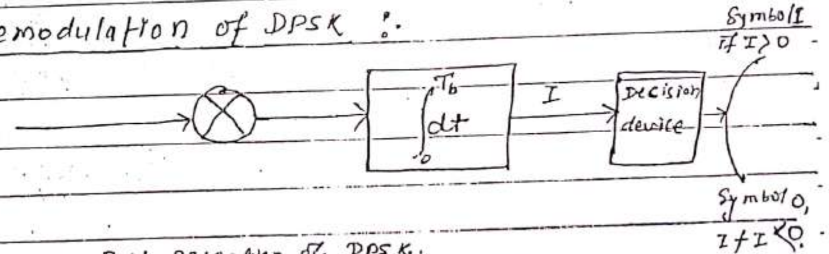

Fig:- Detection of DPSK.

For detection of the differentially encoded PSK i.e (DPSK), we can use the receiver arrangement as shown in Fig. above. The received signal DPSK signal is applied to one i/p of the multiplier. To the other i/p of the multiplier a delayed version of the received DPSK signal time interval T_b is applied. The o/p of difference is proportional to $\cos(\phi)$,

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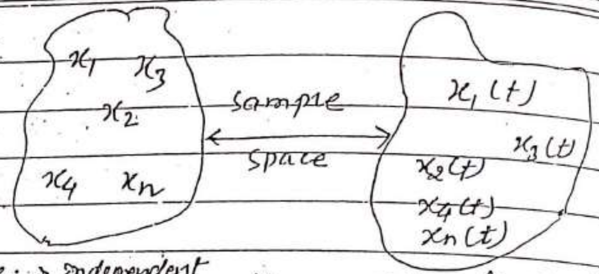
here ϕ is the difference betn the carrier phase angle of the received DPSK signal and its delayed versions measured in the same bit interval.
when $\phi = 0$, the integrator o/p is +ve i.e. symbol '1'
when $\phi = \pi$, the integrator o/p is -ve i.e symbol '0'
In the absence of noise, the receiver can reconstruct the transmitted binary data exactly.

7. Define Matched filter. Explain the approximation of the matched filter for a rectangular pulse using a single pole RC low pass filter with variable band width.

Ans:- Refer 2070 chaitra Quest. No:- 9.

8. What do you mean random process? Explain white noise with its psdf and auto correlation function.

Ans: A random process $X(t)$ is an ensemble of signals. i.e. A random variable which is function of time is called random process.

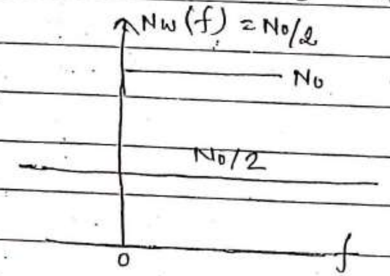


$x_i \rightarrow$ independent of time. $x(t)$ = function of time

white noise:

In communication system the noise analysis is based on ideal white noise. power spectrum density of white noise is independent the frequency. This means that white noise has flat spectrum density $N_w(f)$ over $-\infty < f < \infty$.

$$N_w(f) = N_0/2 \; ; \; -\infty < f < \infty \leftarrow \text{bilateral case.}$$
$$= N_0 \; ; \; 0 \le f \le \infty \leftarrow \text{unilateral case}$$



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The average or mean value of noise is 0.
If the probability of occurence of white noise is specified by a Gaussian distribution factor (function) it is called white Gaussian noise.

Auto correlation function of white noise may be obtained by simply taking the inverse fourier transform.

$$R_{WN}(\tau) = \frac{N_0}{2}\delta(\tau) \quad --- (a)$$

$R_{WN}(\tau) = 0$ for $\tau \neq 0$; two different samples of WN no matters how close they are in time shift $(\tau \to 0)$ are uncorrelated.
i.e. extremely random.

9. With necessary derivations, explain the threshold effect in envelope detector for DSB-FC modulation in analog communication system.

Ans:→ The i/p to the demodulator is the sum of signal Noise.

$$x_i(t) = \{A_c + x_m(t)\}\cos w_c t + n_i(t) \quad ---(i)$$

then, the signal power at the i/p is:

$$P_{S_i} = \frac{A_c^2}{2} + x_m^2(t) \quad --- (a)$$

Noise power at i/p is

$$P_{N_i} = \overline{n_i^2}(t) \quad --- (b)$$

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$$\text{So, } SNR_i = \frac{A_c^2 + x_m^2(t)}{2\, n_i^2(t)} \quad --- (ii)$$

For envelop detection:

The i/p signal can be expressed as:

$$x_i(t) = \{A_c + x_m(t)\}\cos w_c(t) + n_c(t)\cos w_c t + n_s(t)\sin w_c t$$

$$= \{A_c + x_m(t) + n_c(t)\}\cos w_c t + n_s(t)\sin w_c t$$

$$x_i(t) = V(t)\cos\{w_c t + \phi(t)\} \quad --- \text{eq}^n (1)$$

$$V(t) = \sqrt{(A_c + x_m(t) + n_c(t))^2 + n_s^2(t)}$$

$$\phi(t) = -\arctan\left[\frac{n_s(t)}{A_c + x_m(t) + n_c(t)}\right]$$

Let us assume that the noise is very small.

i.e. $A_c + x_m(t) \gg n_i(t)$

$(A_c + x_m(t) + n_c(t))^2 \gg n_s^2(t)$

$\therefore V(t) = A_c + x_m(t) + n_c(t)$

& $\phi(t) = 0$

The O/P of the ideal envelope detector is

$$V(t) = A_c + x_m(t) + n_c(t)$$

Since A_c is the DC component and is filtered out by LPF.

So, signal O/P power is

$$P_{so} = \overline{x_m^2(t)} \quad \text{--- eqn (2)}$$

& Noise O/P power is.

$$P_{No} = \overline{n_c^2(t)} \quad \text{--- eqn (3)}$$

$$SNR_o = \frac{\overline{x_m^2(t)}}{\overline{n_c^2(t)}} \quad \text{--- eqn (4)}$$

The detection gain γ is expressed as:-

$$\gamma = \frac{SNR_o}{SNR_i} = \frac{\dfrac{\overline{x_m^2(t)}}{\overline{n_c^2(t)}}}{\dfrac{A_c^2 + \overline{x_m^2(t)}}{2\overline{n_i^2(t)}}}$$

$$\therefore \gamma = \frac{2\overline{x_m^2(t)}}{A_c^2 + \overline{x_m^2(t)}} \quad \text{--- eqn (5)}$$

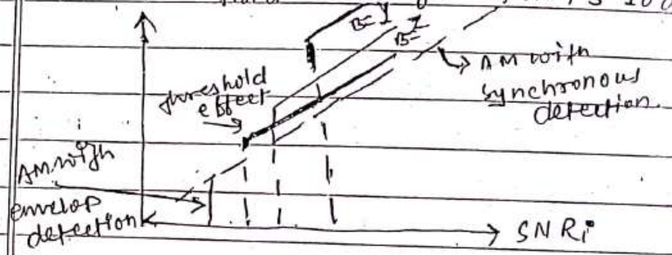
The detection ~~of~~ gain (γ) in FM is proportional to β^2 so for given SNR_i the rise in β will increase γ or SNR_o. But with increase in β will considerably increase the system BW at the Carson's rule rate,

$$B_{FM} = 2(\beta+1) f_m \quad \text{--- eqn (6)}$$

Increase in bandwidth will subsequently increases in i/p noise power and decreases SNR_i.

As assumed earlier the carrier amplitude A_c is greater than the Noise so this assumption will no longer remain valid. If we increase β indiscriminately i.e. at some point of increase of β SNR_i will be so low that the signal as in case of large noise AM signal will be ~~mutil~~ mutilated by the noise causing ~~receptio~~ reception impossible.

The threshold level for fm is 10 dB.



Befor reaching the threshold level the FM system produces noise clicks. As SNR_i is further decreased then individual clicks are produced which are converted to cracking sound, with further reduced in SNR_i and finally threshold level is reached, the receiver signal is completely mutilated by the noise. threshold effect exhibits locking property to Noise.

10. Derive the expression of error probability for binary PAM signal

Ans ⇒ Refer :.. 2069 chaitra Question No. 10

11. Explain Convolution coding with Example.

Ans:- The fundamental hardware unit for the convolution encoding is a taped shift resister with (L+1) stages, as shown in figure 16.46. Here, g_0, g_1 --- etc are the tap gains which are nothing but binary digits 0s or 1s. A tap gain of 0 represents an open ckt whereas a tap gain of 1 represents a short ckt.

The message bits enter one by into the tapped shift resister, which are then combined by mod-2 addition to form the encoded bit x. Therefore, we have

$$x = m_L g_L \oplus \text{------} \oplus m_1 g_1 \oplus m_0 g_0 \quad (1)$$

$$x = \sum_{i=0}^{L} m_i g_i \text{ (mod-2 addition)}$$

The name convolutional encoding comes from the fact that eqn (1) has a form of binary convolution which analogous to the convolutional intergral. The message bit m_0 in figure below represents the current I/P where as the bits m_1 to m_L represent the past I/P or state of the shift resister. From eqn (1), it is clear that a message bit x depends on the current message bit m_0 and the state of the shift register defined by the previous L message bits.

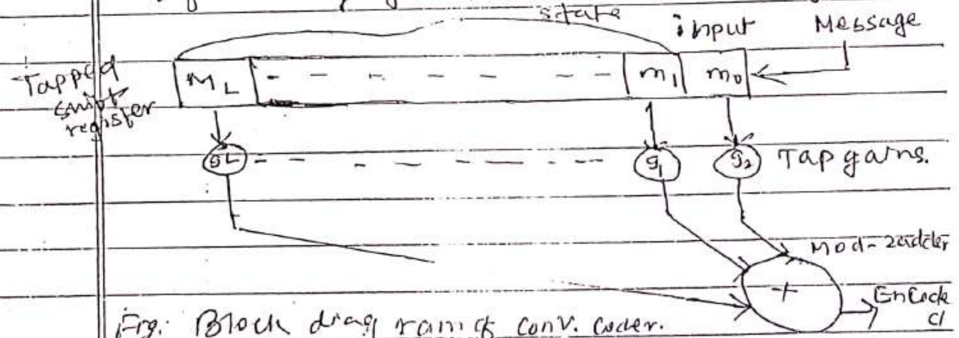
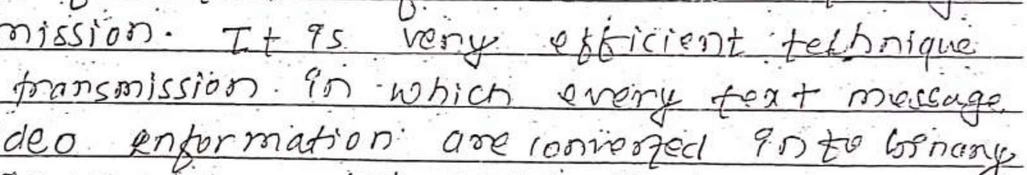


Fig. Block diagram of conv. coder.

2073 Chaitra

1. Discuss the importance of source coding in digital communication system. A discrete memoryless source emits five symbols with probability $p = \{0.3, 0.25, 0.2, 0.15, 0.1\}$, Find coding efficiency for both fixed and shanno - Fano coding and compare results. [6+4]

⇒ Source coding is the process of conversion of the o/p of the DMS into a sequence of binary symbol (BCD). It ~~helps~~ is generally done to compress the data at transmitting end and expand at the receiving end. which helps to reduce the redundancy present in the original information for efficient storage or transmission. It is very efficient technique for data transmission in which every text message audi, video information are converted into binary bits either '1' or '0' and their combinations to represent the original information. It improves the transmission efficiency with great factor.



given,

probabilities of symbol, $P = \{0.3, 0.25, 0.2, 0.15, 0.1\}$

For fixed coding (let BCD coding)

So, $L = 4$.

$$H(X) = -\sum_{i=1}^{5} p(X_i)\log_2[p(X_i)]$$

$$= -[0.3\log_2 0.3 + 0.25\log_2 0.25 + 0.2\log_2 0.2 + 0.15\log_2 0.15 + 0.1\log_2 0.1]$$

$$= 2.228$$

Then, efficiency of Fixed coding:

$$\eta_{FC} = \frac{H(X)}{L} = \frac{2.228}{4} * 100\%$$

$$\therefore \boxed{\eta_{FC} = 55.7\%} \quad \text{--- (1)}$$

Now, For shannon-Fano Coding:

Symbol x_i	Probabilities $P(x_i)$	step I	step II	step III	Code word	Length of code
x_1	0.3	0	0		00	2
x_2	0.25	0	1		01	2
x_3	0.2	1	0		10	2
x_4	0.15	1	1	0	110	3
x_5	0.1	1	1	1	111	3

~~H(X)~~ entropy remains same,

So, $H(X) = 2.228$.

$$L = \sum_{i=1}^{5} L_i * P(X_i) = [2\times0.3 + 2\times0.25 + 2\times0.2 + 3\times0.15 + 3\times0.1]$$

$$= 2.25$$

coding efficiency of shanno-Fano,

$$\eta_{SFC} = \frac{H(X)}{L}$$

$$= \frac{2.228}{2.25} \times 100\%$$

$$\therefore \boxed{\eta_{SFC} = 99.02\%} \quad \text{--- (11)}$$

From (1) & (11), it is clear that $\eta_{SFC} > \eta_{FC}$

Hence, shanno-Fano coding is better than Fixed length coding.

2. Define the Aperture and Aliasing effects? A signal $g(t) = 10\cos(20\pi t)\cos(200\pi t)$ is sampled at the rate of 250 samples per second, then (i) determine the spectrum of the resulting sampled signal, (ii) Specify the cut-off frequency of the ideal recustruction filter so as to recover $g(t)$ from its sampled version, and (iii) determine the Nyquist rate for $g(t)$. [7+3]

Ans:- The real sampling pulse is flat topped (in flat topped sampling) with finite duration τ. The net result is that

$$x_s(t) = \sum_{k=-\infty}^{\infty} x(kT_s)\, p(t - kT_s)$$

where $p(t)$ is the sampling pulse of duration τ.

In other words, finding convolution for Flat-topped pulse and the ideally sampled signal can derive the real sampled signal:

$$x_s(t) = [p(t)] * [\sum_{k=-\infty}^{\infty} x(kT_s)\, \delta(t - kT_s)]$$

and, the expectrum of the sampled signal will be:

$$X_s(f) = P(f)\, X_\delta(f) = P(f)\, [f_s \sum_{k=-\infty}^{\infty} X(f - kf_s)]$$

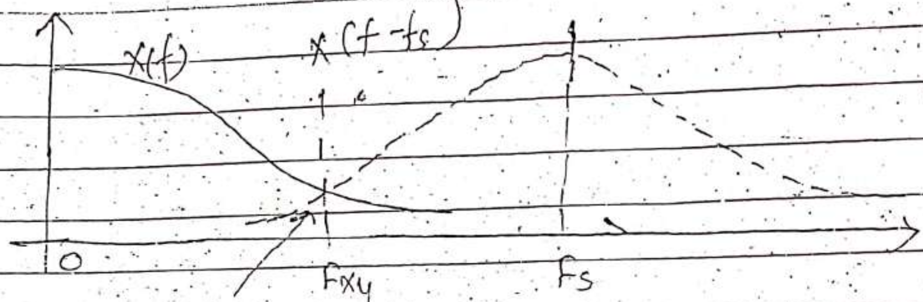
Since $P(f)$ is a sinc function, the primary effect of flat-topped sampling is the attenuation of high frequency components of the message signal. This effect is also called Aperture effect. This effect can be neutralised by using equalizing filter with a transfer function.

$$H_{eq}(f) = \frac{1}{P(f)}$$

But if $\tau << T_s$, then $P(f)$ is more or less constant over message frequency band and the apperture effect can be neglected.

Aliasing effect :-

Real signal encountered in real life are usually time limitted but not band limited. Example may be a pulse of finite duration, whose spectrum is theoretically unlimited. Thus there will be always aliasing effect.



Aliasing effect

To overcome this effect, a filter called pre-alias filter is used to attenuate frequency components of the message spectrum higher than the band of interest and by selecting Fs moderately higher than the Nyquist rate.

Next part:

$x(t) = 10 \cos(20\pi t) \cdot \cos(200\pi t)$

$= 5 [\cos 220\pi t + \cos 180\pi t]$

by comparing with standard equation, we get.

$\omega_1 = 220\pi$ and $\omega_2 = 180\pi$

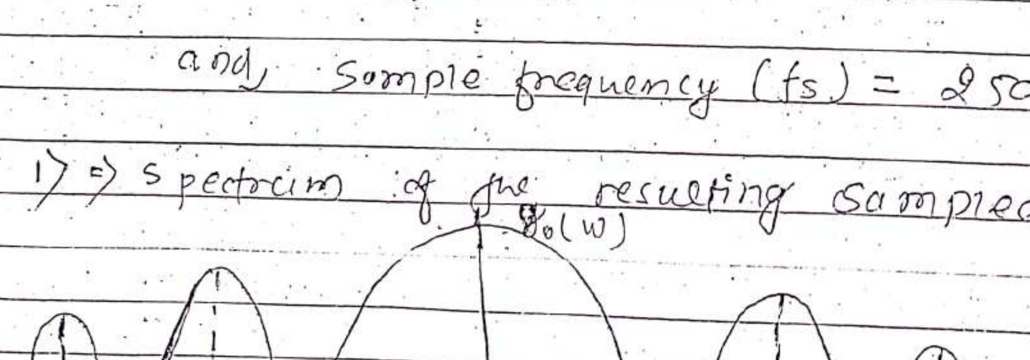
$f_1 = 110$ Hz, $F_2 = 90$ Hz

since, $F_1 > F_2$,

$F_m = F_1 = 110$ Hz

and, sample frequency $(f_s) = 250$ Hz

1) => Spectrum of the resulting sampled signal



ii) ⇒ Cutt-off frequency of the ideal recunstruction filter so as to recover g(t);

$$F_s = 2F_m$$
$$= 2 \times 110 = 220\,Hz.$$

(iii) ⇒ Nyquist Rate $= \frac{1}{2F_m} = \frac{1}{2\times110}$
$$= 4.545 \times 10^{-3} \text{ sec}$$
$$= 4.545 \text{ ms sec.}$$

3. Differentiate between uniform and non-uniform quantization. The information in analog waveform with maximum frequency 4KHz is to be transmitted over a 16-level PCM system.
a) what would be the maximum number of bits per sample?
b) what is the minimum sampling rate and bit rate?

Ans ⇒ Difference betn Uniform and Non-Uniform quantization

	Uniform Quantization	Non-Uniform Quantization
i)	Step size remains same throughout the i/p range.	i) The step size varies according to the i/p signal values.
ii)	over the complet range of i/ps quantization error is also same.	ii) over complete i/p range, quantization error varies according to the i/p signal.
iii)	For small signal SQNR becomes very low.	iii) For small signal SQNR, remains high by ↓ step size 'Δ'.

Next part :-

$F_m = 4KHz$

Quantization level $(q) = 16$

So, $16 = 2^v$

$\therefore v = 4$ bits.

a) ⇒ No. of bits per sample $(v) = 4$ bits.

b) ⇒ sampling rate $F_s \geq 2F_m$.

for Min sampling rate,

$F_s = 2F_m = 2 \times 4 \ KHz$

$= 8 KH$

and, bit rate $(r) = v F_s$

$= 4 \times 8 KHz$

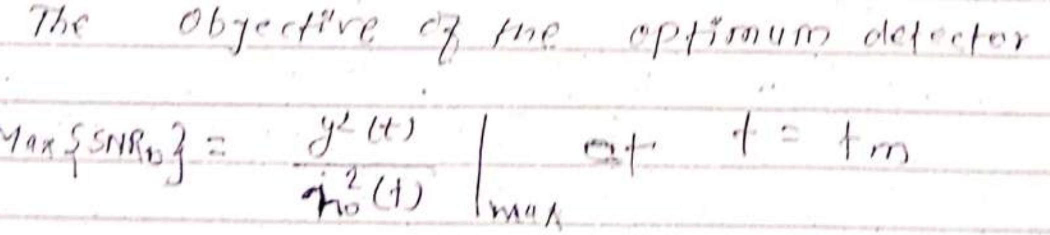
$= 32 KHz$.

Q. What do you mean by optimum detector? Find the impulse response of optimum detector in the presence of additive white noise.

Ans:- A Detector is a decision making device that examines the entire duration of pulses and gives decision whether the pulse is present or absent.

Hence, optimum detector of the pulse is a device that has least probability of errors in making decision in favor of 1 or 0.

The basic idea is to pass the received signal through a network (or filter) that suppress the noise and give sharp peak to the signal at the decision making instant, thus creating Maximum S/N ratio at decision making instance.



The Objective of the optimum detector is to make:

$$Max\{SNR_0\} = \left.\frac{y^2(t)}{n_0^2(t)}\right|_{max} \quad at \quad t = t_m$$

Now, the o/p the optimum detector can be derived by taking inverse of Fourier ~~transf~~ transform of product of $X(\omega)$ and $H(\omega)$:

$$y(t) = F^{-1}[X(\omega)\,H(\omega)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(\omega) * X(\omega)\, e^{j\omega t}\, d\omega$$

And for $t = t_m$:

$$y(t_m) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(\omega)\,X(\omega)\, e^{j\omega t_m}\, d\omega$$

The o/p noise power at the decision making instance can be expressed as:

$$\overline{n_o^2(t_m)} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{N}{2}|H(\omega)|^2\, d\omega$$

$$= \frac{N}{4\pi}\int_{-\infty}^{\infty} |H(\omega)|^2\, d\omega$$

Finally, the o/p signal to noise ratio will be:

$$SNR_o = \frac{y^2(t_m)}{\overline{n_o^2(t_m)}}$$

$$= \frac{1}{N\pi}\,\frac{\left|\int_{-\infty}^{\infty} H(\omega)\,X(\omega)\, e^{j\omega t_m}\, d\omega\right|^2}{\int_{-\infty}^{\infty} |H(\omega)|^2\, d\omega}$$

To maximize the SNR_o we can apply Schwarz's inequality principle, which states:

$$\frac{\left|\int_{-\infty}^{\infty} F_1(\omega)\,F_2(\omega)\, d\omega\right|^2}{\int_{-\infty}^{\infty} |F_1(\omega)|^2\, d\omega} \le \int_{-\infty}^{\infty} |F_2(\omega)|^2\, d\omega$$

and holds true only if

$$F_1(\omega) = kF_2^*(\omega)$$

where k is an arbitrary constant and $F_2^*(\omega)$ is complex conjugate of $F_2(\omega)$.

By substituting $F_1(\omega)$ by $H(\omega)$ and $F_2(\omega)$ by $X(\omega)\exp(j\omega t_m)$ we get,

$$\left|\int_{-\infty}^{\infty} H(\omega)X(\omega)e^{j\omega t_m}\,d\omega\right|^2 \leq \int_{-\infty}^{\infty}|H(\omega)|^2\,d\omega \int_{-\infty}^{\infty}|X(\omega)|^2 d\omega$$

$$\text{or}\quad \frac{1}{\pi n}\,\frac{\left|\int_{-\infty}^{\infty} H(\omega)X(\omega)e^{j\omega t_m}\right|^2}{\int_{-\infty}^{\infty}|H(\omega)|^2\,d\omega} \leq \frac{1}{\pi m}\int_{-\infty}^{\infty}|X(\omega)|^2 d\omega$$

$$\text{or,}\quad SNR_0 \leq \frac{1}{\pi n}\int_{-\infty}^{\infty}|X(\omega)|^2\,d\omega$$

The maximum value of the SNR would be:

$$SNR_0\Big|_{max} = \frac{1}{\pi n}\int_{-\infty}^{\infty}|X(\omega)|^2\,d\omega$$

The above expression remains valid only when the prerequisite condition of the schwarz's inequality remains valid, i.e.

$$H(\omega) = K * \{X(\omega)\,e^{j\omega t_m}\} = K * X^*(\omega)\,e^{-j\omega t_m}$$

The impulse response of the optimum corresponding to the transfer function $H(\omega)$ will be:

$$h(t) = FT^{-1}[H(\omega)] = FT^{-1}\left[k * X(-\omega)\, e^{-j\omega t_m}\right]$$

As the inverse FT of $X(-\omega)$ is $x(-t)$ and $e^{-j\omega t_m}$ gives time shift of t_m, the impulse response will be:

$$h(t) = k\, x(t_m - t)$$

Assuming $k = 1$ for conveniences, the impulse response of the optimum detector network (filter) is fair replica of in coming signal shifted by t_m.